Commitment of Electric Power Generators

under Stochastic Market Prices

Jorge Valenzuela [1]

Department of Industrial and Systems Engineering

Auburn University

211 Dunstan Hall

Auburn University, AL 36849-5346 USA

jvalenz@eng.auburn.edu

Mainak Mazumdar

Department of Industrial Engineering

University of Pittsburgh

1048 Benedum Hall

Pittsburgh, PA 15261 USA

mmazumd@pitt.edu

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[1] Corresponding author.

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Abstract

A formulation for the commitment of electric power generators under a deregulated electricity market is proposed. The problem is expressed as a stochastic optimization problem in which the expected profits are maximized while meeting demand and standard operating constraints. First, we show that when an electric power producer has the option of trading electricity at market prices, an optimal unit commitment schedule can be obtained by considering each unit separately. Therefore, we describe solution procedures for the self-commitment of one generating unit only. This description is given for three certainty-equivalent formulations of the stochastic problem. The procedures involve application of optimization methods, statistical analysis, and asymptotic probability computations. The optimization problem uses Schweppe's definition of hourly spot price to drive self-commitment decisions. Under the assumption of perfect market competition, the volatility of hourly spot price of electricity is represented by a stochastic model, which highlights its dependence on demand, generating unit reliabilities, and temperature fluctuations. The exact computations become very time-consuming for large systems; we therefore use several approximation methods (normal, Edgeworth series expansion, and Monte Carlo simulation) for computing the required probability distributions. Dynamic programming is used to solve the stochastic optimization problem. Numerical examples show that for a market consisting of 150 generating units, the self-commitment problem can be accurately solved in a reasonable time.

1. INTRODUCTION

Electric power consumption varies with time reflecting the predictably cyclical nature of human activities. The demand for electricity is higher during the day and early evening, weekdays and the summer or winter seasons as compared to the late night and early morning, weekends and fall or spring seasons. Also, electricity is a non-storable commodity and needs to be produced at the same rate at which it is consumed. In order to run the electric power generation system economically so as to reliably meet the demand, it is thus necessary to switch the generating units on and off at appropriate times. The generating units cannot however be turned on and off in a haphazard manner. Besides the start-up costs, one also needs to consider certain operating constraints that dictate how frequently and in what manner the units can be turned on and off. They are, for example: minimum up time, minimum down time, minimum capacity, maximum capacity, and ramping rate. The decision problem of optimally scheduling the operation of these machines is known as the unit commitment problem (UCP). [Wood and Wollenberg (1996)].

The UCP has both combinatorial and continuous non-linear optimization components, and the problem is usually very complex to solve because of the non-linear objective function and many variables and constraints. Much of the computational difficulty of this problem arises from the coupling constraint requiring that the total production from all the generating units under consideration equal electricity demand for every hour. The number of generating units and the number of hours in the time horizon roughly determine the problem size. Well-known mathematical programming techniques such as integer programming, dynamic programming, branch and bound, Benders decomposition,

and Lagrangian relaxation have been used to solve the UCP. For small problems, they can provide the optimal solution in a reasonable amount of time. However, for large problems, the computational time required to find the optimal solution becomes prohibitive. In such cases, the solution space is only partially searched and therefore there is no guarantee that the optimal solution can be found. Meta-heuristic methods such as simulated annealing, tabu search, and genetic algorithms have also been used for solving these large and highly complex problems. The interested reader can refer to Sheble and Fahd (1994) and Sen and Kothari (1998) for a synopsis of the literature on the topic of unit commitment.

Within a regulated framework, an electric utility serves the customers of a certain region under tariffs calculated to guarantee the recovery of its costs. In this situation, a power generating utility solves the UCP to obtain an optimal production schedule of its units to meet customer demand. The optimal schedule is found by minimizing the production cost over a given time interval while satisfying the demand and the set of operating constraints. The minimization of the production costs assures maximum profits because the power generating utility has no option but to reliably supply the prevailing demand. The price of electricity over this period is predetermined; therefore, the decisions on the operation of the individual units have no effect on the firm's revenues. Under deregulation [Galiana and Ilić (1998) and Federal Energy Regulatory Commission (1996)] the price of electricity is however no longer predetermined. The unit commitment decisions in this situation are based on the expected market price of electricity rather than on the demand although these variables are usually correlated. Thus far, the hourly market prices of electricity have shown evidence of being highly volatile [Citizens Power

(1997)]. The UCP now requires a stochastic formulation that includes a representation of the electricity market.

In a recently published monograph, Allen and Ilić (1999) have described the new formulation of the unit commitment problem that will be needed for the deregulated environment. They have attempted to obtain the stochastic characteristics of the market prices directly from available data on market-clearing prices, load, and covariates (mainly, temperature) that affect the prices. In other relevant literature, Takriti *et al.* (2000) have introduced a stochastic model for the UCP in which the uncertainty in the demand and prices of fuel and electricity are modeled using a set of possible scenarios. However, generating representative scenarios and assigning them appropriate probabilities remains a challenge. Tseng (2001) uses Ito processes to model the prices of electricity and fuel. These three approaches require observation of market prices and fitting a model that can describe their expected behavior. These models may require many years worth of data, which may not be available any time soon. Deregulated markets have been in operation for just a few years. Besides, under deregulation the cost structure of the supply side of an electricity market could change drastically. Generating units also have the potential to fail. The implication is that the set of generating units participating in a particular market at a given week or month can be quite different from those that participated in previous periods. To overcome these difficulties, we have taken a bottom up engineering economic approach to forecasting market prices based on process representations of electricity production and consumption. The main advantage of bottom up models is that they can be used to represent non-stationary systems (e.g., in which fuel prices differ from the past), and can more easily be used to consider scenarios

in which the system's structure has been changed (e.g., entry of additional generators). Our price model assumes that an electricity market for a region can be seen as one large electric power system. Under perfect competition, the hourly spot price is equivalent to the hourly marginal cost of the power generation system (Schweppe *et al*. 1988). We represent the generating units participating in the market via a production-costing model [Baleriaux *et al*. (1967) and Kahn (1995)] and deduce from it the statistical behavior of the hourly marginal cost of the system. Rather than fitting a statistical model to observed electricity prices, we determine the stochastic behavior of the prices via modeling two stochastic processes (supply and demand) underlying the hourly spot price of electricity. The production-costing model, which has been extensively used by the regulated utility industry for the purpose of forecasting costs of electricity production, uses the assumption that the generating units are dispatched in accordance with an economic merit order. That is, in order to meet the demand, the unit with the lowest marginal cost is loaded first, followed by the next unit with the lowest cost, and so on. Although this assumption neglects unit commitment constraints, it is generally believed that this approximation provides very good estimates. Additional justifications for the use of this model to represent market prices are given in section 2.2.

The main advantage of this bottom up modeling approach is that the stochastic processes corresponding to the supply side of the market use parameters that pertain to the individual generating units only. These parameters such as mean time to failure, mean time to repair, and capacity are insensitive to decisions made in an attempt to alter the price of electricity. Initial estimates of these parameters can remain valid during long periods and therefore no calibration with historical data is required. Similarly, initial

estimates of parameters of the stochastic process corresponding to the demand side of the market can also remain valid during a long period of time. The reason is that the demand for electricity is more susceptible to changes in the ambient temperature, hour of the day, and day of week rather than to changes in the market structure.

Assuming perfect competition and absence of market power, we equate the spot price of electricity for a given hour to the marginal cost of the last unit (marginal unit) used to meet the demand prevailing at this hour. Recent developments in California as well much of the literature on price models may cast doubt on this assumption. Thus, the price model depicted here should be regarded as a first approximation. Eventually, it will be necessary to introduce in our model the oligopolistic structure of the electricity market. However, when the supply situation is not too tight in relation to the demand and/or the number of generators participating in the market is not too small, this model should provide quite accurate results [Rudkevich *et al*. (1998)]. We determine the probability distribution function of the hourly spot price based on the stochastic process governing the marginal unit. This in its turn depends on the demand and the alternating renewal processes describing the availabilities of the generating units participating in the market. In our stochastic model of the marginal unit, we make the same assumptions as used in the probabilistic production-costing model proposed by Ryan and Mazumdar (1990) which was adopted by Lee, Lin and Breipohl (1990) for the purpose of computing the variance of production costs over a given time interval. This model represents a stochastic process based enhancement of the original Baleriaux model of production costing. It assumes that the operating state of each generating unit participating in the market follows a two-state continuous-time Markov chain. The two states are "available"

and "unavailable". The assumption that times to failure and repair are exponentially distributed is commonly used to model the reliability of a component in the power systems literature [Billinton and Allan (1992)]. The information on the mean time to repair, mean time to failure, capacity, and marginal cost of each unit required to characterize these processes is assumed available. The hourly demand is represented by a Gauss-Markov process [Breipohl *et al*. (1992)]. Valenzuela and Mazumdar (2000) have recently reported on the statistical analysis of hourly load data covering a region of the Eastern United States. They have shown that when the effect of temperature is suitably subtracted from the hourly load, such a process can represent the demand quite accurately.

We also propose a formulation for the unit commitment problem for an electric power producer, which owns a number of generating units in a deregulated market and consider computationally efficient procedures to solve it. We express the decision problem as a stochastic optimization problem in which the decisions on the operation of the individual generators do not affect the price of electricity (lack of market power). The objective is to maximize expected profits, and the decisions are required to meet standard operating constraints. We first show in section 2 that when the market price of electricity is considered exogenous to the unit commitment decisions and the demand constraints are the only coupling constraints, the optimization problem (for a generation company acting as a price-taker in a PoolCo-type electricity market) decomposes in a straightforward manner into as many sub-problems as the number of generating units owned by the company. Therefore, the optimal solution of a UCP with M units can be found by solving M uncoupled sub-problems. In the recent literature, it has been tacitly assumed that the

UCP under deregulation can be solved by considering one unit at a time [Allen and Ilić (1999), Rajaraman *et al*. (2000)]. Here, we furnish a formal proof of this fact that highlights the implicit conditions under which this assumption holds. Much of the computational difficulty of the standard UCP arises from the coupling constraint requiring the total production from *M* units to equal electricity demand for every hour. The feature of decomposability into sub-problems for the deregulated market considerably reduces the computational burden. This section gives dynamic programming solutions for three different models representing the UCP.

