Econ Theory
DOI 10.1007/s00199-009-0501-x

RESEARCH ARTICLE

Optimism and firm formation

Luca Rigotti · Matthew Ryan · Rhema Vaithianathan

Received: 11 November 2008 / Accepted: 11 September 2009

Abstract This paper analyses firm formation and innovation in an economy where agents differ with respect to their optimism in the face of ambiguity. Individuals choose between starting a firm or working in one; and also between employing a traditional technology or a new technology about which little is known. In the face of ambiguity, decision-makers are either optimistic or pessimistic. We study the *innovation-proof equilibria* of the economy: wages clear all labor markets when agents make optimal occupational choices, and no mutually beneficial opportunity for innovation remains unexploited. In equilibrium, optimists are more likely to form firms, but also more likely to be *workers* in firms using the ambiguous technology. This phenomenon sheds new light on the relationship between firm culture and technology. We find that three types of firms emerge in equilibrium: entrepreneurial firms, where both owners and workers are optimists operating a highly ambiguous technology; traditional firms, where an optimistic owner employs a pessimistic worker and uses a less ambiguous technology; and bureaucratic firms where both owners and workers are pessimists employing a well-known technology. We also suggest how the relative scarcity of the optimists may help to explain the commonly observed S-shaped diffusion profile for successful innovations.

Keywords Ambiguity · Optimism/pessimism · Entrepreneur · Belief function · Innovation · Diffusion

JEL Classification C62 · D21 · D81 · L23 · O33

L. Rigotti (✉)
Fuqua School of Business, Duke University, 1 Towerview Drive, Durham, NC 27708, USA
e-mail: luca.rigotti@duke.edu

M. Ryan · R. Vaithianathan
Department of Economics, University of Auckland, PB 92019, Auckland, New Zealand

Published online: 25 September 2009

1 Introduction

A hallmark of entrepreneurial behavior is a willingness to take on gambles with imprecisely known odds. The entrepreneur's tolerance of—even affinity for—ambiguity is well documented in the psychology literature, and figures prominently in the work of economists such as Knight (1921) and Kirzner (1997). This contrasts with Kihlstrom and Laffont's (1979) well-known characterization of entrepreneurs as agents with relatively low levels of risk aversion. Schumpeter (1934) denied that risk-taking was part of the entrepreneurial function, and experimental psychologists have shown that owner-managers are no less risk-averse than salaried managers (Wärneryd 1988). Instead, studies find that entrepreneurs are distinguished by traits such as an "internal locus of control", which implies confidence that their talent and effort can turn imprecise odds in their favor (Keenan and McBain 1979; Busenitz 1999; Bhidé 2000; Hansemark 2003).

Bhidé (2000) conducted face-to-face interviews with the founders of 100 successful start-ups. He selected fast-growing, innovative companies, excluding entrepreneurs motivated solely by a desire for self-employment. He concludes that entrepreneurs are distinguished by their confidence in the face of ambiguity.

> " [T]he low ambiguity aversion of the individuals who start promising businesses derives from (or is a manifestation of) exceptionally high levels of self-confidence [...] The self confidence of entrepreneurs however, appears so strong that they are prepared to start a business where they do not have any objective advantage over their rivals." (Bhidé 2000, p. 98)

Self-confidence sometimes appears to be over-confidence. Cooper et al. (1988, p. 103) observe that "entrepreneurs' perceptions of their own odds for success display a noteworthy degree of optimism". In their survey, a remarkable 33% of new business owners assessed their odds of success at 10 out of 10; and only 4% assessed the odds to be less than 5 out of 10. On the empirical evidence available at the time of Cooper et al.'s survey (1988), the average failure rate of US firms within 5 years of start-up lay somewhere between 50 and 71%. Interestingly, 68% of respondents agreed with the statement that their odds of success were "better than those for any other firm like yours". These entrepreneurs clearly have highly optimistic beliefs about their probability of success in business, and this belief is tied to their personal involvement in the business rather than the nature of the business itself.

While this sort of optimism is acknowledged as an important psychological factor in empirical studies of entrepreneurship,[1] theoretical models with optimistic agents are rare. An optimist believes that "whatever I choose will turn out for the best". A pessimist, on the other hand, believes that "my decisions are always ill-fated". Thus, optimists believe they do not need an umbrella because it never rains when they go out; a pessimist takes his umbrella in the belief that, by doing so, rain will be prevented. In this paper, we model such "beliefs" as a particular assignment of probabilities to states, amongst many plausible candidate assignments, with which to evaluate a

[1] For example, Evans and Leighton (1989), de Meza and Southey (1996), Bernardo and Welch (2001), and Moskowitz and Vissing-Jørgensen (2002).

specific action. In this sense, beliefs are choice-dependent rather than fixed—different actions may be evaluated using different beliefs—capturing the notions of pessimism and optimism highlighted above. Nevertheless, beliefs are always confined within rational limits, so that neither optimists nor pessimists are mistaken or irrational.[2] Formally, although the available information leaves room for ambiguity about the relative likelihoods of events, precise upper and lower bounds on these likelihoods are objectively defined.

Decision models for such environments have been axiomatized by Jaffray (1989, 1991, 1994), Hendon et al. (1994) and Jaffray and Wakker (1994). Information is quantified by a *belief function* that summarizes the *set* of probability distributions consistent with the available information about outcomes. When evaluating alternatives, individuals select probabilities from this set in an act-contingent manner: extreme optimists make choices according to the most favorable probability, while extreme pessimists make choices according to the least favorable. In this sense, probabilistic "beliefs" are choice-contingent. In our model, since extreme optimists choose the most favorable belief for their particular circumstances, the observed frequencies of success across optimists engaged in different ventures will necessarily fall short of average optimistic expectations. This is consistent with the evidence cited above but does not rely on optimists having inferior information or possessing an inferior capacity to understand objective information about risk.

In our model of occupational choice, agents are heterogeneous in optimism/pessimism. The economy consists of a range of industries with more or less precise revenue distributions. The risks associated with any given technology are described by a belief function and its implied set of return distributions. The larger the set, the more imprecise the risks. There are two types of agent: optimists, whom we refer to as *Bulls*, and pessimists, the *bears*. Bulls evaluate returns using the most favorable distribution from the relevant set, while bears employ the least favorable prior.

A firm consists of an owner-worker pair operating a particular technology. Due to limited liability constraints, workers may not receive a risk-free wage. If the promised wage is higher than the firm's revenue, the worker is only entitled to realized revenue. Technology adoption is also endogenous: agents who choose to own firms must select a technology. In equilibrium, some technologies may remain idle. The model's predictions about technology and occupational choices are based on the notion of an *innovation-proof equilibrium*. This requires that all labor markets clear, and that no-one can profitably adopt an idle technology (i.e., "innovate"), which seems the natural notion of equilibrium for an entrepreneurial economy. Our first result yields existence of innovation-proof equilibria, which is a non-standard result.

