# Pulse Modulation

- **What if the Carrier Signal were a Pulse Train Instead of a Sinusoid?**
- **Three Approaches**
  - **Pulse Amplitude Modulation (PAM)**
  - **Pulse Width Modulation (PWM)**
  - **Pulse Position Modulation (PPM)**
- **Each of these Approaches Uses a Discrete Signal to Carry an Analog Signal**

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# Bandwidth of a Pulse Train

- $V(f) = V\tau[\sin(\pi f\tau)/(\pi f\tau)]$
- **$\tau$ is the Duration of the Pulse**
- **$V$ is the Amplitude of the Pulse**
- **The $N^{th}$ Harmonic of a Pulse Train**
  - $V_n = (V\tau/T)[\sin(nx)/(nx)] = (V\tau/T)\text{sinc}(\tau/T)$
  - **$T$ is the Interval Between Pulses (*i.e.*, the Period)**
  - $x = \pi\tau/T$

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# Bandwidth of a Pulse Train

- **Find the "Zeros"**
  - **We Need the Location of sin($nx$) = 0**
  - **This Occurs When $n\pi\tau/T = \pi$**
  - **That is, When $n/T=1/\tau$ (By Substitution)**
  - **Let $f_0=1/T$, So Zeros Occurs at $nf_0 = 1/\tau$**
  - **The First Zero Occurs at $n$=1, or at the Frequency $f=1/\tau$**
- **Note The Following**
  - **Most of the Signal Energy (92%) is Contained in the Frequency Between 0 and $f=1/\tau$**
  - **Thus, we can Ignore the Higher Frequency Components**





# Example

- **$V$ = 5V**
- **$T$ = 25µsec**
- **$\tau$ = 5µsec**
- **Spectrum**
  - **Calculate 1st zero: $f_0=1/\tau=1/5\mu sec=200KHz$**
  - **Calculate 2nd zero: $f_1=2/\tau=400KHz$**

# Pulse Amplitude Modulation (PAM)

- **Modulate a Pulse Stream with a Signal**
  - **Used in Dimension PBX's**
  - **Type of a AM system**
- **Categories**
  - **Natural PAM**
    - **Top of Pulse Conforms to the Signal Shape**
    - **Makes Mathematics Easy**
  - **Flat Top PAM**
    - **More Practical**
    - **Approaches Natural PAM for Narrow Pulses**





# Bandwidth of PAM

- $v(t) = m(t)p(t)$
  - $m(t)$ **is the Modulating Waveform**
  - $p(t)$ **is the Pulse Train**
- **Fourier Equivalent of a Pulse Train**
  - $p(t) = V\tau/T + (2V\tau/T)[\text{sinc}(x)\cos\omega t + \text{sinc}(2x)\cos(2\omega t) + \ldots]$
  - **Where** $x = \pi\tau/T$
- **Thus,**
  - $v(t) = m(t)V\tau/T + m(t)(2V\tau/\text{T})[\text{sinc}(x)\cos\omega t + \text{sinc}(2x)\cos(2\omega t) + \ldots]$
  - **This is in the Same General Form of the AM Signal:**
    - $v(t)=m(t)V\tau/T + [(2V\tau/\text{T})\text{sinc}(x)]m(t)\cos\omega t + [(2V\tau/\text{T})\text{sinc}(2x)]m(t)\cos(2\omega t) + \ldots$
    - $X_s(f)=c_0M(f) + c_1[M(f-f_s)+M(f+f_s)] + c_2[M(f-2f_s)+M(f+2f_s)] + \ldots$
    - **Where** $c_n = 2Vf_s\tau[\sin(\text{n}\pi f_s\tau)/\text{n}\pi f_s\tau)] = f_s\tau\ \text{sinc}(nf_s\tau)$

