A SUFFICIENT CONDITION FOR THE KOLMOGOROV 4/5 LAW FOR STATIONARY MARTINGALE SOLUTIONS TO THE 3D NAVIER-STOKES EQUATIONS

FRANZISKA WEBER

We prove that statistically stationary martingale solutions of the 3D Navier-Stokes equations on the torus subjected to forcing with white noise satisfy the Kolmogorov 4/5 law (in an averaged sense and over a suitable inertial range) using only the assumption that the kinetic energy is $o(\nu)$ as $\nu \to 0$ (where ν is the inverse Reynolds number). This plays the role of a weak anomalous dissipation. No energy balance or additional regularity is assumed (aside from that satisfied by all martingale solutions from the energy inequality). If the force is statistically homogeneous, then any homogeneous martingale solution satisfies the spherically averaged 4/5 law pointwise in space. An additional hypothesis of approximate isotropy in the inertial range gives the traditional version of the Kolmogorov law. We demonstrate a necessary condition by proving that energy balance and an additional quantitative regularity estimate as $\nu \to 0$ imply that the 4/5 law (or any similar scaling law) cannot hold.