We then use this model to address the correlation between occupation and individual traits.[3] Although workers cannot be fully insured by their owner-managers, we show

[2] In Manove (2000), by contrast, optimistic entrepreneurs over-estimate the productivity of all firms in a given sector relative to the "realists". Closely related is the finance literature on "overconfident" investors, who overestimate the precision of signals about investment returns (e.g., García et al. 2007).

[3] Other models have correlated occupational choice with risk-aversion (Kihlstrom and Laffont 1979), or entrepreneurial ability (Lucas 1978; Holmes and Schmitz 1990; Laussel and Le Breton 1995; Calvo and Wellisz 1980).

that there can be a partial equilibrium tendency for Bulls to precede bears into ownership roles. Nevertheless, general equilibrium effects may outweigh this tendency. Since Bulls' comparative advantage is in tolerance of imprecise risks, they may get higher returns from working in a firm that uses an innovative technology than from owning a firm in a traditional industry where the distribution of returns is precisely known. In other words, workers in start-ups exposed to vague and potentially large bankruptcy risks must share the entrepreneurial spirit of owners.

Finally, we use a simple two-sector example to perform a comparative static exercise to assess the effects of increasing precision of information about a new technology. This may be done parametrically through the specification of the belief function. We examine the equilibrium implications for take-up of the new technology, wages in the "new" and "old" industries, and occupational choices. The example highlights the importance of recognizing the *individual* as entrepreneurial decision-maker. Initially, new industry firms have Bullish owners splitting expected returns equally with their similarly Bullish workers. With a relative scarcity of Bulls, and all roles in the new industry filled by optimists, only a few firms form around the innovative technology in the early stages. However, once ambiguity is sufficiently low, Bull workers are replaced with bears. The displaced Bull workers start their own firms, and the new industry expands more rapidly.

The remainder of the paper is organized as follows. The next section discusses the use of belief functions to characterize ambiguity, and the decision-making rules employed by Bulls and bears. Section 3 defines the notion of an innovation-proof equilibrium for an entrepreneurial economy, and establishes a general existence result. Observations on the correlation between ambiguity tolerance and occupational choice are offered in Sect. 4. Section 5 presents a simple two-sector example which illustrates equilibrium characteristics and comparative static effects. Section 6 concludes. An Appendix contains the lengthier proofs.

2 Individual decision-making

In this section, we describe the decision-making criteria used by agents in our economy. We consider environments in which risks are imprecisely specified, but determined by objective information. Jaffray (1991) refers to this as a situation of *imprecise risk*. Optimism and pessimism are confined within the bounds determined by this commonly known and objective information.

2.1 Imprecise risk

The set of *payoff-relevant states* is Θ, a non-empty and finite set with cardinality $|\Theta|$. The objects of choice are *acts*: money-valued random variables $f : \Theta \to \mathbb{R}_+$. Let (S, Σ, p) be a measure space and $\Gamma : S \twoheadrightarrow \Theta$ a measurable non-empty-valued correspondence. We call S the set of *fundamental states* and Γ the *information correspondence*. These two objects embody the objective information about Θ as follows: if the fundamental state $s \in S$ is realized, the available information implies that the payoff-relevant state lies in the set $\Gamma(s)$, but nothing more than this.

Following Dempster (1967) and Shafer (1976), information about payoff-relevant states is quantified using a *belief function*.[4] In particular, the belief function induced by (S, Σ, p) and Γ is a mapping $\underline{v} : 2^{\Theta} \to [0, 1]$ that a assigns a lower bound to the probability of each payoff-relevant event (i.e., each subset of Θ) as follows:

$$\underline{v}(E) := p(\{s \in S \mid \Gamma(s) \subseteq E\}) \tag{1}$$

Thus, $\underline{v}(E)$ is the *lower probability* of E—the smallest probability one can assign to event E given the available information. One can also define

$$\overline{v}(E) := p(\{s \in S \mid \Gamma(s) \cap E \neq \emptyset\})$$

to be the *upper probability* of E—the largest probability one can attach to E given the available information. The probability of E therefore lies in the interval $[\,\underline{v}(E),\ \overline{v}(E)]$. The function (1) suffices to summarize this interval since

$$\overline{v}(E) = 1 - \underline{v}(E^c) \tag{2}$$

The *core* of the belief function $\underline{v}$ is the set

$$\text{core}(\underline{v}) := \{\pi \in \Delta(\Theta) \mid \pi(E) \geq \underline{v}(E) \ \ \forall E \subseteq \Theta\} \tag{3}$$

where $\Delta(\Theta)$ is the unit simplex in $\mathbb{R}^{\Theta}$ and $\pi(E)$ is shorthand for $\sum_{\theta \in E} \pi_{\theta}$. This set describes the probability measures on Θ that are consistent with the available information.[5]

Example 1 Ellsberg (1961) A ball will be drawn at random from an urn containing 90 balls, of which 30 are red, while the remaining 60 are either yellow or black. Let

$$\Theta = \{\text{red}, \text{yellow}, \text{black}\}$$

denote the payoff-relevant state space and let $\pi(E)$ denote the probability that the ball's color lies in the set $E \subseteq \Theta$. The available information allows us to precisely determine $\pi(\emptyset)$, $\pi(\Theta)$, $\pi(\{\text{red}\})$ and $\pi(\{\text{yellow, black}\})$, but the probabilities of the remaining events are imprecise. All one knows about $\pi(\{\text{yellow}\})$ and $\pi(\{\text{black}\})$, for example, is that each lies in the interval $[0, \frac{2}{3}]$. Similarly, the probabilities of events $\{\text{red, yellow}\}$ and $\{\text{red, black}\}$ lie in $[\frac{1}{3}, 1]$. These upper and lower bounds are precisely determined by the objective information. The lower probabilities are as follows:

4 See Srivastava and Mock (2002) for a straightforward introduction to belief functions, and Mukerji (1997) for a useful discussion in the context of economic decision-making.

5 This terminology acknowledges the analogous concept in cooperative game theory. However, there is some risk of confusion, as Shafer (1976) uses *core* to describe the set

$$\bigcup\{E \subseteq \Theta \mid p(\Gamma^{-1}(E)) > 0\}.$$

$$\underline{v}(\emptyset) = \underline{v}(\{\text{ yellow}\}) = \underline{v}(\{\text{black}\}) = 0$$
$$\underline{v}(\{\text{red}\}) = \underline{v}(\{\text{red, black}\}) = \underline{v}(\{\text{red, yellow}\}) = \frac{1}{3}$$
$$\underline{v}(\{\text{yellow, black}\}) = \frac{2}{3}$$
$$\underline{v}(\Theta) = 1$$

To verify that this is a well-defined belief function, one needs to find a measure space (S, Σ, p) and information correspondence Γ that generate $\underline{v}$. There are many possibilities; for example, let $S = \{1, 2\}$, $\Sigma = 2^S$, $p(\{1\}) = 1-p(\{2\}) = \frac{1}{3}$ and define

$$\Gamma(s) = \begin{cases} \{\text{red}\} & \text{if } s = 1 \\ \{\text{yellow, black}\} & \text{if } s = 2 \end{cases}$$