Our model differs from other stochastic formulations of the unit commitment problem with regard to the load and operating constraints, the manner in which the spot market is modeled, and the solution technique. The model proposed by Carpentier *et al.* (1996) does not include start-up costs and uses the Augmented Lagrangian Relaxation technique to decompose the problem. Although, they do not include the feature of buying from or selling energy to a spot market in their model to correct energy imbalances, they add a penalty term associated with the discrepancy between generation and demand. In their model, the penalty represents the cost of bringing on-line expensive generation. Another stochastic model has recently been proposed by Takriti and *et al.* (2000). Their model includes fuel constraints in addition to the operating constraints considered in our formulation. The spot market is modeled as two generating units. One unit represents the transaction of buying from the pool and the other unit selling to the pool. They have assumed that the cost functions of these two units are linear with different cost coefficients, e.g. they assume that the price at which energy is bought is different at which it is sold. Unlike our model, load constraints are here set as inequalities (total

power produced greater than or equal to the demand). They have used Lagrangian relaxation and Bender's decomposition to solve the optimization problem.

Section 3 describes the stochastic model based on which the probability distribution for the marginal unit at different hours is obtained. This leads to the derivation of the probability distribution for the hourly spot price. The model incorporates information on the demand as well as how the prevailing temperature influences it. The dynamic programming algorithm that we use to solve the unit commitment problem requires the computation of the bivariate probability function of the marginal units at two different hours. This computation is however not easy. The difficulty arises from the need to account for the stochastic behavior of the demand for electricity and the 2^N possible availability states of a market comprising N generating units. Experimental results for a power generating system of 15 units showed that the exact computation of the bivariate probability function is prohibitively time-consuming. Therefore, we evaluate several approximation methods including the normal, cumulant based approximation (Edgeworth or Gram-Charlier expansions), large deviation, and Monte Carlo approximation methods. These approximations have been used in a univariate context with much success in power-system reliability and production costing computations [Stremel *et al.* (1980), Caramanis *et al.* (1983), Rau *et al.* (1980) and Mazumdar (1988)] in which similar difficulties arise. Iyengar and Mazumdar (1998) have provided bivariate generalizations of these formulas and used them for approximate evaluation of composite power system reliability. The application of the method of cumulants requires computation of the cumulants of several orders for the appropriate distributions. To evaluate the large deviation approximations, the solution of a non-linear system of equations becomes

necessary. Section 4 describes the numerical example used to describe the market. In section 5, we compare the computational accuracy and efficiency of the different approximation schemes that were examined.

2. A STOCHASTIC FORMULATION OF THE UNIT COMMITMENT

PROBLEM FOR A POWER PRODUCER UNDER DEREGULATION

We consider an electric power producer with M generating units and assume that the producer wishes to schedule its units to maximize profit over a short time period of length T hours. We assume that the producer obtains its revenues by selling power at pre-negotiated prices based on long-term bilateral contracts and selling electric power at market prices to the power pool that serves as a market. That is, if at a particular hour the power supplier produces electricity with its generating units, it is then willing to take the price prevailing in the market at this hour (price-taker). We also assume lack of market power, e.g. the power supplier's decisions do not affect the market prices. The M generating units are assumed to remain available during the time interval of interest. Recently other formulations have been proposed for the unit commitment problem for the new deregulated environment, which make somewhat different assumptions. For example, Baillo *et al.* (2000) have proposed a model that explicitly considers the ability of the generating company to affect prices with its decisions on the amount of its power output.

2.1. Modeling the Operation of Generating Units

In determining an optimal commitment schedule, there are two decision variables, $P_{k,t}$ and $v_{k,t}$. $P_{k,t}$ denotes the amount of power to be generated by unit k at time t, and $v_{k,t}$ is a control variable whose value is chosen to be "1" if the generating unit k is committed at hour t and "0" otherwise. (Of course, if $v_{k,t} = 0$, then $P_{k,t} = 0$). The cost of the power produced by the generating unit k depends on the amount of fuel consumed and is

typically approximated by a quadratic cost function $CF_k(p)$ [Wood and Wollenberg (1996)]:

$$CF_k(p) = a_{0,k} + a_{1,k}p + a_{2,k}p^2 , \tag{1}$$

where p is the amount of power generated. The start-up cost at hour t, which for thermal units depends on the temperature of the boilers, is given by a known function $S_{k,t}(x_{k,t-1})$. The value of $x_{k,t-1}$ specifies the consecutive time intervals during which the unit has been on (+) or off (-) at the end of the hour $t-1$. For example, $x_{1,7}$ equaling +4 indicates that at the end of hour 7 the generating unit number 1 has been up for 4 hours. In addition, a generating unit must satisfy certain operating constraints. The power produced by a generating unit must be within certain limits. When the kth-generating unit is running, it must produce an amount of power between P_k^{min} and P_k^{max} (MW). If the generating unit is off, it must stay off for at least t_k^{dn} hours, and if it is on, it must stay on for at least t_k^{up} hours.

2.2. Decomposition into Sub-problems

The objective function is total profit, revenue minus cost, over the interval $[1,T]$. The revenue during hour t is obtained from supplying the quantity stipulated under the long-term bilateral contracts and by selling surplus energy (if any) to the power pool at the market price, m_t ($/MWh). As we have mentioned in section 1, m_t is a stochastic process and therefore the total profit over the interval $[1,T]$ is a random variable. The cost includes those of producing the energy, buying shortfalls (if needed) from the power pool, and the startup costs. Defining the amount to be sold under the bilateral contract by l_t (MWh), the contract price by R ($/MWh) and the amount of energy bought or sold from

the market by E_t, we solve the stochastic optimization problem by maximizing the expected profit over the period $[1,T]$. (A positive value of E_t indicates that E_t (MWh) is bought from the power pool and a negative value indicates that $-E_t$ (MWh) is sold to the pool.) The objective function (maximum expected total profit) is given by:

$$\underset{v_{k,t},P_{k,t},E_t}{\text{Max}} \; \text{E}\left\{\sum_{t=1}^{T} \{l_t R - m_t E_t - \sum_{k=1}^{M}[\text{CF}_k(P_{k,t}) + S_{k,t}(x_{k,t-1})(1-v_{k,t-1})]v_{k,t}\}\right\}. \tag{2}$$

Here, we are making the assumption that the process of buying from or selling to a power pool in response to a price signal does not require a positive lead-time. This assumption is supported by the example of several markets that have evolved during recent years in which imbalances are adjusted on a real time basis. For example, the Independent System Operator of the California market (Cal-ISO) has established a real-time market, in which imbalances can be corrected on ten minutes notices based on spot prices. At any given time, Cal-ISO dispatchers adjust generation to match California's demand. Market traders receive either payments (at spot prices) for extra generation they supply or are billed (at spot prices) for the extra energy they need to meet the demand of their customers. Other markets such as the Pennsylvania-New-Jersey-Maryland (PJM) and that of England and Wales also provide mechanisms for near real-time clearing and settlement of the imbalances between the contractual and physical positions.

Since the quantity $l_t R$ is a constant, the optimization problem reduces to:

$$\underset{v_{k,t},P_{k,t},E_t}{\text{Max}} \; \text{E}\left\{\sum_{t=1}^{T} \{-m_t E_t - \sum_{k=1}^{M}[\text{CF}_k(P_{k,t}) + S_{k,t}(x_{k,t-1})(1-v_{k,t-1})]v_{k,t}\}\right\} \tag{3}$$

subject to the following constraints   (for $t=1,\ldots,T$ and $k=1,\ldots,M$)

Load:
$$E_t + \sum_{k=1}^{M} P_{k,t} = l_t \tag{4}$$

Capacity limits:
$$v_{k,t} P_k^{\min} \le P_{k,t} \le v_{k,t} P_k^{\max} \tag{5}$$

Minimum down time: $v_{k,t} \le 1 - \mathrm{I}(-t_k^{dn} + 1 \le x_{k,t-1} \le -1) \tag{6}$

Minimum up time: $v_{k,t} \ge \mathrm{I}(1 \le x_{k,t-1} \le t_k^{up} - 1) \tag{7}$

where $\mathrm{I}(x) = \begin{cases} 0 & \text{if } x \text{ is false} \\ 1 & \text{if } x \text{ is true} \end{cases}$, $P_{k,t} \ge 0$, E_t unrestricted in sign, and $v_{k,t} \in \{0,1\}$.

The quantity $x_{k,t}$ must satisfy the state equation:

$$x_{k,t} = \begin{cases} \mathrm{Min}\{t_k^{up}, x_{k,t-1} + 1\} & \text{if } x_{k,t-1} > 0 \text{ and } v_{k,t} = 1 \\ +1 & \text{if } x_{k,t-1} < 0 \text{ and } v_{k,t} = 1 \\ \mathrm{Max}\{-\mathrm{Max}\{t_k^{dn}, t_k^{cold}\}, x_{k,t-1} - 1\} & \text{if } x_{k,t-1} < 0 \text{ and } v_{k,t} = 0 \\ -1 & \text{if } x_{k,t-1} > 0 \text{ and } v_{k,t} = 0 \end{cases} \tag{8}$$

and the equality:
$$v_{k,t} = \begin{cases} 1 & \text{if } x_{k,t} > 0 \\ 0 & \text{if } x_{k,t} < 0 \end{cases} \tag{9}$$

The parameter t_k^{cold} is the number of hours that it takes for the boiler to cool down. Additional constraints such as ramp constraints that account for the fact that units cannot change their output too rapidly can be easily added to this model. For example, ramp constraints can be written as: $P_{k,t} \le P_{k,t-1} + \Delta_k^{inc}$ and $P_{k,t} \ge P_{k,t-1} - \Delta_k^{dec}$, where Δ_k^{inc} and Δ_k^{dec} are the maximum ramp rates (in MW/h) for increasing and decreasing energy output, respectively.

After substituting in the objective function, $E_t = l_t - \sum_{k=1}^{M} P_{k,t}$, obtained from Equation 4,

we can rewrite Equation 3 as follows:

$$\underset{v_{k,t},P_{k,t}}{\text{Max}} E\left\{\sum_{t=1}^{T}\left\{-m_t[l_t - \sum_{k=1}^{M} P_{k,t}] - \sum_{k=1}^{M}[\text{CF}_k(P_{k,t}) + S_{k,t}(x_{k,t-1})(1-v_{k,t-1})]v_{k,t}\right\}\right\}, \quad (10)$$

which after removing the constant terms is equivalent to:

$$\underset{v_{k,t},P_{k,t}}{\text{Max}} E\left\{\sum_{t=1}^{T}\sum_{k=1}^{M} m_t P_{k,t} - [\text{CF}_k(P_{k,t}) + S_{k,t}(x_{k,t-1})(1-v_{k,t-1})]v_{k,t}\right\}, \quad (11)$$

subject to the operating constraints (5), (6), and (7). Because these constraints refer to individual units only, Equation 11 shows that the optimization problem is now separable by individual units. The optimal solution can be found by solving M de-coupled sub-problems. Thus, the sub-problem for the kth unit ($k=1,..,M$) is:

$$\underset{v_{k,t},P_{k,t}}{\text{Max}} E\left\{\sum_{t=1}^{T} m_t P_{k,t} - [\text{CF}_k(P_{k,t}) + S_{k,t}(x_{k,t-1})(1-v_{k,t-1})]v_{k,t}\right\}, \quad (12)$$

subject to operating constraints of the kth unit. Equation 12 is similar to the sub-problem obtained in the standard version of the UCP [Bard (1988)] using the Lagrangian relaxation method, except that the values of the Lagrange multipliers are now replaced by the market price of electricity and the expected value is being maximized.

