Math 0420 Homework #5

Due date: Oct. 04, 2016

Problem 0.
Do the following problems in the book:
P140: 4, 5, 7, 10.

Problem 1.

1. Consider \( f(x) = 1 - \sqrt[3]{x^2} \). We have \( f(-1) = f(1) = 0 \), but \( f'(x) \neq 0 \) in \([-1, 1]\). Does it contradict the Rolle’s theorem? Explain why.

2. Let \( f \in C([a, b]) \) be differentiable on \((a, b)\). For any \( c \in (a, b) \), is it always possible to find two points \( x_1, x_2 \in [a, b] \) such that \( x_1 < c < x_2 \) and

\[
 f'(c) = \frac{f(x_1) - f(x_2)}{x_1 - x_2} ?
\]

Explain your conclusion.

Problem 2.
Prove the following inequalities.

1. \( |\arctan x - \arctan y| \leq |x - y|, \quad \forall \ x, y \in \mathbb{R}. \)

2. \( \sin x < x < \tan x, \quad \forall \ 0 < x < \frac{\pi}{2}. \)

3. \( \frac{2}{\pi} x < \sin x, \quad \forall \ 0 < x < \frac{\pi}{2}. \)