This induces lower probabilities that exactly match $\underline{v}$ as above.[6]

The construction of a belief function makes explicit the degree of precision of the underlying information. For example, suppose that for a given (S, Σ, p), information correspondences $\hat{\Gamma} : S \twoheadrightarrow \Theta$ and $\Gamma : S \twoheadrightarrow \Theta$ satisfy $\hat{\Gamma}(s) \subseteq \Gamma(s)$ for all $s \in S$. Then $\hat{\Gamma}$ describes a situation with *more precise information*—or *less imprecise risk*—than Γ. At one extreme, if Γ is singleton-valued (a function), then the upper and lower probabilities of events coincide and core($\underline{v}$) is a singleton. We refer to this as a situation of *pure* (or *precise*) *risk*. At the other extreme, if $\Gamma(s) = \Theta$ for every $s \in S$, then we have *pure uncertainty*: core($\underline{v}$) $= \Delta(\Theta)$. In between are situations like the Ellsberg example. Thus, for a given (S, Σ, p), there is a natural partial order on information correspondences that we may exploit to study the comparative static effects of imprecise risk. We shall do so in Sect. 5.

2.2 Decision-making: Bulls and bears

We distinguish two types of agents—*Bulls* and *bears*—according to how they use imprecise risk information. When evaluating an act, Bulls apply expected utility with respect to the most *optimistic* element of core($\underline{v}$) while bears use the most *pessimistic* probability in that set. We assume all agents have the same strictly increasing von Neumann–Morgenstern utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$. If $\mathbb{E}_\pi[g]$ denotes the expected

[6] Although artificial in many respects, this construction is somewhat natural. The information provided does not allow us to identify precise probabilities for each state in Θ, but it does allow us to assign probabilities to elements of the *coarsened* state space

$$\Theta' = \{\{\text{red}\}, \{\text{ yellow, black}\}\}$$

The construction above sets $S = \Theta'$, while Γ describes the coarsening process.

value of $g : \Theta \to \mathbb{R}_+$ with respect to the probability $\pi \in \Delta(\Theta)$, Bulls therefore evaluate an act f using the *upper expected utility*

$$\overline{\mathbb{E}}[u \circ f] := \max_{\pi \in \text{core}(\underline{v})} \mathbb{E}_\pi[u \circ f]$$

while bears employ the *lower expected utility*

$$\underline{\mathbb{E}}[u \circ f] := \min_{\pi \in \text{core}(\underline{v})} \mathbb{E}_\pi[u \circ f].$$

If core$(\underline{v})$ is a singleton—the case of *pure risk*—both types of agent evaluate acts using expected utility with von Neumann–Morgenstern utility function u. Therefore, u encapsulates the risk attitude of the decision-maker in the usual way. The behavioral rules followed by Bulls and bears are different extensions of expected utility to allow for the possibility of imprecise risk information.

The behavioral rules above imply choice-contingent "beliefs" because the π that maximizes $\mathbb{E}_\pi[u \circ f]$ typically varies with f, and likewise the π that minimizes $\mathbb{E}_\pi[u \circ f]$. Here, we think of "beliefs" not in an axiomatic sense but as the probability in core$(\underline{v})$ that generates the upper/lower expectation of the act being evaluated.[7] For instance, a Bull evaluating the returns to betting on yellow in the Ellsberg example would assign probability $2/3$ to {yellow}, but if the same Bull were betting *against* yellow, she would assign probability 0 to {yellow}. Conversely for a bear.

Bulls therefore evaluate wage labor with as much optimism as they evaluate the returns to firm ownership. This optimism does not arise explicitly from any natural entrepreneurial talent on their part, as it does in other models of entrepreneurship.[8] A short example shows how an ambiguous venture may arise, and how Bulls and bears might react to it.

2.2.1 An illustrative example

Scientists working in New Guinea discover a medication for asthma. This medication is 100% effective on subjects possessing gene γ, and completely ineffective otherwise. Gene γ has been identified in some subjects from a remote New Guinea hill tribe. No tests have yet been performed on any other groups for the presence of this gene. There is effectively no evidence on the distribution of this gene in, for example, the U.S. population. An entrepreneur is considering whether or not to leave her job in order to introduce the new asthma medication into the U.S. market. Profit from this new drug is a known function of the *average disposable income* (s_1), the *average level of asthma* (s_2), and the *proportion of the population possessing gene* γ (s_3). Let

---

7 A similar terminology is used in Rigotti et al. (2008).

8 See Holmes and Schmitz (1990), Laussel and Le Breton (1995), Lazear (2004), or Grüner (2003). However, Heath and Tversky (1991) argue that individual competence (such as general knowledge and general skills) may induce an affinity for ambiguity. In that sense, Bullishness may be motivated by greater general knowledge and skills—even though the true distribution of occupational returns is unrelated to these competencies.

$$S = S_1 \times S_2$$

be the fundamental states. Each $s \in S$ identifies an ordered pair containing the average disposable income and average index of prevalence of asthma in the population. The payoff–relevant states are

$$\Theta = S_1 \times S_2 \times S_3$$

where the third component of each ordered triplet designates the proportion of the population possessing gene γ.[9]

The probability p^S over states in S is known by virtue of market survey data. The information mapping implied by the information available to a potential entrepreneur is therefore

$$\Gamma((s_1, s_2)) = \{s_1\} \times \{s_2\} \times S_3$$

for each $(s_1, s_2) \in S$. Let $\underline{v}$ denote the lower probability (belief function) induced by Γ on Θ and note that

$$\text{core}(\underline{v}) = \{\pi \in \Delta(\Theta) \mid \pi((s_1,\ s_2,\ s_3)) \in [0,\ p^S((s_1,\ s_2))]\ \ \forall (s_1,\ s_2,\ s_3) \in \Theta\}$$

Let the profit of the venture be given by the function $f : \Theta \to \mathbb{R}$ defined as

$$f((s_1, s_2, s_3)) = s_1 s_2 + s_3$$

Hence

$$\overline{\mathbb{E}}[u \circ f] = \mathbb{E}_{p^S}[u(s_1 s_2 + 1)]$$

and

$$\underline{\mathbb{E}}[u \circ f] = \mathbb{E}_{p^S}[u(s_1 s_2)]$$

In this case, a Bullish potential entrepreneur evaluates the prospects for the enterprise *as if* believing that *every* U.S. consumer possesses gene γ. A bearish potential entrepreneur evaluates the enterprise *as if* believing that *no one* in the U.S. consumer population possesses gene γ.