2.3. Spot Market Model under Perfect Competition

The basic theory of spot market pricing of electricity, under perfect competition, was developed by Schweppe *et al.* (1988). They stated that the hourly spot price could be viewed as the sum of individual components accounting for: (a) fuel and maintenance

costs, (b) quality of supply, which is the amount of price raised in order to ration limited supply since otherwise demand would excess supply, and (c) costs to compensate for transmission losses and capacity limitations. If transmission and distribution network costs are ignored, a mathematical expression for the optimal spot price at time t is:

$$\rho_t = \frac{\partial G_{FM}[u(t)]}{\partial u(t)} + \frac{\partial G_{QS}[u(t)]}{\partial u(t)} \tag{13}$$

$G_{FM}(.)$ is the total fuel and maintenance cost of generation and $G_{QS}(.)$ is the generation quality of supply costs incurred to provide reliable energy to customers. The quantity $u(t)$ is the load at time t. The two terms in the right-hand side of Equation 13 are the dominant costs. When generation capacity limits are being approached, the cost of quality of supply becomes predominant. In the following formulation, we assume that the supply is not too tight in relation to the demand, and the spot price is obtainable from the first term of Schweppe's equation. To determine the probability distribution of the hourly spot price, we use the Ryan-Mazumdar production-costing model and Schweppe's definition in Equation 13. Under these conditions, the spot price at a specific hour t, can be taken to be equal to the marginal cost ($/MWh) of the last unit used to meet the demand prevailing at this hour. Support for this conclusion is also obtained from Rudkevich *et al.* (1998) who have shown from a Nash equilibrium standpoint that when the number of units participating in the market is not too small the market-clearing price can be approximated by the marginal cost of this last unit. The Ryan-Mazumdar production-costing model assumes that the generating units are dispatched in accordance with an economic merit order determined by their marginal costs. The last unit, in the loading order, used to meet the demand is the marginal unit and is represented by the index $J(t)$. For instance, if we assume that the market consists of the eight units listed in Table 1 and the demand is 950

MW, the marginal unit would be the 4th unit ($J(t)$ is equal to 4). The model also assumes that the marginal operating cost of each unit is a constant quantity denoted by d_i ($/MWh), marginal cost of the ith unit in the loading order.

The model we use to forecast prices (fixed loading order, constant marginal cost) may appear to be inconsistent with how we solve the single generator's unit commitment problem (integer programming problem with quadratic cost function.) The reason for this is that constructing a market model considering every market participant's optimal 0-1 decisions under uncertainty is not practical, so we have developed a continuous, computable approximation. If we were to develop a market model that explicitly represented integer decisions, then we would have to face the very real problem of duality gap. Under these circumstances, the market model could not guarantee that it could find a set of market clearing prices.

When the optimization sub-problem is solved for a particular unit, we assume that the market consists of N generating units (N will be much larger than M). The generating unit for which the sub-problem is solved is excluded from the market. Excluding a unit from the market does not influence the spot price because of the existence in all likelihood of a number of generating units with almost equal marginal costs, ready to produce if any of the infra-marginal (or marginal) units is unavailable. Since the objective functions from the optimization sub-problems (Equation 12) have a similar structure for each value of k ($k=1,..,M$), we will drop the sub-index k from the variables $P_{k,t}$, $x_{k,t}$, and $v_{k,t}$. If the ith unit has a marginal cost d_i ($/MWh), the market price, m_t, is equal to $d_{J(t)}$. The quantities $J(t)$ and $d_{J(t)}$ are random variables that depend on the prevailing demand and the operating states of the generating units. We re-write Equation 12 for one generating unit as follows:

$$\underset{v_t,P_t}{\text{Max}} \ \text{E}\left\{\sum_{t=1}^{T} P_t d_{J(t)} - [CF(P_t) + S_t(x_{t-1})(1 - v_{t-1})]v_t\right\}, \tag{14}$$

where we have replaced m_t by $d_{J(t)}$ and removed the index k. Equation 14 is also subject to the operating constraints: capacity limits, minimum up time, and minimum down time of this generating unit.

In the next subsections, we describe three certainty-equivalent models for this stochastic optimization problem. Model 0 is a naive model that assumes that both commitment decisions (v_t) and operating decisions (P_t) are made at the start of the day, and decisions are not adjusted in response to price conditions. Model 1 assumes that commitment decisions (v_t) are made ahead of time and adhered to for the entire day, but operating decisions (P_t) can be adjusted in real time as prices are realized. Model 2 assumes that both v_t and P_t can be adjusted in real time in response to price movements. Model 1 is how most units operate; however, these three models permit the evaluation of the option value of the asset as we move from model 0 to model 1 to model 2. The expected profit should be higher as we move from model 0 to 1 to 2. Model 0 is the least computation intensive while model 2 is the most. Although we believe that model 0 should not be used in practice, we retain it to ascertain how its results compare with those of models 1 and 2.

At the time of determining the commitment schedule for the next T hours, hour zero, the demand for electricity at previous hours is assumed known. Additionally, we assume that the last unit used to meet the demand at hour zero is also known and it is denoted by j_0. Since the marginal unit at time t is correlated with the marginal unit at any other time s, knowing the index of the marginal unit at time zero reduces the variance of the

marginal unit at future hours. If the marginal unit at hour zero is not known, then we use simple expectation instead of conditional expectation.

2.4. Model 0: Expected-Value Solution

We first provide the expected value solution of Equation 14, in which the random variable $d_{J(t)}$ is replaced by its expected value given that j_0 is known. The objective function is:

$$\text{Max } E[\text{profit}\,|\,j_0]=\underset{v_t,P_t}{\text{Max}}\sum_{t=1}^{T}P_t E[d_{J(t)}\,|\,j_0]-[\text{CF}(P_t)+S_t(x_{t-1})(1-v_{t-1})]v_t \qquad (15)$$

This equation is subject to the operating constraints (5), (6) and (7). We solve this maximization problem using dynamic programming. We define the function $r_t(v_t)$ by the following equation:

$$r_t(v_t)=\underset{P_t}{\text{Max}}\{[P_t\sum_{j=1}^{N}d_j\,\Pr(J(t)=j\,|\,J(0)=j_0)-\text{CF}(P_t)]v_t\}\qquad t=1,\ldots,T \qquad (16)$$

This function denotes the maximum profit at hour t excluding start-up costs given the expected value of the spot price at this hour. The value of v_t denotes whether the unit is "on" or "off" at hour t. Notice that v_t is determined by x_t. We also define the recursive function $F_t(x_{t-1})$ to be the optimum expected profit from hour t to hour T of operating the generator that is in state x_{t-1} at the end of hour t-1. Thus, the expression for hour t is:

$$F_t(x_{t-1})=\underset{v_t}{\text{Max}}\{r_t(v_t)-v_t(1-v_{t-1})S_t(x_{t-1})+F_{t+1}(x_t)\} \qquad (17)$$

subject to the operating constraints. Since both the revenues and costs of decisions taken beyond the scope of the model are irrelevant, we set the expected incoming profit at time T+1 to be zero ($F_{T+1}(x_{T+1})$=0).

2.5 Model 1

Here, we assume that the owner of the generating unit makes unit commitment decisions (v_t) ahead of time for the entire period $[1,T]$ and adheres to them, but operating decisions (P_t) are adjusted in real time as prices are realized. The objective function is:

$$\text{Max E[profit}|j_0]=\underset{v_t,P_t}{\text{Max}}\sum_{t=1}^{T}\{E[P_t d_{J(t)}-CF(P_t)|j_0]-S_t(x_{t-1})(1-v_{t-1})\}v_t \qquad (18)$$

Here, we also use dynamic programming to obtain the optimal solution. We define the function $g_t(v_t,j)$ by the following equation:

$$g_t(v_t,j)=\underset{P_t}{\text{Max}}\{[P_t d_j-CF(P_t)]v_t\} \qquad\qquad t=1,\ldots,T \qquad (19)$$

This function denotes the maximum profit at hour t excluding start-up costs given that the jth unit is determining the spot price. For a quadratic fuel cost function, the value of $g_t(v_t,j)$ is:

$$g_t(v_t,j)=[P_t^*(d_j-a_1)-a_0-a_2 P_t^{*2}]v_t , \qquad (20)$$

where $P_t^* = \text{Min}\{\text{Max}(\dfrac{d_j-a_1}{2a_2},P^{\min}),P^{\max}\}$, a_0, a_1, and a_2 are the coefficients of the fuel cost function (see Equation 1). Notice that P_t^* depends on the value of d_j, which is a random variable. In this model, the expression for $F_t(x_{t-1})$ is:

$$F_t(x_{t-1})=\underset{v_t}{\text{Max}}\{[\sum_{j=1}^{N}\Pr[J(t)=j\mid J(0)=j_0]g_t(v_t,j)]-v_t(1-v_{t-1})S_t(x_{t-1})+F_{t+1}(x_t)\} \quad (21)$$

The optimal commitment schedule is given by the solution of $F_1(x_0)$. Equation 21 is subject to the operating constraints (5), (6), and (7), which do not include ramp rate limits. Ramp rate constraints can be particularly important under this model for large units. Ramp rate constraints can be accommodated by adding the ramp rate constraints

and considering the variable P_t as a state variable. The profit to go function would be in this case $F_t(P_{t-1}, x_{t-1})$, a function of P_{t-1} and x_{t-1}.