# Example

- $V$=5V
- $T$=5μsec, or $f_s$=200,000/sec
- $\tau$ =1μsec
- First Zero, $f_0=1/\tau=1/10^{-6}$=1MHz
- $f_s\tau$ = .2, $f_s\tau$ = .628
- $c_0$ = (10)(.2)[sinc(0)]=2
- $c_1$ = (10)(.2)[sin(.628)/.628]=2(.935)=1.87
- $c_2$ = 2[sinc(.4)]=2(.757)=1.514





# Form of the Harmonic Terms

- $V_n m(t)\cos(n\omega_s t)$, Where
  - $\omega_s = 2\pi/T_s$, or the Angular Sampling Frequency
  - and $V_n=(V\tau/T)[\sin(nx)/(nx)]$, $x=\pi\tau/T$
- Observation:
  - $m(t)$ is Completely Contained in DC Component
  - Thus, Low Pass Filtering can be Used for Demodulation

# Comments on PAM

- Natural vs. Flat-Topped Sampling
  - Flat Tops Introduce Distortion
  - If $\tau$ is Small, the Distortion is Minimal
- Nyquist's Sampling Theorem can be Demonstrated with PAM
  - Consider the Frequency Domain
  - Let *T* Decrease
  - What Happens to the Spectrum of the Modulating Signal?
- PAM can be Used in Time Division Multiplexing
  - Take PAM Samples of Several Signals
  - Interleave PAM Samples on a Transmission Channel
  - Separate Them at the Destination





# Pulse-time modulation

- Analagous to Angle Modulation
- Types
  - Pulse Width Modulation
  - Pulse Position Modulation

# Pulse Width Modulation (PWM)

- Width of the Pulse is Proportional to the Signal
- Measured With Respect To
  - Leading Edge
  - Trailing Edge
  - Center of the Pulse





# Spectrum of PWM

- Cannot be Computed Exactly
- Recall That
  - A Pulse Train has the Following Equivalent,
    - $p(t)=V\tau/T + (2V\tau/T)[\text{sinc}(x)\cos\omega t + \text{sinc}(2x)\cos(2\omega t) + \ldots]$
    - Where $x=\pi\tau/T$
  - The Harmonic Terms are of the Form: $V_n\cos(n\omega_s t)$, where
    - $\omega_s = 2\pi/T_s$, or the Angular Sampling Frequency
    - $V_n=(V\tau/T)[\sin(nx)/(nx)]$
- In PWM
  - $\tau$ Varies with Amplitude
  - Therefore, the Spectrum Depends on the Value of $\tau$ at Any Instance

# Pulse Position Modulation (PPM)

- **Use Narrow, Uniform Pulses**
- **Position of Pulse is Proportional to the Amplitude of the Modulating Signal**
- **Spectrum of PPM**
  - $p(t)=V\tau/T + (2V\tau/T)[\mathrm{sinc}(x)\cos\omega t + \mathrm{sinc}(2x)\cos(2\omega t) + \ldots]$
  - **Here,** $T=T_0+m(t)\Delta T$
  - **The Spectrum Resembles PWM, in Principle**