2.3 Discussion and related literature

Our paper is related to familiar ideas from the literature on ambiguity. Consider the following formula for evaluating acts in the context of either ambiguity or imprecise risk:

[9] To ensure finiteness, assume that incomes and the asthma index are bounded, and that all figures are rounded to a fixed number of decimal places.

$$\lambda\overline{\mathbb{E}}[u \circ f] + (1-\lambda)\underline{\mathbb{E}}[u \circ f] \tag{4}$$

where $\lambda \in [0, 1]$. Jaffray (1989, 1994) and Hendon et al. (1994) provide axiomatic foundations for this rule. It is usually referred as the Arrow-Hurwicz criterion, after Arrow and Hurwicz (1972), and λ has been called the pessimism-optimism index (see Luce and Raiffa 1957, chap. 13).[10] In our model, Bulls and bears employ this criterion with $\lambda = 1$ and $\lambda = 0$ respectively.[11] Although one may think otherwise, restricting attention to these extremes does not distort our qualitative results. They would be unchanged if one assumed that Bulls use (4) with $\lambda^B \in (\frac{1}{2}, 1)$ while bears use $\lambda^b = 1 - \lambda^B \in (0, \frac{1}{2})$. In that case,

$$\lambda^B\overline{v}(E) + \left(1-\lambda^B\right)\underline{v}(E) = 1 - \left[\lambda^b\overline{v}\left(E^c\right) + \left(1-\lambda^b\right)\underline{v}\left(E^c\right)\right] \quad \text{for any } E \subseteq \Theta$$

so that Bulls and bears continue to employ "dual" or "conjugate" evaluations of event probabilities. Consider the operator

$$\mathbb{E}^{\lambda}[g] := \lambda\overline{\mathbb{E}}[g] + (1-\lambda)\underline{\mathbb{E}}[g]$$

Although $\lambda^b\overline{v} + \left(1-\lambda^b\right)\underline{v}$ may not be a belief function, the operators $\mathbb{E}^{\lambda^B}$ and $\mathbb{E}^{\lambda^b}$ behave just like ordinary upper and lower expectations, so we could replace $\overline{\mathbb{E}}$ and $\underline{\mathbb{E}}$ with $\mathbb{E}^{\lambda^B}$ and $\mathbb{E}^{\lambda^b}$ (respectively) without substantively affecting any of our results.[12]

We model imprecise risk as objective: one can determine probability intervals for each event based on objective information, and all observers agree on these intervals. Belief functions or sets of probabilities are used to describe this objective information. Many theories have been developed to study the extreme case of pure uncertainty, in which such objects arise in the description of individual responses to uncertainty. They describe subjective, rather than objective, phenomena. In Schmeidler (1989),

[10] Following Jaffray and Wakker (1994), we keep the state space Θ explicit in the description of acts. For the extension to infinite state spaces, see Philippe et al. (1999). The recent contribution by Olszewski (2007) axiomatizes criterion (4) for objective ambiguity described by subsets of $\Delta(\Theta)$ that need not be the cores of belief functions. Vierø (2009) extends the Olszewski model to an Anscombe and Aumann (1963) environment in which sets of lotteries may be obtained in the second stage.

[11] The idea that optimism/pessimism corresponds to higher/lower λ is consistent with psychological propensities described by other studies of entrepreneurship. de Meza and Southey (1996), for example, suggest that entrepreneurs will be drawn disproportionately from those of an optimistic disposition. They argue that an optimistic disposition is very much a personal characteristic and that there is a symmetric distribution of "biased beliefs" amongst the population. Entrepreneurs, they claim, are drawn from the more optimistic end. Arabsheibani et al. (2000) provide confirmatory evidence using data from the British Household Panel Survey (1990–1996), which asks respondents to assess whether they are "better off, worse off or about the same financially" as one year ago, and whether they expect to be "better off, worse off or about the same financially" one year hence. By considering two successive years of the panel, one can check expectations against realizations. The self-employed were 4.6 times as likely to forecast an improved financial position but experience a deterioration than to forecast a deterioration but experience an improvement. For employees, the ratio was 2.9. Arabsheibani et al. conclude that "Entrepreneurs do seem to be driven by wishful thinking" (Arabsheibani et al. 2000, p. 40).

[12] In Rigotti et al. (2008) we consider the case of agents who are heterogeneous with respect to λ but who are all somewhat pessimistic in that all have $\lambda < \frac{1}{2}$.

for example, agents use subjective *capacities* (of which belief functions are a special case) to evaluate acts, applying a Choquet expected utility (CEU) criterion. A CEU decision-maker is *uncertainty-averse* if her subjective capacity is convex. In Gilboa and Schmeidler's (1989) maxmin expected utility (MEU), agents construct subjective sets of probabilities and use lower expectations to evaluate acts.[13] An extensive literature studies variants and generalizations of CEU and MEU.[14] There is also a small literature on belief functions as subjective constructs in the characterization of choice under pure uncertainty, including Jaffray and Phillipe (1997) and Ghirardato (2001).

Since a belief function is a convex capacity, Schmeidler's (formal) notion of uncertainty aversion is somewhat related to our (informal) notion of pessimism. Schmeidler also defines the complementary concept of uncertainty appeal which is equivalent to convexity of the conjugate capacity. Thus, uncertainty appeal is the analog of our notion of optimism.[15] Here the similarity ends, however, since the capacities in CEU are subjective, and therefore uncertainty aversion/appeal is conceptually distinct from pessimism/optimism.[16]

Because the objective level of ambiguity is fixed, undertaking comparative statics in the precision of information (as we do) has no analog in the CEU model. In a different sense, however, MEU and CEU are more flexible than the model we consider. In these models, objective information does not exclude any element of $\Delta(\Theta)$—there is pure uncertainty—but agents may subjectively exclude some probabilities. Our model rules this out. A more general framework would admit greater variety in the ambiguity of objective information than do Gilboa and Schmeidler, but would also allow a greater range of subjective responses to objective ambiguity than do we.[17]

Bewley (1989) uses Knightian decision theory to describe entrepreneurs. There, preferences are not necessarily complete and individuals use a set of subjective probability distributions to evaluate state-contingent outcomes. Strict preference obtains when one choice has higher expected utility than the alternative for all probability distributions in the set. If the inequality changes direction for different probabilities, the

---

13 All MEU decision-makers are uncertainty-averse according to Schmeidler's definition, and all uncertainty-averse CEU preferences are within the MEU class.

14 In parts of this literature, the term "ambiguity" is used where we would use "pure uncertainty": to indicate that objective information places no restriction on the probability of any given event (except, of course, for $\emptyset$ and Θ). Elsewhere in this literature, the term "ambiguous" is used to describe an event whose (subjective) probability is not uniquely determined.

15 Schmeidler's definitions of uncertainty aversion/appeal are expressed within an Anscombe–Aumann environment. Wakker (2001) uses *pessimism* and *optimism* to describe conditions on preferences within a Savage framework (in which consequences need not be lotteries) equivalent to convexity of the capacity or its conjugate in a CEU representation.

16 Olszewski (2007, pp. 568–569) observes: "There seems to be no clear way of relating the two approaches, but there is probably a common feeling that they capture a slightly different notion of ambiguity".