Notice that to solve models 0 and 1, the following conditional probabilities need to be computed.

$$\Pr[J(t) = j | J(0) = j_0] = \frac{\Pr[J(t) = j \text{ and } J(0) = j_0]}{\Pr[J(0) = j_0]} \tag{22}$$

Thus, the joint probability distribution of $J(0)$ and $J(t)$, and the marginal probability distribution of $J(0)$ are needed. The marginal probability distribution of $J(0)$ can be computed from the bivariate distribution using the following equation:

$$\Pr[J(0) = j_0] = \sum_{j=1}^{N} \Pr[J(t) = j \text{ and } J(0) = j_0] \tag{23}$$

2.6 Model 2

Model 2 assumes that both v_t and P_t can be adjusted in real time in response to price movements. Unlike model 1, the power producer here determines, at each hour, whether to self-commit its generating unit the next hour by maximizing the future expected total profit over a period of length T. Thus, the objective function is:

$$\text{Max E[profit} | j_0] = \underset{v_t, P_t}{\text{Max}} \left\{ \sum_{t=1}^{T} E_{J(t)|J(t-1)} \{P_t d_{J(t)} - [CF(P_t) + S_t(x_{t-1})(1 - v_{t-1})]v_t\} \right\} \tag{24}$$

This equation is subject to the operating constraints. We define the recursive function $F_t(j_{t-1}, x_{t-1})$ to be the optimum total expected incoming profit from hour t to hour T of operating the generator that is in state x_{t-1} when the marginal unit at time $t-1$ is j_{t-1}. Thus, the expression for hour t is:

$$F_t(j_{t-1}, x_{t-1}) = \text{Max}_{v_t} \{ \sum_{j_t=1}^{N} \{ g_t(v_t, j_t) + F_{t+1}(j_t, x_t) \} \Pr[J(t) = j_t \mid J(t-1) = j_{t-1}] - v_t(1 - v_{t-1})S_t(x_{t-1}) \}$$

(25)

As in the previous model we set $F_{T+1}(j_{T+1}, x_{T+1}) = 0$. The value of $g_t(v_t, j_t)$ is obtained as

before from equation (20). The optimal commitment policy is given by the solution of

$F_1(j_0, x_0)$. To solve model 2, we need to compute the following conditional probabilities.

$$\Pr[J(t+1) = j \mid J(t) = i] = \frac{\Pr[J(t+1) = j \text{ and } J(t) = i]}{\Pr[J(t) = i]}$$

(26)

Thus, the joint probability distribution of $J(t)$ and $J(t+1)$, and the marginal probability

distribution of $J(t)$ are needed.

3. STOCHASTIC MODEL FOR THE MARGINAL UNIT

In the stochastic model for the marginal unit we use the production-costing model proposed by Ryan and Mazumdar (1990) which itself is based on the model proposed by Baleriaux *et al*. (1967). This model has been used in estimating the mean and variance of production cost [Mazumdar and Kapoor (1995), Ryan and Mazumdar (1990), Lee, Lin and Breipohl (1990), Shih, Mazumdar, and Bloom (1999)] and the marginal cost [Shih and Mazumdar (1998)] of a power generating system.

3.1. Stochastic Model of the Market

For a market with N generating units, the model uses the following assumptions:

1. The generators are dispatched each hour in a fixed, merit order of loading based on marginal costs, which depends only on the demand and the availability of the generating units. Thus the unit with the lowest marginal cost is loaded first followed by the next most inexpensive unit, and so on. Unlike the unit that is being considered for being committed we do not take into account the operating histories of the generating units participating in the market.

2. The ith unit in the loading order has a capacity c_i (MW) and marginal cost d_i ($/MWh).

3. After adjusting for the variations in the ambient temperature and periodicity, the demand at time t, $u(t)$, is assumed to follow a Gauss-Markov process [Breipohl *et al*. (1992) and Valenzuela *et al*. (2000)] with u(t) and u(r) following a bivariate normal distribution for any pair (r,t) with $E[u(t)]=\theta_t$ and $Cov[u(r), u(t)]=\sigma_{r,t}$, where θ_t and $\sigma_{r,t}$ are assumed to be known. (Data analysis given in Valenzuela and Mazumdar (2000) validates this assumption.)

4. The operating state of each generating unit i follows a two-state continuous-time Markov chain in the steady state, $Y_i(t) \in \{0,1\}$, with failure rate λ_i and repair rate μ_i. The steady state unavailability index or the forced outage rate q_i is related to these quantities by the equation $q_i = \lambda_i /(\lambda_i + \mu_i)$. We define $p_i = 1 - q_i$.

5. For $i \neq j$, $Y_i(r)$ and $Y_j(t)$ are probabilistically independent for all values of r and t. Each $Y_i(t)$ is independent of $u(t)$ for all values of t.

3.2. Bivariate Probability Distribution of the Marginal Unit

To derive an analytical expression for the bivariate probability mass function of the marginal units $J(r)$ and $J(t)$, we first note that the events $\{ u(t) - \sum_{i=1}^{n} c_i Y_i(t) > 0 \}$ and $\{ J(t) > n \}$ are self-evidently the same, given the definition of loading order, for all values of t. The bivariate probability mass function of the marginal units $J(r)$ and $J(t)$, can be obtained as follows:

$$\begin{aligned}
\Pr[J(r) = m, J(t) = n] = & \Pr[J(r) > m-1, J(t) > n-1] \\
& -\Pr[J(r) > m, J(t) > n-1] \\
& -\Pr[J(r) > m-1, J(t) > n] \\
& +\Pr[J(r) > m, J(t) > n].
\end{aligned} \tag{27}$$

Since the events $\{ u(r) - \sum_{i=1}^{m} c_i Y_i(r) > 0 \text{ and } u(t) - \sum_{i=1}^{n} c_i Y_i(t) > 0 \}$ and

$\{ J(r) > m \text{ and } J(t) > n \}$ are equivalent, the following equality is obtained:

$$\Pr[J(r) > m \text{ and } J(t) > n] = \Pr[u(r) - \sum_{i=1}^{m} c_i Y_i(r) > 0 \text{ and } u(t) - \sum_{i=1}^{n} c_i Y_i(t) > 0] \tag{28}$$

Therefore, to compute the bivariate probability mass function of $J(r)$ and $J(t)$, we need to

compute the probability that $\{ u(r) - \sum_{i=1}^{m} c_i Y_i(r) > 0$ and $u(t) - \sum_{i=1}^{n} c_i Y_i(t) > 0 \}$, which we

denote by $p_{mn}(r, t)$. For models 0 and 1, we need to evaluate just $p_{mn}(0, t)$, while for

model 2 we need $p_{mn}(r, r+1)$ $(1 \leq m, n \leq N, 0 \leq r \leq T\text{-}1,$ and $1 \leq t \leq T)$.

3.3. Exact Computation of $p_{mn}(r, t)$

The exact value of $p_{mn}(r, t)$ can be computed using conditional probabilities. The

probability is conditioned by the values of $Y_i(r)$, $i=1,..,m$ and $Y_j(t)$, $j=1,..,n$. In order that

we may use steady-state formulations, we assume that the firm knows the index of the

marginal unit at time zero only but it does not know the state of the individual generating

units, $Y_i(t)$. First we define the vectors $Y_r=[Y_1(r), Y_2(r),. ,Y_m(r)]$ and $Y_t=[Y_1(t), Y_2(t),....$

$,Y_n(t)]$. We also define an outcome of Y_r by y_r and an outcome of Y_t by y_t, where y_r

$=[y_1(r), y_2(r),....,y_m(r)]$ and $y_t =[y_1(t), y_2(t),.... ,y_n(t)]$. Denote the sample space of $[Y_r, Y_t]$

by $\Omega = \Omega_r \times \Omega_t$. Notice that there are 2^{m+n} individual outcomes in the sample space Ω.

Defining

$$X_m(r) = u(r) - \sum_{i=1}^{m} c_i y_i(r) \quad \text{and} \quad X_n(t) = u(t) - \sum_{j=1}^{n} c_j y_j(t) , \tag{29}$$

where $y_i(r)$ and $y_j(t)$ are fixed, we obtain the expression for $p_{mn}(r, t)$ to be:

$$p_{mn}(r, t) = \sum_{y_r \in \Omega_r} \sum_{y_t \in \Omega_t} \Phi_2 \left(z_m(r), z_n(t), \rho_{mn}(r,t) \right) \times \Pr[\mathbf{Y}_r = \mathbf{y}_r, \mathbf{Y}_t = \mathbf{y}_t] \tag{30}$$

Here, $\Phi_2(x,y,\rho)$ is the cumulative distribution function of the bivariate normal

distribution with zero means, unit variances, correlation coefficient ρ, and

$$z_m(r) = \frac{1}{\sigma_r}\left\{\theta_r - \sum_{i=1}^{m} c_i y_i(r)\right\} \tag{31}$$

$$z_n(t) = \frac{1}{\sigma_t}\left\{\theta_t - \sum_{j=1}^{n} c_j y_j(t)\right\} \tag{32}$$

$$\rho_{mn}(r,t) = \frac{\sigma_{r,t}}{\sigma_r \sigma_t} \tag{33}$$

The probability $\Pr[\mathbf{Y_r}{=}\mathbf{y_r}, \mathbf{Y_t}{=}\mathbf{y_t}]$ can be computed from the following equation:

$$\Pr[\mathbf{Y}_r = \mathbf{y}_r, \mathbf{Y}_t = \mathbf{y}_t] = \left[\prod_{i=1}^{\min(m,n)} P_{y_i(r),y_i(t)}\right] \times \left[\prod_{i=\min(m,n)+1}^{\max(m,n)} \Pr[Y_i(w) = y_i(w)]\right], \tag{34}$$

where if $m \leq n$, the value of w is equal to t; otherwise, w is equal to r. The expression $P_{y_i(r),y_i(t)}$ is the joint probability of the random variables $y_i(r)$ and $y_i(t)$ of the generating unit i. It is given by Ross (2000), chapter 6:

$$P_{y_i(r),y_i(t)} = \begin{cases} q_i(q_i + p_i e^{-\delta_i|t-r|}) & \text{if } y_i(r) = 0 \text{ and } y_i(t) = 0 \\ p_i q_i(1 - e^{-\delta_i|t-r|}) & \text{if } y_i(r) = 0 \text{ and } y_i(t) = 1 \\ p_i q_i(1 - e^{-\delta_i|t-r|}) & \text{if } y_i(r) = 1 \text{ and } y_i(t) = 0 \\ p_i(p_i + q_i e^{-\delta_i|t-r|}) & \text{if } y_i(r) = 1 \text{ and } y_i(t) = 1 \end{cases}, \tag{35}$$

where $\delta_i = \lambda_i + \mu_i$.

