University of Pittsburgh

The DIETRICH School of Arts & Sciences

Workshop on PDEs in Fluid Dynamics and Applications

Department of Mathematics, University of Pittsburgh

May 8–9, 2021

Program

All talks will be given online via ZOOM: Join Zoom Meeting https://pitt.zoom.us/j/96049270479; Meeting ID: 960 4927 0479.

Sponsors: Mathematics Research Center (MRC) of the University of Pittsburgh.

Organizers: Ming Chen and Dehua Wang.

Workshop on PDEs in Fluid Dynamics and Applications
University of Pittsburgh, May 8–9, 2021

❋ *All talks are online via Zoom:*
Join Zoom Meeting https://pitt.zoom.us/j/96049270479;
Meeting ID: 960 4927 0479

	Saturday, 05/08	**Sunday, 05/09**
❖ Morning Session		
09:00-09:45am	Gui-Qiang Chen	Athanasios Tzavaras
10:00-10:45am	Graziano Guerra	Wen Shen
11:15-12:00pm	Robert Strain	Yuan Lou
12:00-1:00pm	*Lunch break*	
❖ Afternoon Session		
01:00-01:45pm	Alberto Bressan	Juncheng Wei
02:00-02:45pm	Yan Guo	David Ambrose
03:15-04:00pm	Mikhail Feldman	Ronghua Pan
04:15-05:00pm	Cheng Yu	Free discussion

Saturday, May 8

❖ Morning Session

- **09:00-09:45**: Gui-Qiang Chen, Oxford University
  Cavitation and concentration in entropy solutions to the compressible Euler equations and related nonlinear PDEs

- **10:00-10:45**: Graziano Guerra, Università di Milano
  The Cauchy problem for a non strictly hyperbolic 3×3 system of conservation laws arising in polymer flooding

- **11:15-12:00**: Robert Strain, University of Pennsylvania
  On the relativistic Boltzmann equation with long range interactions

12:00-01:00 pm: Lunch Break

❖ Afternoon Session

- **01:00-01:45**: Alberto Bressan, Penn State University
  Ill-posedness and generic singularities for 2-D pressureless gases

- **02:00-02:45**: Yan Guo, Brown University
  Gravitational collapse of Newtonian stars

- **03:15-04:00**: Mikhail Feldman, University of Wisconsin at Madison
  Weak-strong uniqueness and stability for the semigeostrophic system

- **04:15-05:00**: Cheng Yu, University of Florida
  Infinitely many solutions to the isentropic system of gas dynamics

Sunday, May 9

❖ Morning Session

- **09:00-09:45**: Athanasios Tzavaras, King Abdullah University of Science and Technology
  From Euler flows with friction to gradient flows
- **10:00-10:45**: Wen Shen, Penn State University
  Variations on the theme of scalar conservation law
- **11:15-12:00**: Yuan Lou, Shanghai Jiao Tong University/Ohio State University
  Basic reproduction number and principal eigenvalue

12:00-01:00 pm: Lunch Break

❖ Afternoon Session

- **01:00-01:45**: Juncheng Wei, University of British Columbia
  Singularity formations in liquid crystal flows, Landau-Liftschitz and Keller-Segel-Navier-Stokes system
- **02:00-02:45**: David Ambrose, Drexel University
  Existence (and non-existence) results for a mean field games model of household wealth
- **03:15-04:00**: Ronghua Pan, Georgia Tech University
  Isentropic approximation
- **04:15-05:00**: Free discussion

THE END.

Workshop on PDEs in Fluid Dynamics and Applications
University of Pittsburgh, May 8–9, 2021

Abstracts

David Ambrose, *Drexel University*

Title: Existence (and non-existence) results for a mean field games model of household wealth

Abstract: We consider a mean field games model of household wealth introduced by Achdou, Buera, Lasry, Lions, and Moll. In this problem, a large number of similar households must decide how much of their income to allocate between consumption and savings, and the decisions are coupled through the determination of the interest rate. The mean field games system for this situation consists of a forward parabolic equation for the distribution of the agents and a backward parabolic equation for the value function being optimized by a representative household, although the parabolic effects are only with respect to one of the two independent variables. The model is completed through the specification of initial, terminal, boundary, and moment conditions. Prior existence theories for the mean field games system of PDEs tend to rely on structure such as additive separability of the Hamiltonian or uniform parabolic effects, and these features are not present in this model. Relaxing a moment condition, we arrive at a problem for which we can prove existence and uniqueness of solutions. Central to the work is the question of how to calculate the interest rate; our relaxation together with conditions on the support of solutions gives a formula for the interest rate.