17 A recent paper by Gajdos et al. (2008) provides an axiomatic generalization of MEU in this direction. Objective information determines a set of probabilities P from $\Delta(\Theta)$. Agents choose a subjective subset $\varphi(P) \subseteq P$ and then apply the MEU criterion with respect to this subset. Pessimism can be attenuated by the selection $\varphi(P)$: selecting a smaller set (in the sense of set inclusion) will reduce the scope for pessimism. The "size" of the subjectively selected set determines what Gajdos et al. (2008) call the agent's degree of *aversion to bet imprecision*. This model, however, does not capture optimism since it focuses on uncertainty aversion rather than uncertainty appeal.

two alternatives are not comparable. When faced with a new investment opportunity, Bewley suggests an entrepreneur could be an individual who has a tendency to seize it even if investing and not investing are incomparable alternatives. In spirit, this entrepreneur is somewhat similar to our Bulls. In Bewley (1989), however, there is no analysis of the interaction between entrepreneurial and non-entrepreneurial types.[18] Moreover, his theory, unlike ours, does not consider occupational choices or firm formation.

2.4 Some technicalities

We collect here some important facts about belief functions and upper/lower expectations that will be needed in the sequel. Proofs of claims may be found in Schmeidler (1986, 1989). In particular, Schmeidler discusses a generalization of a belief function called a *capacity*. A belief function is a (special type of) *convex* (*2-monotone*) *capacity* (Chateauneuf and Jaffray 1989).

Throughout, we fix a belief function $\underline{v}$ and consider the upper and lower expectation operators defined from $\underline{v}$.

Remark 1 (Algebra of expectation) Clearly, $\underline{\mathbb{E}}[f] \leq \overline{\mathbb{E}}[f]$ for any f. Whenever $\text{core}(\underline{v})$ is a singleton (the case of pure risk), these two operators are identical and coincide with $\mathbb{E}_\pi$, where π is the single element of $\text{core}(\underline{v})$. Otherwise, neither operator is additive. In fact, for any f and g,

$$\underline{\mathbb{E}}[f+g] \geq \underline{\mathbb{E}}[f] + \underline{\mathbb{E}}[g]$$

and

$$\overline{\mathbb{E}}[f+g] \leq \overline{\mathbb{E}}[f] + \overline{\mathbb{E}}[g]$$

A sufficient condition for additivity is *comonotonicity* of f and g:

$$[f(\theta) - f(\theta')][g(\theta) - g(\theta')] \geq 0$$

for all $\theta, \theta' \in \Theta$. In other words, two random variables are comonotone provided they never strictly disagree about the ranking of any two states.

Remark 2 (Choquet expectation) Schmeidler (1986, 1989) describes the operation of Choquet integration with respect to a capacity. This operation coincides with standard (Lebesgue) integration when the capacity is a probability measure. We have that

$$\underline{\mathbb{E}}[f] = \mathbb{E}_{\underline{v}}[f]$$

and

$$\overline{\mathbb{E}}[f] = \mathbb{E}_{\overline{v}}[f]$$

[18] Rigotti (2000) studies the financing problem of such decisive entrepreneurs faced by more cautious bankers.

where $\mathbb{E}_v$ denotes the *Choquet expected value* of f with respect to v, and $\overline{v}$ is the upper probability obtained from the belief function $\underline{v}$ via (2). It is often useful to think about upper and lower expected value operators as Choquet expectations.

Remark 3 (Extreme points of the core) For every bijection $\rho : \Theta \to \{1, 2, \ldots, |\Theta|\}$, there is a probability $\pi^{\rho} \in \text{core}(\underline{v})$ such that

$$\pi^{\rho}(\{\theta \mid \rho(\theta) > k\}) = \min_{\pi \in \text{core}(\underline{v})} \pi(\{\theta \mid \rho(\theta) > k\}) \qquad k = 1, 2, \ldots, |\Theta| - 1 \tag{5}$$

Given ρ we also define ρ^* as follows:

$$\rho^*(\theta) = (|\Theta| + 1) - \rho(\theta)$$

for each $\theta \in \Theta$. Thus, ρ^* orders the states in the opposite sequence to ρ. It is now easy to observe that

$$\underline{\mathbb{E}}[f] = \mathbb{E}_{\pi^{\rho}}[f] \tag{6}$$

and

$$\overline{\mathbb{E}}[f] = \mathbb{E}_{\pi^{\rho^*}}[f] \tag{7}$$

for any f that is comonotone with ρ. This gives yet another useful characterization of the upper and lower expectation operators.

3 The economy

The economy consists of a continuum of agents, indexed by the unit interval. Agents indexed $[0, \alpha)$ are Bulls, where $\alpha \in (0, 1)$, while the remainder are bears. The mass of agents in any Borel subset B of $[0, 1]$ is equal to its Lebesgue measure, denoted $\text{Leb}(B)$. Agents share a common von Neumann–Morgenstern utility function $u : \mathbb{R}_+ \to \mathbb{R}_+$, which we assume to be strictly increasing. There are I productive technologies. Each technology requires the human capital of two agents—an owner and a worker—in order to be productive. A firm is therefore a coalition of two agents operating one of these technologies. Many firms may operate the same technology.[19]

The set of firms employing technology i is called *industry* i. A firm's revenue depends on its technology, the mass of firms in each industry, plus some stochastic factors. Formally, the revenue of a firm in industry i is a mapping $(\theta, \delta) \longmapsto R_i(\theta, \delta) \in \mathbb{R}_{++}$, where $\theta \in \Theta$ is the payoff-relevant state, and δ is a vector with $\delta_i \in \left[0, \frac{1}{2}\right]$ being the mass of firms in industry i.[20] The dependence of revenues on δ captures the effects of output market competition without specifying how this competition works.

[19] Choices of non-labor inputs are suppressed in the analysis.

[20] Since each firm requires two agents and total mass of agents is unity, the mass of any industry must lie in $\left[0, \frac{1}{2}\right]$.

The model is therefore compatible with price-taking or strategic behavior in these markets. It should be noted, however, that we assume exogeneity of demand-side conditions: firm revenues are independent of the equilibrium incomes of agents in the model.

The stochastic properties of the random variable $R_i\,(\cdot, \delta)$ are determined by a fundamental measure space $(S,\ \Sigma,\ p)$ and an information correspondence $\Gamma : S \twoheadrightarrow \Theta$. We take S to be the unit interval $[0, 1]$, Σ the Borel σ-algebra on S, and p the Lebesgue measure on (S, Σ), unless otherwise specified.