To explain Equation 34, we assume $n < m$. The random vectors $\mathbf{Y_r}$ and $\mathbf{Y_t}$ denote, respectively, the state of the first n and m generating units in the loading order. In Equation 34, the first term calculates the joint probability of the states of the first n common generating units. The second term calculates the probability of the states of the remaining generating units. The multiplication of these probabilities follows from the

assumption of independence among generating unit states. It is obvious that the computation of the exact value of the probabilities in Equation 30 will be extremely time-consuming for large N. Three approximation methods are next examined: normal, large deviation, and Edgeworth approximations.

3.4. Normal Approximation of $p_{mn}(r,t)$

Using the central limit theorem, the joint distribution of $X_m(r)$ and $X_n(t)$ (unlike Equation 29, $y_i(r)$ and $y_j(t)$ are random variables here) can be approximated by a bivariate normal distribution. Thus,

$$p_{mn}(r,t) \cong \int\limits_{a_m(r)}^{\infty} \int\limits_{a_n(t)}^{\infty} \phi_2(z_1, z_2; \rho_{mn}(r,t)) dz_1 dz_2 \tag{36}$$

where

$$a_j(s) = -E[X_j(s)] / \sqrt{V[X_j(s)]} \tag{37}$$

$$E[X_j(s)] = \theta_s - \sum_{i=1}^{j} c_i p_i \tag{38}$$

$$V[X_j(s)] = \sigma_s^2 + \sum_{i=1}^{j} c_i^2 p_i q_i \tag{39}$$

$$\rho_{mn}(r,t) = Cov[X_m(r), X_n(t)] / \sqrt{V[X_m(r)] \times V[X_n(t)]} \tag{40}$$

$$Cov[X_m(r), X_n(t)] = \sigma_{r,t} + \sum_{i=1}^{\min(m,n)} c_i^2 p_i q_i e^{-\delta_i |t-r|} \tag{41}$$

3.5. Edgeworth (Gram-Charlier) Approximation of $p_{mn}(r,t)$

The normal approximation may not be very accurate when computing the tail probabilities. Moreover, for small values of j, the central limit theorem may be inaccurate. An alternative approach is to make small corrections to the normal approximation by using the Edgeworth expansion. The use of the Edgeworth expansion in evaluating power generating system reliability indexes is known as the method of cumulants [Stremel et $al.$ (1980) and Rau et $al.$ (1980)]. The approximate expression for $p_{mn}(r,t)$ using the bivariate Edgeworth expansion for the joint distribution function of $X_m(r)$ and $X_n(t)$ in which only the terms involving cumulants up to the third order is retained is given by:

$$p_{mn}(r,t) \cong \int\limits_{a_m(r)}^{\infty} \int\limits_{a_n(t)}^{\infty} \phi_2[z_1,z_2;\rho_{mn}(r,t)] \times \left\{ 1 + \frac{1}{6}\frac{K_{30}}{K_{20}^{3/2}} H_{30}(z_1,z_2;\rho_{mn}(r,t)) \right.$$
$$+ \frac{1}{2}\frac{K_{21}}{K_{20}K_{02}^{1/2}} H_{21}(z_1,z_2;\rho_{mn}(r,t)) + \frac{1}{2}\frac{K_{12}}{K_{20}^{1/2}K_{02}} H_{12}(z_1,z_2;\rho_{mn}(r,t)) \qquad (42)$$
$$\left. + \frac{1}{6}\frac{K_{03}}{K_{02}^{1/2}} H_{30}(z_1,z_2;\rho_{mn}(r,t)) \right\} dz_1 dz_2$$

Here, K_{kl} is the cumulant of order (k,l) of $[X_m(r), X_n(t)]$, and H_{ij} the bivariate Hermite polynomial. Expressions for these quantities are given in [Valenzuela (2000)]. When only cumulants up to the third order are considered, the Gram-Charlier and the Edgeworth series give identical expansions.

3.6. Large Deviation Approximation of $p_{mn}(r,t)$

When tail probabilities are involved, a better approximation is given by the large deviation approximation [Mazumdar and Gaver (1984), Iyengar and Mazumdar (1998)].

Exponential tilting is used here to convert the tail region into a central region and then approximate the tilted distribution by a bivariate normal distribution.

Let the exponentially tilted distribution of $\mathbf{X}=[X_m(r), X_n(t)]$ for a given vector $\mathbf{S}=(s_1, s_2)$ be:

$$dF^{\mathbf{S}}(\mathbf{X}; m, n, r, t) = e^{\mathbf{S'X} - K(\mathbf{S}; m, n, r, t)}\, dF(\mathbf{X}; m, n, r, t) \tag{43}$$

where $F(x; m, n, r, t)$ is the bivariate c.d.f. of the random vector $\mathbf{X}$, and

$$K(\mathbf{S}; m, n, r, t) = \ln E[e^{\mathbf{S'X}}] \tag{44}$$

Then, the value of $p_{mn}(r,t) = \displaystyle\int_0^\infty\!\!\int_0^\infty dF(\mathbf{X}; m, n, r, t)$ can be expressed as a function of the distribution function $F^{\mathbf{S}}(\mathbf{X}; m, n, r, t)$ as

$$p_{mn}(r,t) = e^{K(\mathbf{S}; m, n, r, t)}\int_0^\infty\!\!\int_0^\infty e^{-\mathbf{S'X}} dF^{\mathbf{S}}(\mathbf{X}; m, n, r, t) \tag{45}$$

Next, the central limit theorem is used to approximate $F^{\mathbf{S}}(\mathbf{X}; m, n, r, t)$ by a normal distribution $\Phi_2[\mathbf{X}; \mathbf{B}, \mathbf{\Sigma}]$ with appropriately determined mean vector $\mathbf{B}$ and covariance matrix $\mathbf{\Sigma}$. Thus, we obtain

$$p_{mn}(r,t) \cong e^{K(\mathbf{S}; m, n, r, t)}\int_0^\infty\!\!\int_0^\infty e^{-\mathbf{S'X}} d\Phi(\mathbf{X}; \mathbf{B}, \mathbf{\Sigma}) \tag{46}$$

The constant vector $\mathbf{S}$ is now chosen so that the lower limits of the integrals is the expected value of the random variable $\mathbf{X}^{\mathbf{S}}$ whose distribution function is $F^{\mathbf{S}}(\mathbf{X}; m, n, r, t)$. This reduces to the following system of equations (assuming $m > n$):

$$\theta_r + \sigma_r^2 s_1 + \sigma_{r,t} s_2 - \sum_{i=1}^{n} \frac{c_i p_i g_{r,t}(q_i,q_i,\delta_i)e^{-s_1 c_i} - c_i p_i f_{r,t}(p_i,q_i,\delta_i)e^{-(s_1+s_2)c_i}}{q_i f_{r,t}(q_i,p_i,\delta_i) + p_i g_{r,t}(q_i,q_i,\delta_i)e^{-s_1 c_i} + q_i g_{r,t}(p_i,p_i,\delta_i)e^{-s_2 c_i} + p_i f_{r,t}(p_i,q_i,\delta_i)e^{-(s_1+s_2)c_i}}$$

$$-\sum_{i=n+1}^{m} \frac{c_i p_i e^{s_1 c_i}}{q_i + p_i e^{-s_1 c_i}} \hspace{4cm} = 0 \tag{47}$$

$$\theta_t + \sigma_t^2 s_2 + \sigma_{r,t} s_1 - \sum_{i=1}^{n} \frac{c_i p_i g_{r,t}(p_i,p_i,\delta_i)e^{-s_2 c_i} - c_i p_i f_{r,t}(p_i,q_i,\delta_i)e^{-(s_1+s_2)c_i}}{q_i f_{r,t}(q_i,p_i,\delta_i) + p_i g_{r,t}(q_i,q_i,\delta_i)e^{-s_1 c_i} + q_i g_{r,t}(p_i,p_i,\delta_i)e^{-s_2 c_i} + p_i f_{r,t}(p_i,q_i,\delta_i)e^{-(s_1+s_2)c_i}} = 0 \tag{48}$$

where $f_{r,t}(a_i,b_i,\delta_i) = a_i + b_i e^{-\delta_i|r-t|}$ and $g_{r,t}(a_i,b_i,\delta_i) = a_i - b_i e^{-\delta_i|r-t|}$

Let $\mathbf{S}_0 = [s_1, s_2]$ denote the roots of equations 47 and 48. Then, Equation 46 can be rewritten as

$$p_{mn}(r,t) \cong e^{K(\mathbf{S}_0;m,n,r,t)} \int_0^\infty \int_0^\infty e^{-\mathbf{S}_0'\mathbf{X}} d\Phi(\mathbf{X};\mathbf{0},\mathbf{\Sigma}) \tag{49}$$

Ordering terms, completing squares, and evaluating the integrals gives:

$$p_{mn}(r,t) \cong e^{K(s_1,s_2;m,n,r,t)+G(s_1,s_2;m,n,r,t,\rho)} \Phi_2(\alpha_1,\alpha_2,\rho(s_1,s_2;m,n,r,t)) \tag{50}$$

Expressions for $\rho(\cdot)$, $G(\cdot)$, α_1, and α_2 have been derived in [Iyengar and Mazumdar (1998)] to be

$$\rho(s_1,s_2;m,n,r,t) = \frac{K_{11}(s_1,s_2;m,n,r,t)}{\sqrt{K_{02}(s_1,s_2;m,n,r,t) K_{20}(s_1,s_2;m,n,r,t)}} \tag{51}$$

$$G(\cdot) = \frac{1}{2[1-\rho^2]} \Big\{ [\sigma_m(r)s_1 + \rho\sigma_n(t)s_2]^2 + [\rho\sigma_m(r)s_1 + \sigma_n(t)s_2]^2 +$$
$$2\rho[\rho\sigma_m(r)s_1 + \sigma_n(t)s_2][\sigma_m(r)s_1 + \rho\sigma_n(t)s_2] \Big\} \tag{52}$$

$$\alpha_1 = \sigma_m(r)s_1 + \rho\sigma_n(t)s_2 \tag{53}$$

$$\alpha_2 = \rho \sigma_m(r) s_1 + \sigma_n(t) s_2 \tag{54}$$

where

$$\sigma_m(r) = \sqrt{K_{20}(s_1, s_2; m, n, r, t)}, \sigma_n(t) = \sqrt{K_{02}(s_1, s_2; m, n, r, t)}, \text{ and } \rho = \rho(s_1, s_2; m, n, r, t)$$