Alberto Bressan, *Penn State University*

Title: Ill-posedness and generic singularities for 2-D pressureless gases

Abstract: The talk will review some examples of ill-posedness for the so-called "sticky particles" equations, in two space dimensions.

Some recent analysis of generic singularities will also be discussed. (This is joint work with Geng Chen and Shoujun Huang.)

Gui-Qiang Chen, *Oxford University*

Title: Cavitation and concentration in entropy solutions to the compressible Euler equations and related nonlinear PDEs

Abstract: In this talk, we will discuss the intrinsic phenomena of cavitation/decavitation and concentration/deconcentration in entropy solutions to the compressible Euler equations and related nonlinear PDEs, which are fundamental to understand the well-posedness and solution behavior of nonlinear hyperbolic systems of conservation laws. We will start to discuss the formation process of cavitation and concentration in the entropy solutions of the one-dimensional isentropic Euler equations with respect to the initial data and the vanishing pressure limit. Then we will analyze a longstanding fundamental problem in fluid dynamics: Does the concentration occur generically so that the density develops into a Dirac measure at the origin generically in spherically symmetric entropy solutions of the multi-dimensional compressible Euler equations? We will report our recent results

and approaches developed for solving this longstanding open problem for the Euler equations and related nonlinear PDEs, including the Euler-Poisson equations, and discuss its close connections with entropy methods and theory of divergence-measure fields. Further related topics, perspectives, and open problems in this direction will also be addressed.

Mikhail Feldman, *University of Wisconsin at Madison*

Title: Weak-strong uniqueness and stability for the semigeostrophic system

Abstract: The semigeostrophic (SG) system is a model of large scale atmosphere/ocean flows. Solutions of this system are expected to contain singularities corresponding to the atmospheric fronts, and need to be understood in the appropriate weak sense. We will survey the results on existence of weak solutions. Then we will describe recent results on weak-strong uniqueness for SG system, and on convergence of smooth solutions of incompressible Euler system with Coriolis force to a sufficiently regular solution of SG system in 2D and 3D. In both results main assumptions on the strong solution are the boundedness of the velocity field and the uniform convexity of the Legendre-Fenchel transform of the modified pressure. Both results are obtained by the relative entropy techniques. This talk is based on joint works with M. Cullen and A. Tudorascu.

Graziano Guerra, *Università di Milano*

Title: The Cauchy problem for a non strictly hyperbolic 3×3 system of conservation laws arising in polymer flooding

Abstract: We study the Cauchy problem of a 3×3 system of conservation laws modeling two-phase flow of polymer flooding in rough porous media with possibly discontinuous permeability function. The system loses strict hyperbolicity in some regions of the domain where the eigenvalues of different families coincide, and BV estimates are not available in general. For a suitable 2×2 system, a singular change of variable introduced by Temple [3, 5] could be effective to control the total variation [4]. An extension of this technique can be applied to a 3×3 system only under strict hypotheses on the flux functions [1]. Through an adapted front tracking algorithm we show the existence of solutions for the Cauchy problem under mild assumptions on the flux function, using a compensated compactness argument.

REFERENCES:

[1] Giuseppe Maria Coclite and Nils Henrik Risebro. Conservation laws with time dependent discontinuous coefficients, *SIAM J. Math. Anal.*, **36** (4):1293–1309, 2005.

[2] Graziano Guerra andWen Shen. The Cauchy problem for a non strictly hyperbolic 3×3 system of conservation laws arising in polymer flooding, *Commun. Math. Sci.*, 2021. To appear.

[3] Eli L. Isaacson and J. Blake Temple. Analysis of a singular hyperbolic system of conservation laws, *J. Differential Equations*, **65** (2):250–268, 1986.

[4] Wen Shen. On the Cauchy problems for polymer flooding with gravitation, *J. Differential Equations*, **261** (1):627–653, 2016.

[5] Blake Temple. Global solution of the Cauchy problem for a class of 2×2 nonstrictly hyperbolic conservation laws, *Adv. in Appl. Math.*, **3** (3):335–375, 1982.

Yan Guo, *Brown University*

Title: Gravitational collapse of Newtonian stars

Abstract: We present recent constructions of blowup solutions to the Euler-Poisson system for describing gravitational collapse of Newtonian stars. This is a part of joint research program with Hadzic and Jang.