Although each random variable $R_i\,(\cdot, \delta)$ is defined on the same state space Θ, this does not imply that each industry is subject to the same source of revenue uncertainty. For example, each $\theta \in \Theta$ may be a vector in $\mathbb{R}^I$, and R_i depend only on the ith component θ_i. In this case, the uncertainty in industry i depends on the precision of the available information about the i^{th} component of θ, and this may be quite different to the uncertainty surrounding the j^{th} component.[21]

Summarizing the discussion so far:

Definition 1 An *economy* is an object

$$\mathcal{E} = \left\{ (S,\ \Sigma,\ p)\ ,\ \Theta,\ \Gamma,\ \{R_i\}_{i=1}^{I}\ ,\ \alpha,\ u \right\}$$

where (i) Γ is a measurable correspondence from S to Θ; (ii) R_i gives the revenue of a typical firm in industry i as a function of (w, δ); and (iii) $\alpha \in (0, 1)$ divides the unit interval into Bulls, agents with indices in the sub-interval $[0, \alpha)$, and bears, agents with indices in the sub-interval $[\alpha, 1]$.

Given an economy $\mathcal{E}$, we shall refer to the upper $(\overline{v})$ and lower $(\underline{v})$ probability on Θ for $\mathcal{E}$. These are constructed from (S, Σ, p) and Γ in the usual way.

The model endogenously determines industry wages, occupational choices and firm formation. Each agent has $2I$ occupational options: wage-earning or firm ownership in one of the I industries. Each agent must choose a single occupation: she cannot divide her time amongst a portfolio of jobs. Workers earn a wage, while firm owners receive revenue less wages paid. Wage contracts are subject to limited liability. That is, we make the following institutional assumption:

Assumption 1 An employment contract in industry i specifies a wage level $w_i \geq 0$ and contains the following limited liability clause: if state $\theta \in \Theta$ is realized, the employer pays the worker

$$\min \{w_i,\ R_i\,(\theta, \delta)\}$$

[21] Correlation in these random shocks would affect agents' occupation choices only if they could divide their time among several jobs. Since agents can work in only one firm, correlation is irrelevant to individual decision-making in our model. Similarly, it is also irrelevant whether shocks to industry i are common or i.i.d. across firms.

Limited liability implies that wage earning need not eliminate income risk.[22] Assumption 1 also imposes restrictions on the form of risk sharing that is feasible through labor contracts. Limited liability explains why different wages may be posted in different industries: the same posted wage may induce different state-contingent incomes in different industries. However, since the default contingencies are identical for firms in the same industry, there are no intra-industry wage differentials.

To evaluate the returns from the various occupational options, agents need to know the vector $w = (w_1, w_2, \ldots, w_I)$ of wages for each industry, and the vector $\delta = (\delta_1, \delta_2, \ldots, \delta_I)$ of industry sizes. Given (w, δ), a Bull anticipates expected utility

$$\overline{\mathbb{E}}\left[u\left(R_i\left(\theta, \delta\right) - \min\left\{w_i,\ R_i\left(\theta, \delta\right)\right\}\right)\right]$$

from owning a firm in industry i, and

$$\overline{\mathbb{E}}\left[u\left(\min\left\{w_i,\ R_i\left(\theta, \delta\right)\right\}\right)\right]$$

from being a worker in the same industry. The bears' payoff expectations are defined similarly, with lower expectations in place of upper expectations.

Let $\mathcal{O} = \{1, 2, \ldots, 2I\}$ be a set of indices for occupations in $\mathcal{E}$. We shall index the occupation of being a firm owner in industry i by $(2i - 1) \in \mathcal{O}$, and the occupation of being a wage earner in industry i by $2i \in \mathcal{O}$. Given an economy $\mathcal{E}$ and vectors (w, δ), we define the optimal occupation choices as follows.

Definition 2 The Bulls' *optimal occupation set* $BR^{\mathcal{E},B}(w, \delta) \subseteq \mathcal{O}$ is the set of occupations that maximize the upper expected utility of income given (w, δ). Let $u^{\mathcal{E},B}(w, \delta)$ denote the upper expected utility of occupations in $BR^{\mathcal{E},B}(w, \delta)$.

Definition 3 The bears' *optimal occupation set* $BR^{\mathcal{E},b}(w, \delta) \subseteq \mathcal{O}$ is the set of occupations that maximize the lower expected utility of income given (w, δ). Let $u^{\mathcal{E},b}(w, \delta)$ denote the lower expected utility of occupations in $BR^{\mathcal{E},b}(w, \delta)$.

In addition to the vectors (w, δ), an equilibrium must also specify the occupation of each agent. This is described using an *allocation function*.

Definition 4 An *allocation function* for the economy $\mathcal{E}$ is a Lebesgue measurable function $\phi : [0, 1] \to \mathcal{O}$.

An equilibrium specifies wages, the mass of firms in each industry and individual occupations such that, when all agents anticipate (w, δ) and each chooses his or her occupation optimally, all labor markets clear and the common expectations of δ are confirmed. Formally:

Definition 5 The triplet (w, δ, ϕ) is an *equilibrium* of $\mathcal{E}$ if

(i) $\phi(j) \in BR^{\mathcal{E},B}(w, \delta) \quad \forall j \in [0, \alpha)$;

[22] Although we have not done so, it would be straightforward to elaborate the model so that agents also have heterogeneous endowments of initial wealth, and must draw on personal wealth to pay wages if necessary (as in Kihlstrom and Laffont 1979).

(ii) $\phi(j) \in BR^{\mathcal{E},b}(w, \delta) \quad \forall j \in [\alpha, 1]$; and
(iii) $\text{Leb}\left[\phi^{-1}(2i)\right] = \text{Leb}\left[\phi^{-1}(2i-1)\right] = \delta_i$ for each $i \in \{1, 2, \ldots, I\}$.[23]

In equilibrium some industries may fail to operate; this happens if the vector δ has some zero components. If $\delta_i = 0$ in equilibrium, how should we interpret the common wage expectation w_i? A rational potential entrant into industry i should assess the wage costs of operating technology i as the minimum wage necessary to attract some other agent away from his or her current occupation. However, there is no reason why w_i should correspond to this wage rate under Definition 5. There may be equilibria in which technologies remain idle because of unreasonable expectations about the wages needing to be paid in order to adopt them.[24] This can be avoided with a natural refinement of Definition 5.

We wish to exclude the possibility of a profitable *innovation*. For any idle technology, it should not be possible for an entrepreneur to improve upon her equilibrium payoff by adopting this technology, and paying a wage equal to the lowest salary necessary to attract some other agent away from his or her current occupation.

Definition 6 Consider an economy $\mathcal{E}$, and let (w, δ, ϕ) be an equilibrium of $\mathcal{E}$. This equilibrium is *innovation-proof* if there does **not** exist an industry i with $\delta_i = 0$, a potential entrepreneur $k \in [0, 1]$, a potential worker $k' \in [0, 1]$ $(k \neq k')$, and a wage level $\hat{w}_i > 0$ such that

(a) $\mathbb{E}_{v_{k'}}\left[u\left(\min\left\{\hat{w}_i, R_i(\theta, \delta)\right\}\right)\right] \geq u^{\mathcal{E},\beta_{k'}}(w, \delta)$ where $\beta_{k'} \in \{b, B\}$ is the type of agent k' and $v_{k'}$ denotes the corresponding upper or lower probability; and
(b) $\mathbb{E}_{v_k}\left[u\left(R_i(\theta, \delta) - \min\left\{\hat{w}_i, R_i(\theta, \delta)\right\}\right)\right] > u^{\mathcal{E},\beta_k}(w, \delta)$ where $\beta_k \in \{b, B\}$ is agent k's type and v_k denotes the corresponding upper or lower probability.