The bivariate normal approximation to $F^S(\mathbf{X}; m, n, r, t)$ is not likely to be very accurate in the tails. However, if s_1 and s_2 are positive the multiplier, $\exp(-s_1 x_1 - s_2 x_2)$, reduces the error of the bivariate normal approximation in the tails in Equation 49. If either s_1 or s_2 turns out to be negative, the probability of the complement of the region is approximated instead. Three cases are identified: s_1 is positive and s_2 is negative; s_1 is negative and s_2 is positive; s_1 and s_2 are negative. The same change of variable previously used in the one-dimensional case is also used here, which gives the following equations:

Case 1: if s_1 is positive and s_2 is negative redefine s_2 as $-s_2$ use Equation 55:

$$p_{mn}(r, t) \cong \Pr[X_m(\mathrm{r}) \geq 0] - e^{K(\mathbf{S}_0; m, n, r, t) + G(\mathbf{S}_0; m, n, r, t, \rho)} \Phi_2(\alpha_1, -\alpha_2, -\rho) \tag{55}$$

Case 2: if s_1 is negative and s_2 is positive redefine s_1 as $-s_1$ and use Equation 56

$$p_{mn}(r, t) \cong \Pr[X_m(\mathrm{r}) \geq 0] + \Pr[X_n(\mathrm{t}) \geq 0] + e^{K(\mathbf{S}_0; m, n, r, t) + G(\mathbf{S}_0; m, n, r, t, \rho)} \Phi_2(-\alpha_1, \alpha_2, -\rho) \tag{56}$$

Case 3: if both s_1 and s_2 are negative redefine s_1 as $-s_1$ and s_2 as $-s_2$ and use Equation 57

$$p_{mn}(r, t) \cong \Pr[X_n(\mathrm{t}) \geq 0] - e^{K(\mathbf{S}_0; m, n, r, t) + G(\mathbf{S}_0; m, n, r, t, \rho)} \Phi_2(-\alpha_1, -\alpha_2, \rho) \tag{57}$$

3.7. Monte Carlo Approximation of $p_{mn}(r, t)$

We used Monte Carlo simulation as a benchmark when the exact calculations could not be made. The inputs of the Monte Carlo procedure are the stochastic model of the

load and the characteristics of the generating units that comprise the marketplace. Each run generates a sample $\mathbf{J}_k=[j_k(0), j_k(1),\ldots,j_k(T)]$ of the marginal units for each hour during the time horizon of T hours. In the Monte Carlo procedure, $\mathbf{U}_k=[u_k(0), u_k(1),\ldots, u_k(T)]$ represents a sample of an hourly load profile during hours 0 to T. The sequence $[t_1,t_2,\ldots,t_{j..}]^i$ corresponds to the sampled up and down times of generator i ($i=1,..,N$). If t_j is the time at which the generator i fails, then t_{j+1} denotes the time that the generator i will be repaired and made available. The sequence $\mathbf{S}_q=\{[t_1,t_2,..]^1, [t_1,t_2,..]^2,\ldots,[t_1,t_2,..]^N\}$ groups the sampled up and down times of all N generators covering the period $[0,T]$. The number of runs is denoted by K. Since we have used the Monte Carlo simulation as a benchmark, we wanted to perform an exhaustive simulation; thus, we arbitrary chose the value of K to be 200,000. The bivariate probability mass functions at hours r and t are given by the matrix $\mathbf{P}_{rt}$. The entry $\mathbf{P}_{rt}[j_r, j_t]$ denotes the probability that the marginal unit at time r is j_r and the marginal unit at time t is j_t.

Steps of the Monte Carlo Procedure

1. Read the parameters pertaining to capacity, cost, failure and repair rates for each generating unit as well as the estimated parameters for the load model.

2. Repeat for $k=1$ to K.

   2.1 Obtain a load vector $\mathbf{U}_k$ by sampling each hour's load for T hours based on the known values of the load for earlier hours and the load model.

   2.2 Sample at time $t=0$, the initial state (up or down) of each generating unit by drawing random samples from a Bernoulli distribution with parameter p_i ($i=1,2,...,N$.)

2.3 Obtain $\mathbf{S}_q$ by generating successive up and down times until time T. For each generating unit i draw random samples of uptime from an exponential distribution with parameter λ_i, and draw independent samples of downtime from an exponential distribution with parameter μ_i (i=1,2,...,N.)

2.4 Using the predetermined loading order among the available units and the sample load, determine $\mathbf{J}_k$, the vector of marginal units at each hour during the time interval $[0,T]$.

2.5 Using the sample values of $\mathbf{J}_k$, increment the corresponding counters of the marginal units, $\mathbf{P}_{rt}[j_k(\mathrm{r}),\ j_k(t)]$ (r,t=0,..,T).

3. Obtain the bivariate probability mass function of the marginal units at time r and time t dividing the values of $\mathbf{P}_{rt}$ by K (r,t=0,..T)

4. ELECTRICITY MARKET

For the purpose of forecasting the hourly spot prices, we assume that a complete description of the electricity market is obtainable from the data on the N power generators participating in the market, the aggregate demand, and the hourly temperature forecast for the day of trading. The description of the power generators includes the order in which they are loaded by the Independent System Operator, their capacities, energy costs, mean times to failure, and mean times to repair. The data for the aggregate demand gives the historically observed ambient temperature and the corresponding demand for each hour in the region served by the marketplace. A data set that gave the actual ambient temperature and the corresponding demand for each hour in a region covering the Northeastern United States during the calendar years 1995 and 1996 was used. The last day of this data set, September 20, 1996, was chosen as the trading day. The actual temperatures on this day were assumed to be the forecast temperatures.

4.1. The Market: Demand

The demand is characterized by the model given in Valenzuela and Mazumdar (2000). Because the time horizon is so short, only the temperature is considered as a predictor variable for the demand. Individual regression equations were fitted for each hour of a 24-hour period in which the hourly demand $u(t)$ is the response and the hourly temperature τ_t ($^\circ F$) is the independent variable. The plots of demand versus temperature at each hour, as the one shown in Figure 1, suggested the following regression equation:

$$u(t) = \beta_{0,t} + \beta_{1,t}\tau_t + \beta_{2,t}(\tau_t - 65)\delta(\tau_t) + x(t) \tag{58}$$

where $\delta(\tau_t)$ is defined as:

$$\delta(\tau_t) = \begin{cases} 0 & \text{if} \quad \tau_t \leq 65 \\ 1 & \text{if} \quad \tau_t > 65 \end{cases} \tag{59}$$

and $x(t)$ is a time series ARIMA $(1,0,0)x(0,1,0)_{120}$, given by:

$$x(t) = x(t\text{-}120) + .879[\ x(t\text{-}1) - x(t\text{-}121)] + z(t) \tag{60}$$

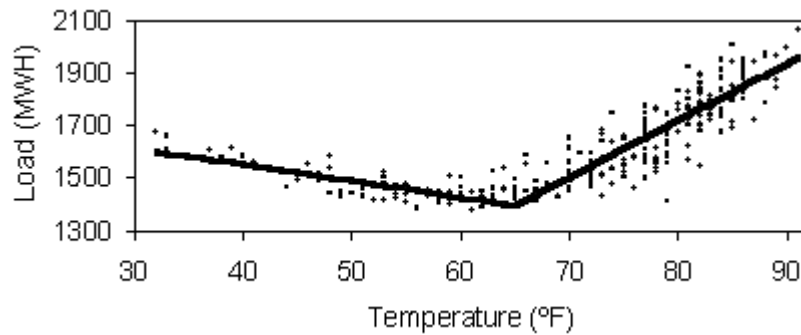$z(t)$ is a Gaussian white noise with mean zero and estimated variance $\hat{\sigma}_z^2 = 2032.55$.

Figure 1: Demand versus Temperature at 12 Noon.

Details of the statistical analysis are given in Valenzuela and Mazumdar (2000).

4.2. The Market: Power Generating System

In order to compare the computing efficiency and accuracy of the various approximations, two hypothetical examples of the electricity market were developed.

System A: Electricity market with eight units

The 8-unit market was built by selecting eight non-identical units from the 32-unit IEEE Reliability Test System (RTS) [APM Subcommittee (1979)]. The resultant system and the relevant characteristics of the units, in their loading order, are given in Table 1. It is assumed that if the available capacity of the system is not sufficient to meet the

demand, electricity can be imported from outside the market at $ 75 per MWh. This system is used for comparing results with exact calculations and validating the Monte Carlo simulation procedure. It is also used as a building block for obtaining markets of larger sizes for which the approximation methods are expected to be much more accurate. We do not expect the approximation procedures to provide an accurate enough result for a system of this small size. When the exact probability distribution of the marginal unit is impractical to compute, the Monte Carlo simulation model is used as a benchmark

Table 1: 8-unit market

Unit i	Capacity (MW)	Mean time to failure $1/\lambda_i$ (hour)	Mean time to repair $1/\mu_i$ (hour)	Marginal cost d_i ($/MWh)
1	400	1100	150	6.00
2	350	1150	100	11.40
3	150	960	40	11.40
4	150	1960	40	14.40
5	200	950	50	22.08
6	100	1200	50	23.00
7	50	2940	60	27.60
8	100	450	50	43.50

(Cost of unserved energy d_9: $ 75 /MWh).

System B: Electricity market with 150 units

This system was constructed by selecting 15 units from the IEEE Reliability Test System and repeating each unit ten times. The data on demand taken from [Valenzuela and Mazumdar (2000)] were also multiplied by a factor of ten. The marginal cost of each unit was modified to provide more generality. Defining C_i as the cumulative capacity of the first i units:

$$C_i = \sum_{j=1}^{i} c_j, \tag{61}$$

the marginal costs of each unit was assumed to be given by the following function:

$$d_i = 6 + 0.00073C_i + 0.000000045C_i^2 \quad \text{for } i=1,..,150 \tag{62}$$

This function was arbitrary created so that we can have a market with marginal costs ranging from \$6.26 per MWh to \$46.60 per MWh. These values were chosen so that they remain close to the original range of the IEEE Reliability Test System. The relevant characteristics of the units, in their loading order, are given in Table 2.