Yuan Lou, *Ohio State University*

Title: Basic reproduction number and principal eigenvalue

Abstract: Basic reproduction number is a dimensionless constant which is used in epidemiology to determine if an emerging infectious disease can spread. Principal eigenvalue, a key concept in spectral theory, is used to reflect certain properties of matrices or differential operators. In this talk we will discuss some recent progress on principal eigenvalues for second-order linear elliptic and time-period parabolic operators, with applications to basic reproduction numbers for some susceptible-infected-susceptible PDE epidemic models.

Ronghua Pan, *Georgia Tech University*

Title: Isentropic approximation

Abstract: In the study of compressible flows, the isentropic model was often used to replace the more complicated full system when the entropy is near a constant. This is based on the expectation that the corresponding isentropic model is a good approximation to the full system when the entropy is sufficiently close to the constant. We will discuss the mathematical justification of isentropic approximation in Euler flows and in Navier-Stokes-Fourier flows. This is based on the joint work with Y. Chen, J. Jia, and L. Tong.

Wen Shen, *Penn State University*

Title: Variations on the theme of scalar conservation law

Abstract: In this talk we visit several variations of a standard conservation law. These include the cases such as: (i) the flux function is discontinuous in space and time, (ii) the flux function is nonlocal and contains an integral term, and (iii) combinations of them. We discuss key challenges, some recent results, and applications to traffic flow.

Robert Strain, *University of Pennsylvania*

Title: On the relativistic Boltzmann equation with long range interactions

Abstract: In this talk I will discuss three recent related results regarding the special relativistic Boltzmann equation without angular cutoff. In the non-relativistic situation without angular cutoff, the change of variables from $v \to v'$ is the main step of the widely used "cancellation lemma". In a first result, in collaboration with James Chapman and Jin Woo Jang, in the special relativistic situation we calculate this very complex ten variable Jacobian determinant and illustrate some numerical results which show that it has a large number of distinct points where it is machine zero. In a second result, in collaboration with Jang, we prove the sharp pointwise asymptotics for the frequency multiplier of the linearized relativistic Boltzmann collision operator that has not been previously established. As a consequence of these calculations, we further explain why the well known change of variables $p \to p'$ is not well defined in the special relativistic context. In the third result, also in collaboration with Jang, I will explain our recent proof

of global-in-time existence, uniqueness and asymptotic stability for solutions nearby the relativistic Maxwellian to the special relativistic Boltzmann equation without any angular cutoff. We work in the case of a spatially periodic box. We assume the generic hard-interaction and soft-interaction conditions on the collision kernel that were derived by Dudynski and Ekiel-Jezewska (in 1985). In this physical situation, the angular function in the collision kernel is not locally integrable, and the collision operator behaves like a fractional diffusion operator. This is the first global existence and stability result for relativistic Boltzmann equation without angular cutoff.

Athanasios Tzavaras, *King Abdullah University of Science and Technology*

Title: From Euler flows with friction to gradient flows

Abstract: I will review some recent works that address the problem of high-friction limit (or small mass approximation) from Euler flows to advection-diffusion systems that are gradient flows. The main technical tool is the relative entropy method applied to systems that are Hamiltonian flows generated by an energy. The computations exploit the variational structure of the problem and are related to the well-known realization of several nonlinear Fokker-Planck equations as gradient flows in Wasserstein. A related problem is the high-friction limit of multicomponent systems and the emergence of the so called Maxwell-Stefan diffusion system from multicomponent Euler systems. Here, we discuss the issue of uniqueness of weak solutions for the Maxwell-Stefan model.

Juncheng Wei, *University of British Columbia*

Title: Singularity formations in liquid crystal flows, Landau-Liftschitz and Keller-Segel-Navier-Stokes system

Abstract: I will discuss the singularity formation problems in some fluid equations including liquid crystal flows, Landau-Lipschtiz equation and Keller-Segel-Navier-Stokes system.

Cheng Yu, *University of Florida*

Title: Infinitely many solutions to the isentropic system of gas dynamics

Abstract: In this talk, I will discuss the non-uniqueness of global weak solutions to the isentropic system of gas dynamics. In particular, I will show that for any initial data belonging to a dense subset of the energy space, there exists infinitely many global weak solutions to the isentropic Euler equations for any $1 < \gamma \leq 1 + 2/n$. The proof is based on a generalization of convex integration techniques and weak vanishing viscosity limit of the Navier-Stokes equations. This talk is based on the joint work with M. Chen and A. Vasseur.