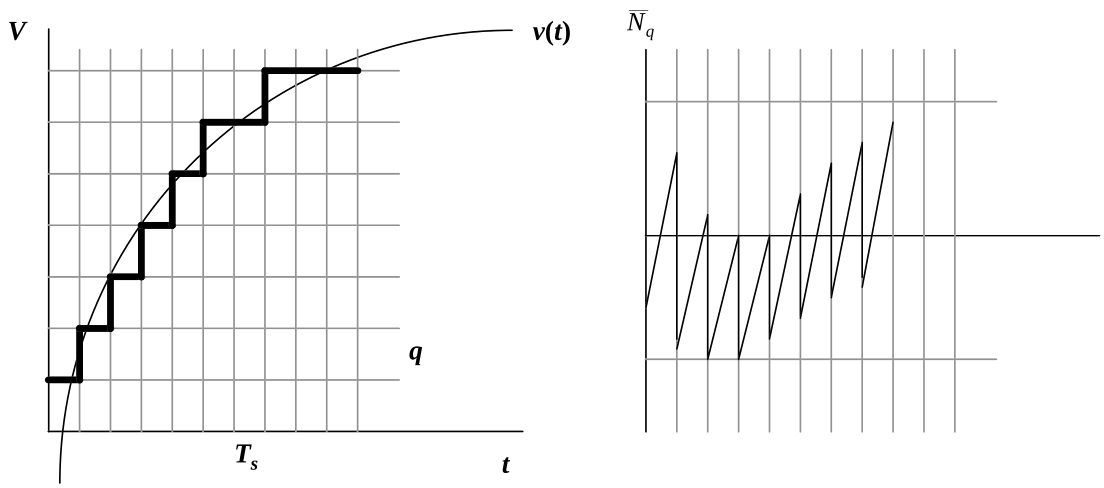


# Pulse Code Modulation (PCM)

- **Essentially a Variant of PAM**
- **Convert Samples into 8 Bit Digital Bytes, *i.e.*, Convert Smooth Analog PAM Samples to Discrete Levels**
  - **This Allows the Use of Digital Transmission Systems**
  - **This Creates Additional *Quantization Noise***

# Quantization Noise in PCM

- **Quantization Noise is a Sawtooth Wave**
  - **Has a Peak Voltage of *q*/2,**
  - ***q* is the Voltage Range of the Quantum**
- **Illustration**

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# Quantization Noise in PCM

- **Compute the Mean Square Quantization Noise Voltage**
  - $\overline{N}_q = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}\left[-\frac{q}{T_0}t\right]^2 dt = \frac{q^2}{T_0^3}\int_{-T_0/2}^{T_0/2}[t]^2 dt = \frac{q^2}{12}$
  - ***q* is the Subrange Voltage Span, *i.e.* Voltage of Each Step**
  - $\overline{N}_q$ **is Small as *q* is Small**
  - **Small *q* Implies Many Steps**
- **RMS Noise Voltage is** $V_{qn} = \sqrt{\overline{N}_q} = \frac{q}{2\sqrt{3}}$
- **Noise Power is** $N_q = \frac{V_{qn}^2}{R} = \frac{q^2}{12R}$

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# Quantization Noise in PCM

- **For a Sinusoid Covering the EntireRange,**
  - $V_s^2 = (1/2)V_p^2 = (1/2)(qM/2)^2 = (qM)^2/8$
  - ***M*** **is Number of Steps in the Conversion**
  - ***q*** **is the Step Voltage**
- **Thus,** $S/N = E_s^2/E_{nq}^2 = [(Mq)^2/8][12/q^2] = (3/2)M^2$
- **Number of Levels**
  - $M = 2^n$
  - ***n*** **is the Number of Bits per Sample**
- **Thus** $S/N = (3/2)(2^{2n})$
- **In Decibels,** $(\text{SNR})_{\text{dB}} = 10 \log(S/N) = 1.761 + 6.02n$





# Practical PCM

- **In the Linear PCM that we Analyzed, Quantization Noise is Constant for all Signal Levels**
- **Low Voltage Signals Suffer a Lower SNR as a Result**
- **Solution**
  - **Decrease the Step Size for Small Voltages and Increase Them for Large Voltages**
  - **This is Called *Companding***

# Companding in PCM

- **North America**
  - **Use μ-Law Companding**
  - $F_{\mu}(x) = \text{sgn}(x)\left(\frac{\ln(1+\mu|x|)}{\ln(1+\mu}\right)$
  - **$F_{\mu}(x)$ is the Compressor Characteristic Function**
  - **Use μ=255**
  - **This Applies for** $-1 \leq x \leq 1$
- **The CCITT Has Defined an Alternate Form**
  - ***A* Law Companding**
  - $F_A(x) = \text{sgn}(x)\left(\frac{1+\ln A|x|}{1+\ln A}\right)$
- **A-Law Companding Gives Flatter SNR for Larger Voltages at the Expense of Poorer SNR for Lower Voltages**