Table 2: 150-unit Market

Units i	Capacity c_i (MW)	MTTF $1/\lambda_i$ (hour)	MTTR $1/\mu_i$ (hour)	Marginal cost d_i (\$/MWh)
1 to 10	350	1150	100	6.26, 6.53, 6.82, 7.11, 7.42, 7.73, 8.06, 8.40, 8.75, 9.11
11 to 20	150	960	40	9.26, 9.42, 9.59, 9.75, 9.92, 10.08, 10.25, 10.43, 10.60, 10.78
21 to 30	150	960	40	10.95, 11.13, 11.32, 11.50, 11.69, 11.87, 12.06, 12.26, 12.45, 12.65
31 to 40	150	960	40	12.84, 13.04, 13.25, 13.45, 13.66, 13.87, 14.08, 14.29, 14.50, 14.72
41 to 50	150	960	40	14.94, 15.16, 15.38, 15.61, 15.83, 16.06, 16.29, 16.52, 16.76, 17.00
51 to 60	150	1960	40	17.24, 17.48, 17.72, 17.96, 18.21, 18.46, 18.71, 18.96, 19.22, 19.48
61 to 70	150	1960	40	19.73, 20.00, 20.26, 20.52, 20.79, 21.06, 21.33, 21.60, 21.88, 22.16
71 to 80	200	950	50	22.53, 22.91, 23.29, 23.67, 24.06, 24.45, 24.84, 25.24, 25.64, 26.05
81 to 90	200	950	50	26.46, 26.87, 27.28, 27.70, 28.13, 28.55, 28.98, 29.42, 29.86, 30.30
91 to 100	200	950	50	30.74, 31.19, 31.64, 32.10, 32.56, 33.02, 33.49, 33.96, 34.43, 34.91
101 to 110	100	1200	50	35.15, 35.39, 35.63, 35.87, 36.12, 36.36, 36.60, 36.85, 37.10, 37.35
111 to 120	100	1200	50	37.60, 37.85, 38.10, 38.35, 38.60, 38.85, 39.11, 39.36, 39.62, 39.88
121-130	100	1200	50	40.13, 40.39, 40.65, 40.91, 41.18, 41.44, 41.70, 41.97, 42.23, 42.50
131-140	50	2940	60	42.63, 42.76, 42.90, 43.03, 43.17, 43.30, 43.43, 43.57, 43.70, 43.84
141-150	100	450	50	44.11, 44.38, 44.66, 44.93, 45.21, 45.48, 45.76, 46.04, 46.32, 46.60

(Cost of unserved energy d_{151}: \$ 75 /MWh).

5. COMPUTATIONAL EFFICIENCY AND ACCURACY

A computer code in C was written to implement the dynamic programming approach for solving the unit commitment problems proposed in section 2. The code is used to evaluate the accuracy and efficiency of the approximation methods in both finding the optimal schedule for turning the unit "on" and "off" and estimating the objective function (maximum expected profit). As we have shown in section 2, when driven by the spot price of electricity, the optimal unit commitment schedule of M generating units can be obtained by solving M de-coupled sub-problems. Therefore, we will give results for scheduling a single generator of a power producer given the information about the electricity market and the known initial conditions for the generating unit to be committed. The characteristics of this generator were patterned after an example taken from Wood and Wollenberg (1996), which are reproduced in Table 3. The fuel-cost function of this unit was modified to be consistent with the range of the energy costs of the individual units comprising the market. The generator is assumed to have been "on" for eight hours. As mentioned earlier, this generator is not included in the set of generators participating in the market.

Table 3: Unit Characteristics

Variable	Generator G1
P^{max}	250 MW
P^{min}	60 MW
t^{up}	5 hours
t^{dn}	3 hours
t^{cold}	5 hours
Initial State (x_0)	+8 hours
Fuel Cost	$585.62 + 16.95p + 0.0042p^2$ \$/h
Start-up Cost	$S_t(x_{t-1}) = \begin{cases} 400\ (\$) & \text{if } -x_{t-1} > t^{cold} \\ 170\ (\$) & \text{if } 1 \le -x_{t-1} \le t^{cold} \\ 0 & \text{otherwise} \end{cases}$

5.1. Computational Efficiency

The processing time is largely determined by the procedure employed for the computation of the probability distribution of the hourly marginal unit. The subroutines that use dynamic programming to solve the three model formulations (models 0, 1, and 2) take less than two seconds irrespective of the approximation method used. The computer time required by the approximation methods is studied by considering markets of different sizes. The power generation system with eight units, System A, is used as a building block to create 10 markets with sizes that vary from 8 to 80 units with increments of eight units. For each case, the probabilities $p_{mn}(0,r)$ (for $r=1,..,24$) required by model 1 are computed and the total processing time is found. The results are plotted in Figure 2. The results show that the large deviation method has the worst computational performance beyond a certain system size. The increase in the computer time for this procedure with increasing system size is a direct result of the computations required to solve the nonlinear equations (47) and (48). The sample size (200,000 replicates) of the Monte Carlo simulation was kept constant for all the problem sizes. Due to the additivity property of the cumulants, the computational time for the Edgeworth approximation increases very little. As expected, the time for the Monte Carlo computations increases with system size although at a much smaller rate compared to the large deviation approximation. These experiments were conducted in a personal computer equipped with a 160 MHZ Pentium Processor with 16 Mbytes of RAM.
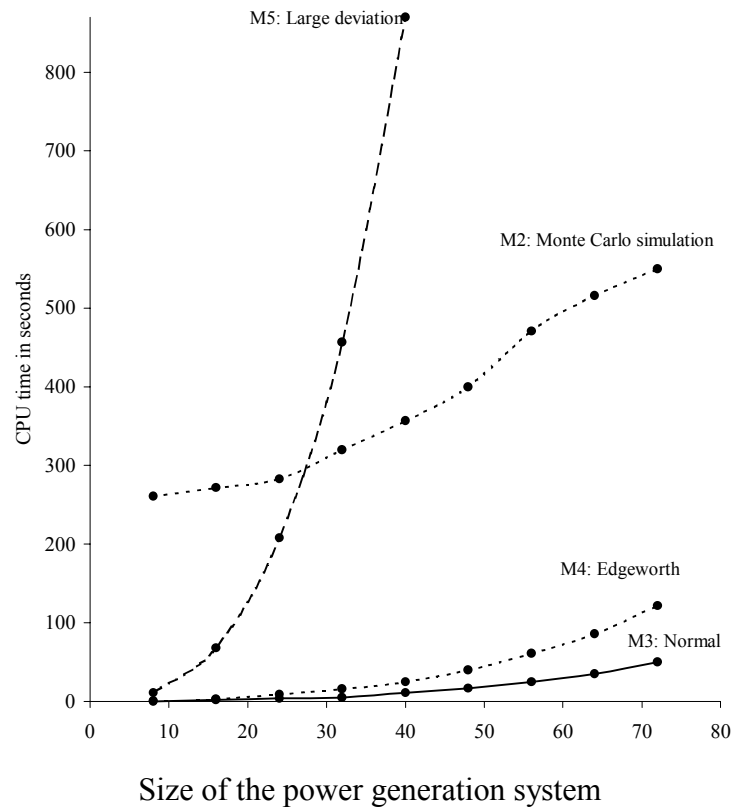
Figure 2: CPU Time versus System Size

5.2. Accuracy of Approximation Methods in the One-unit Commitment Problem

To study the accuracy of the approximation methods in solving the single-unit commitment problem, we consider model 1. The optimal solution of the proposed one-unit commitment problem has three components. The first component is the commitment schedule of the generating unit. The schedule defines the hours at which the unit should be turned "on" or "off". The energy to be generated by the unit at each hour is a second component of the optimal solution. The amount of energy generated at a particular hour depends on the operating state of the generating unit and the spot price at this hour. The third element is the value of the objective function, which is the maximum expected profit over the time horizon. It is important to clearly interpret the accuracy of an optimal

solution to the probabilistic unit commitment problem in which an approximation method is used in terms of these components. The optimal schedule is interpreted as an approximation to the optimal schedule that could in principle (if not in practice) be obtained by using the exact computation of the probability distribution of the hourly marginal unit. The objective function is an estimate of the maximum expected profit that would result when the optimal schedule is executed under the exact probability distribution of the hourly marginal unit. It is possible that each approximation method provides a different solution to the probabilistic unit commitment problem. These solutions may differ based on the solution schedule and/or on the maximum expected profit. Therefore, we define two indices to measure the accuracy of a solution obtained using an approximation method. The first index measures the accuracy of an optimal schedule obtained using an approximation method and the second index measures the accuracy of the estimate of the objective function. Monte Carlo simulation is used as the benchmark procedure.

Let S^m denote the optimal schedule obtained using the approximation method m, and $F_n(S^m)$ denote the maximum expected profit for the schedule S^m when the approximation method n is used in the evaluation of the objective function. We evaluate four different approximation procedures: Monte Carlo simulation, normal, Edgeworth, and large deviation. They are denoted by M2, M3, M4, and M5 respectively. The exact computation procedure is denoted by M1. For instance, S^{M2} describes the optimal schedule obtained using the Monte Carlo simulation output as the approximation to the distribution of the marginal unit. The value of the objective function, which is the maximum expected profit, for this schedule is given by $F_{M2}(S^{M2})$. We assume that the

Monte Carlo output gives a very accurate estimate of the distribution of the marginal unit, and that the solution provided by it is the exact optimal value. Therefore, $F_{M2}(S^m)$ gives the exact maximum expected profit when the schedule S^m is executed and $F_m(S^m)$ gives an estimate of this value. Using this nomenclature, equation 68 gives the relative error of the schedule S^m and equation 69 gives the relative error for the approximation of the maximum expected profit.

$$E_S(m) = \frac{F_{M2}(S^{M2}) - F_{M2}(S^m)}{F_{M2}(S^{M2})} \times 100\% \tag{68}$$

$$E_p(m) = \frac{F_{M2}(S^m) - F_m(S^m)}{F_{M2}(S^m)} \times 100\% \tag{69}$$

Tables 4, 5, and 6 summarize the relative errors of the different approximation methods. These tables show that except for the 8-unit market the approximation methods found the same optimal schedule as the Monte Carlo procedure. Additionally, the relative errors of these methods for the approximations of the maximum expected profit are quite small.