The concept of innovation-proofness represents a conceptual modification of the standard logic of price-taking equilibrium. Potential innovators are assumed not only to know the wages in existing industries, but also the reservation wage necessary to start a firm in any new industry. The (unmodeled) process by which this reservation wage information is promulgated within a market economy is clearly different to the (unmodeled) process by which wages in currently active labor markets are made known. Formulating conjectures about reservation wages in inactive labor markets is fundamental to the process of innovative entrepreneurship. We assume that all agents have perfect knowledge of reservation wage levels in all potential industries, and equal awareness of the revenue functions of inactive industries. Innovation in our model is therefore driven not by differences in agents' awareness of the entrepreneurial opportunities, but by differences in their responses to the ambiguities that surround new technologies.

23 This condition implies $\sum_{i=1}^{I} \delta_i = \frac{1}{2}$.

24 A similar issue arises in Hart (1980) and Makowski (1980), who analyze general equilibrium models where the set of traded commodities is determined endogenously. The market for a particular commodity may not open because the expected price of that commodity is "unreasonable". Likewise, Dubey et al. (2005) study a GEI framework with an endogenous range of traded assets, and the possibility of default. Potential buyers must anticipate the levels of default on different assets, and a market may be inactive on the basis of "unreasonably" pessimistic expectations about price and default risk.

In summary, innovation-proofness ensures that equilibria are robust to lucrative technological innovations when all agents have equal access to information about potential revenue functions, and assess the implicit wage rates in inactive industries at the reservation wage of the cheapest potential worker.[25] Since $\alpha \in (0, 1)$, a non-zero mass of agents would perceive such an opportunity should one exist, and their attempt to seize upon it would disturb the equilibrium.

3.1 Existence of innovation-proof equilibrium

In this section we discuss the existence of innovation-proof equilibrium. To guarantee existence we assume:

Assumption 2 For each i and each $\theta \in \Theta$, the function $R_i(\theta, \delta)$ is continuous in δ.

Theorem 1 *Under Assumptions 1 and 2, every economy $\mathcal{E}$ possesses an innovation-proof equilibrium.*

A detailed proof of Theorem 1 may be found in the Appendix. What follows is a sketch of the basic line of argument.

The proof that an equilibrium in the sense of Definition 5 exists is straightforward. It follows from a standard rational expectations equilibrium problem in (w, δ). The vector giving the Lebesgue measure of agents in each of the $2I$ occupations associated with allocation functions satisfying conditions (i) and (ii) of Definition 5 is a well-behaved function of (w, δ). Thus, a conventional fixed point argument suffices.

To establish the existence of equilibria satisfying innovation-proofness is less standard. To understand why, one needs to see how innovation-proofness might fail. In an equilibrium that is not innovation proof, a technology remains idle if the imputed wage for that (inactive) industry is too high to attract an entrepreneur, but not high enough to lure a worker. How, then, can such a wage be higher than the reservation wage of the "cheapest" potential worker? Precisely when it is above this worker's highest revenue expectation, and just meets his or her reservation utility. Such a wage can be lowered without diminishing its attractiveness, since the worker never expects to receive the full wage anyway. If a potential owner has more optimistic revenue expectations, the lower wage may make the innovation appear strictly more lucrative. Under this scenario, the equilibrium wage rate fails to send a correct signal about the costs of innovation.

The problem just described arises when there are (high) revenue contingencies to which Bulls attach non-zero probability, but bears regard as impossible. The proof of Theorem 1 therefore uses ε-perturbations to the precision of information about risk to ensure that $\underline{v}(E) > 0$ for all non-empty $E \subseteq \Theta$. This guarantees that equilibrium wages in idle sectors never strictly exceed reservation levels, and hence that equilibria of the ε-perturbed economy are innovation-proof. Letting $\varepsilon \to 0$, one obtains a

[25] Consistent with this motivation, innovation-proofness rules out opportunities that are strictly lucrative to the entrepreneur. However, the analysis of the paper goes through under an even stronger refinement: one that precludes any two agents forming a new firm in *any* industry (inactive or otherwise) such that the welfare of this two-agent coalition is Pareto improved.

convergent sub-sequence of equilibria whose limit is an innovation-proof equilibrium of the unperturbed economy.[26]

4 Imprecise risk and firm formation

Kihlstrom and Laffont (1979) study an occupational choice model in which occupational segregation based on risk attitude emerges in equilibrium: all agents in the least risk-averse segment of the population choose to be firm owners. That result obtains under the assumption that wages are not subject to any default: they are completely risk-free. In the following, we establish that in our economy there exists a relationship between occupational choices and optimism under a similar but weaker restriction. For industries in which there is a precise probability attached to the event that revenues are smaller than wages, bears tend to be workers while Bulls tend to be firm owners.[27] Formally:

Proposition 1 *Given an economy $\mathcal{E}$, let i and (w, δ) be such that*

$$\underline{v}\left(\{\theta \mid R_i\left(\theta, \delta\right) \leq z\}\right) = \overline{v}\left(\{\theta \mid R_i\left(\theta, \delta\right) \leq z\}\right) \quad \textit{for all } z \in [0, w_i] \tag{8}$$

Then:

(a) *If bears weakly prefer owning a firm in industry i to working in one, then so do Bulls.*
(b) *If Bulls weakly prefer to work in an industry i firm than to own one, then so do bears.*

The proof of this result is straightforward, so we omit its details. Condition (8) says that there is an unambiguous distribution for industry i revenue up to w_i. This implies

$$\overline{\mathbb{E}}\left[u\left(\min\left\{w_i,\ R_i\left(\theta, \delta\right)\right\}\right)\right] = \underline{\mathbb{E}}\left[u\left(\min\left\{w_i,\ R_i\left(\theta, \delta\right)\right\}\right)\right]$$

Bulls and bears derive the same payoff from working in an industry i firm. Therefore, if there is any ambiguity in the returns to industry i, it only affects firm owners. Proposition 1 follows easily from this observation.

In general, one expects that condition (8) may fail for very new technologies, so workers and owners are both exposed to some imprecise risk. If so, we should not expect a clear segregation of roles according to optimism/pessimism. However, we note the following useful result.

[26] This idea resembles the "trembling-hand" approach employed by Dubey et al. (2005). They induce reasonable price and default expectations for inactive asset markets by introducing ε-traders, who buy and sell small quantities of all assets, meeting all obligations in all states. Letting $\varepsilon \to 0$ does the needful.

[27] In Kihlstrom and Laffont (1979), revenues are larger than wages with (precise) probability one in the only industry present in their model.