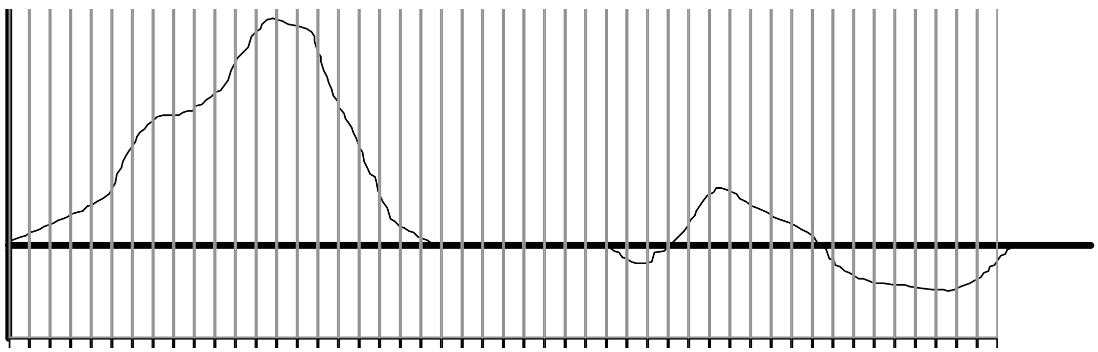


# Improvements in Digital Encoding

- **PCM Encoding is Inefficient**
  - **Does Not Take Advantage of Redundancy**
  - **Alternatives Exist**
- **Adaptive Differential PCM (ADPCM)**
  - **Uses 4 Bits per Sample**
  - **Encodes the *Difference* Between Successive Samples**
  - **Adapts to the Variation in Samples**

# Delta Modulation

- Does Not Sample And Convert Like PCM
- Samples at the Bit Rate
- "1" Indicates the Level is Greater than Before
- "0" Indicates the Level is Less than Before








# Delta Modulation

- Each "1" and "0" Represents a Fixed Voltage Level ($\Delta V$)
- The Receiver Reconstructs the Signal by Adding $\Delta V$ to the Previous Voltage Level for a Received "1", and Subtracting $\Delta V$ for a Received "0"
- Thus, the Maximum Rate of Voltage Change (*i.e.*, Slew Rate) that a Delta Modulation System can Support is $\Delta V / \tau$, where $\tau$ is the Sample Period
- Slew Rates Greater than this Result in *Slope Overload*, which is a Form of Signal Distortion

# Continuously Variable Slope Delta Modulation (CVSD)

- Adaptive Form of Delta Modulation
- Is Less Susceptible to Slope Overload
- The $\Delta V$ Represented by a "1" or a "0" Varies Based on the Number of Successive 1s or 0s Received
  - Many Successive 1s or 0s Increases the $\Delta V$
  - Change Resets $\Delta V$ to Its Nominal Value





# Pulse Modulation Devices

- PAM Transmitter
  - Sample and Hold Circuit
  - Sample the Input Signal and Hold it for the Duration of the Pulse
- PAM Receiver
  - Low Pass Filter
  - Recall that the Entire Modulating Signal was Contained in the Baseband Signal

# Pulse Modulation Devices

- **PWM Transmitter**
  - **PAM Modulator**
  - **Ramp Generator**
  - **Summing Amplifier**
  - **Schmitt Trigger**
- **PWM Receiver**
  - **Line Receiver/Signal Conditioner**
  - **Reference Pulse**
  - **Ramp Generator (Charge Capacitor)**
  - **Summing Amplifier**
  - **Keep Tops: Clipper**
  - **Low Pass Filter**





# Pulse Modulation Devices

- **PCM Transmitter**
  - **Called a *Coder/Decoder (CODEC)***
  - **Generate PAM Samples**
  - **Digitize PAM Samples**
  - **Implement Companding in A/D Converter**
- **PCM Reciever**
  - **Also a Codec**
  - **Perform a D/A Conversion**