Table 4: Accuracy of the Approximation Method for the Expected Profit of Model 1 (Normal Approximation Method)

System size	$F_{M3}(S^{M3})$ ($)	$F_{M2}(S^{M3})$ ($)	$F_{M2}(S^{M2})$ ($)	E_s(M3) (%)	E_p(M3) (%)
8	178227	165360	165946	0.3	7.7
16	168960	168615	168615	0.0	0.2
24	166325	169106	169106	0.0	1.6
32	182471	179922	179922	0.0	1.4
40	179495	179199	179199	0.0	0.1
48	177877	178501	178501	0.0	0.3
56	177046	179217	179217	0.0	1.2
64	183629	183913	183913	0.0	0.1
72	182683	184366	184366	0.0	0.9

Table 5: Accuracy of the Approximation Method for the Expected Profit of Model 1
(Edgeworth Approximation Method)

System size	$F_{M4}(S^{M4})$ ($)	$F_{M2}(S^{M4})$ ($)	$F_{M2}(S^{M2})$ ($)	$E_s(M4)$ (%)	$E_p(M4)$ (%)
8	178636	165946	165946	0.0	7.6
16	158240	168615	168615	0.0	6.1
24	151091	169106	169106	0.0	10.6
32	177718	179922	179922	0.0	1.2
40	175054	179199	179199	0.0	2.3
48	173022	178501	178501	0.0	3.0
56	174467	179217	179217	0.0	2.6
64	182117	183913	183913	0.0	0.9
72	182109	184366	184366	0.0	1.2

Table 6: Accuracy Approximation Method for the Expected Profit of Model 1
(Large Deviation Approximation Method)

System size	$F_{M5}(S^{M5})$ ($)	$F_{M2}(S^{M5})$ ($)	$F_{M2}(S^{M2})$ ($)	$E_s(M5)$ (%)	$E_p(M5)$ (%)
8	167128	165946	165946	0.0	0.7
16	166053	168615	168615	0.0	1.5
24	169249	169106	169106	0.0	0.1
32	180624	179922	179922	0.0	0.3
40	180142	179199	179199	0.0	0.5
48	179727	178501	178501	0.0	0.6
56	179590	179217	179217	0.0	0.2
64	184641	183913	183913	0.0	0.4
72	184163	184366	184366	0.0	0.1

5.3. Unit Commitment Schedule for Models 0 and 1

In this example we consider the 150-unit power generation system, System B. It provides a more realistic representation of the actual markets to be encountered in practice. The problem is to determine an operating schedule of the generator described earlier in Table 3 under models 1 and 2 such that the expected profit over the next 24 hours is maximized. For the purpose of numerical illustration, we assume that the marginal unit at time zero is the 61st unit. It is currently determining the spot price, which is $19.73 /MWh. Table 7 summarizes the unit commitment decisions under model 0 obtained using the different algorithms. The schedule produced by the Monte Carlo

simulation (200,000 replicates) is to turn the generating unit off during the first four hours. Then, the unit is turned back on for the next nineteen hours. It is estimated that the execution of this schedule will generate an expected profit of $37,496. The normal and Edgeworth approximation methods provided a different schedule. However, after running a Monte Carlo simulation with this schedule, we found that the corresponding estimated expected profit was $37,387, and thus this solution is very close to the optimal solution. The estimated expected profits obtained from these algorithms were $37,355 and $37,046, respectively. For model 1, the different methods provided the same schedules (see Table 8).

Table 7: Experimental Results for Model 0 (150-unit market)

Method	Schedule	Expected Profit ($)	CPU Time[*] (sec.)
Monte Carlo Simulation	000001111111111111111111	37,496	65
Normal Approximation	000001111111111111111110	37,355	22
Edgeworth Approximation	000001111111111111111110	37,046	55

[*]CPU time is based on a 900 MHz Pentium III Processor with 256 Mbytes of RAM. It includes computation of probabilities and optimization problem.

Table 8: Experimental Results for Model 1 (150-unit market)

Method	Schedule	Expected Profit ($)	CPU Time[*] (sec.)
Monte Carlo Simulation	000001111111111111111111	37,509	77
Normal Approximation	000001111111111111111111	37,356	23
Edgeworth Approximation	000001111111111111111111	37,229	56

[*]CPU time is based on a 900 MHz Pentium III Processor with 256 Mbytes of RAM. It includes computation of probabilities and optimization problem.

5.4. Unit Commitment Schedule for Model 2

To illustrate the decision process for model 2, we use system A, the electricity market with eight units (see Table 1). The problem is to determine whether to keep the generator

on or to switch it off the next hour as information accumulates over each hour. For purposes of illustration, we assume that the objective is to maximize the expected profit over the next 8 hours. Figure 3 shows the optimal decision policy obtained by solving the UCP using the Monte Carlo method for the probability distribution of the marginal unit. We give the decisions corresponding to the values of x_t within the range {-5,..,-1, 1,..,+5] only because the maximum and minimum values of the variable x_t are, for this problem, determined by the number of hours of a cold start and the minimum up time, which are both 5 hours (see Table 3). The optimal policy dictates that for the next hour the owner should keep its generator on. If at the next hour (hour 1) the index of the marginal unit happens to be greater than four, the generator remains on. Otherwise, the generator is turned off and it must remain off for the next three hours, which is the required minimum down time. The execution of this policy gives an expected total profit over the next eight hours of $32,628.

Next, we solve a problem identical to the one given in Section 5.3 but now using model 2. The objective is to decide whether to commit the generator for the next hour in order to maximize the expected profit over the next 24 hours. We assume the same status of the market as in the previous example. Table 9 summarizes the unit commitment decision obtained using the different approximations. The policy produced by the Monte Carlo simulation (200,000 replicates) dictates, for the next hour (hour one), to turn the generator off. It is estimated that the execution of this action will generate an expected total profit of $38,600 over the period of 24 hours. The normal and Edgeworth approximation methods provide the same action and similar expected profits.
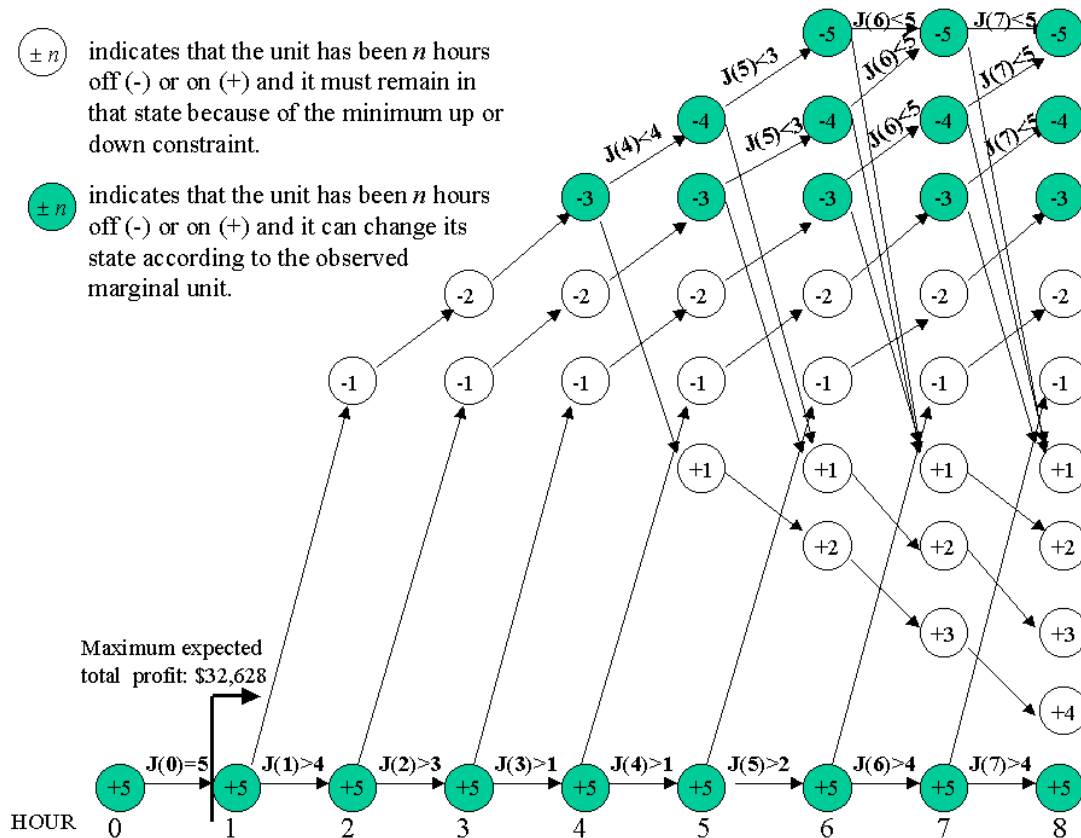
Figure 3: Optimal policy showing the states of the generating unit in an 8-unit market

Table 9: Experimental Results for Model 2 (150-unit market)

Method	Decision for hour 1	Expected Profit ($)	CPU Time[*] (sec.)
Monte Carlo Simulation	OFF	38,600	78
Normal Approximation	OFF	38,599	45
Edgeworth Approximation	OFF	38,592	111

[*]CPU time is based on a 900 MHz Pentium III Processor with 256 Mbytes of RAM. It includes the time of computations of probabilities and optimization part.

Notice that the expected profits increase when we move from table 7 to 9. The reason is that the unit commitment decisions corresponding to Table 9 are being made hour-by-hour, which allows for switching off the generator when the price is unfavorable and vice-versa (obviously, following the minimum down and up time constraints). However, when the unit commitment decisions corresponding to tables 7 and 8 are made for the

complete period of 24 hours, no changes on unit commitment decisions are allowed at any hour within this time interval. The schedules and estimated total profits, however, remain comparable.

6. SUMMARY

In this paper, we have considered a new formulation of the unit commitment problem for the deregulated environment. Under the assumption of perfect competition, we show that when a producer is able to buy from or sell to a pool his excess demand or supply, the unit commitment problem can be solved considering each individual unit separately. The solution method for the new formulation requires the computation of the probability distribution of the spot price of electricity. In order to do so, the power generation system of the marketplace has been modeled using the Ryan-Mazumdar model (1990) of production costing. This model takes into account the uncertainty on the demand and the generating unit availabilities. The probability distribution of the spot price, which is based on Schweppe's definition, is based on the probability distribution of the marginal unit. When more data about the market prices become available, it will certainly be very important to verify whether similar conclusions are obtained from a statistical analysis of market prices.

The exact computation of the probability distribution is computationally prohibitive for large systems. Three approximation methods were evaluated. From the computational experience, it appears that the proposed unit commitment can be accurately solved in a reasonable time by using the normal, Edgeworth, or Monte Carlo approximations. The large deviation approximation does not appear to be feasible for very large markets. In a

sense, it is gratifying that the normal and Edgeworth approximations work so well. In order to apply these procedures we do not need detailed information on the component distributions; only knowledge of the first few moments suffices.

For the example illustrated in this paper, the execution of the solutions (commitment schedules) obtained for each of the three model formulations, models 0, 1, and 2 appear to give similar expected total profits. We intend to further investigate this finding.

ACKNOWLEDGMENT

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