Proposition 2 *Consider an economy $\mathcal{E}$ and industry i such that there exists a partition $\{A, B\}$ of Θ and functions $\underline{r} : \left[0, \frac{1}{2}\right]^I \to \mathbb{R}_{++}$ and $\overline{r} : \left[0, \frac{1}{2}\right]^I \to \mathbb{R}_{++}$ satisfying*

$$\underline{r}\,(\delta) \leq \overline{r}\,(\delta)$$

and

$$R_i\,(\theta, \delta) \;=\; \begin{cases} \underline{r}\,(\delta) & \text{if } \theta \in A \\ \overline{r}\,(\delta) & \text{if } \theta \in B \end{cases}$$

for all δ. Then, in any equilibrium (innovation-proof or otherwise) in which $\delta_i > 0$:

(a) *If bears weakly prefer owning a firm in industry i to working in one, then so do Bulls.*
(b) *If Bulls weakly prefer to work in an industry i firm than to own one, then so do bears.*

Proof We will prove only (a) since case (b) is similar.

First, observe that the random variables

$$u\,(R_i\,(\theta, \delta) - \min\{w_i,\; R_i\,(\theta, \delta)\})$$

and

$$u\,(\min\{w_i,\; R_i\,(\theta, \delta)\})$$

are comonotone—indeed, each is comonotone with $R_i\,(\theta, \delta)$. Recalling (6), there is a bijection $\rho : \Theta \to \{1, 2, \ldots, |\Theta|\}$ such that

$$\underline{\mathbb{E}}\,[u\,(R_i\,(\theta, \delta) - \min\{w_i, R_i\,(\theta, \delta)\})] \;\geq\; \underline{\mathbb{E}}\,[u\,(\min\{w_i, R_i\,(\theta, \delta)\})]$$

if and only if

$$\mathbb{E}_{\pi^\rho}\,[u\,(R_i\,(\theta, \delta) - \min\{w_i, R_i\,(\theta, \delta)\}) - u\,(\min\{w_i, R_i\,(\theta, \delta)\})] \;\geq\; 0 \tag{9}$$

and

$$\overline{\mathbb{E}}\,[u\,(R_i\,(\theta, \delta) - \min\{w_i, R_i\,(\theta, \delta)\})] \;\geq\; \overline{\mathbb{E}}\,[u\,(\min\{w_i, R_i\,(\theta, \delta)\})]$$

if and only if

$$\mathbb{E}_{\pi^{\rho^*}}\,[u\,(R_i\,(\theta, \delta) - \min\{w_i, R_i\,(\theta, \delta)\}) - u\,(\min\{w_i, R_i\,(\theta, \delta)\})] \;\geq\; 0 \tag{10}$$

In particular, we may assume that ρ orders all states in A before any state in B; while ρ^* orders all states in B before any state in A.

Now, for $\delta_i > 0$ to hold in equilibrium, at least one type must be prepared to own an industry i firm. For this to be the case, it is necessary that $\overline{r}(\delta) \geq 2w_i$. If not, then an industry i worker earns strictly more than his or her employer in every state, so inequalities (9) and (10) would both be violated.

We next observe that the function

$$h(x) = u(x - \min\{w_i, x\}) - u(\min\{w_i, x\})$$

is strictly increasing in x when $x \geq w_i$ and $h(x) \gtreqless 0$ as $x \gtreqless 2w_i$. Since $\overline{r}(\delta) \geq 2w_i$ and $\underline{r}(\delta) \leq \overline{r}(\delta)$, it follows that $h(R_i(\theta, \delta))$ is comonotone with $R_i(\theta, \delta)$. Recalling the properties of ρ and ρ^* mentioned at the end of the previous paragraph, inequality (9) clearly implies inequality (10). □

The condition in Proposition 2 says that, for any δ, industry i revenue can take at most two values: it is low if $\theta \in A$ or high if $\theta \in B$. One may think of an enterprise that either succeeds or fails. If this condition is satisfied, then at any equilibrium, optimists have a relatively stronger affinity for ownership roles in industry i than do pessimists: a bear can only employ a Bull in an industry i firm if both are indifferent about which role to take.

In the following section, we shall consider an economy with two industries: a traditional industry ($i = 1$) which uses an unambiguous technology, and an innovative industry ($i = 2$) whose technology satisfies the condition in Proposition 2. Excluding borderline cases, firms in the innovative sector will therefore fall into one of three categories:

Definition 7 Typology of firm structures:

(i) *entrepreneurial* firms, in which both owner and worker are Bulls;
(ii) *traditional* firms, in which a Bull employs a bear; and
(iii) *bureaucratic* firms, in which both owner and worker are bears.

We may think of these different types of firm as embodying distinct corporate cultures. We study the comparative statics of this simple economy as the imprecision surrounding the innovative technology diminishes, which one expects to happen over time. The reduction in imprecision encourages greater take-up of the new technology. We show how this process of diffusion is accompanied by significant firm creation and destruction, and the evolution of firm culture within the innovative sector from entrepreneurial through traditional to bureaucratic.

5 Imprecision and innovation

In this section we consider comparative statics in imprecision. Recall that for a given fundamental measure space (S, Σ, p), the information correspondence Γ determines the imprecision of risk information about the payoff-relevant state. As the graph of Γ gets smaller (in the sense of set inclusion), information becomes more precise: the set core$(\underline{v})$, which contains the distributions on Θ consistent with the objective information, shrinks. Increased precision of information will reduce the effects of optimism and pessimism on choice behavior.

Using a simple example, we explore the effects of reduction of imprecision on innovation-proof equilibrium. From this comparative static exercise, we may draw some lessons about the diffusion of new innovations. One can naturally expect that imprecision diminishes as a new technology is adopted by more and more firms. Hence, understanding how imprecision affects innovation-proof equilibrium sheds light on this aspect of the diffusion process.

A two-state, two-technology example conveys the main ideas and delivers a surprisingly rich picture. We assume one of the technologies is well-known, with a deterministic revenue function (the old industry); while the other is a new innovation exposed to imprecise risks (the new industry). We examine how reductions in the imprecision surrounding the new technology affect innovation-proof equilibrium. What emerges is a natural evolution of firm structure in the new industry. When imprecision is large, firms are entrepreneurial; at medium levels of imprecision, firms are traditional; when imprecision of risk is small, firms are bureaucratic. These transitions are associated with a distinctive pattern in new industry wages. Labor costs are initially high to prevent Bullish workers from leaving to run their own new industry start-ups. Once imprecision has fallen sufficiently, bears are employed in the new industry firms, allowing the Bull owners to realize an expected surplus. The accompanying upswing in adoption of the new technology is reminiscent of the commonly observed S-shaped diffusion profile for (successful) innovations.

5.1 The economy

There are two payoff-relevant states ($\Theta = \{H, L\}$) and two technologies ($I = 2$). We index variables by "o " (old) and "n" (new), instead of 1 and 2, for ease of exposition. The old technology has deterministic revenue function

$$R_o(\theta, \delta) = 10 - 2\delta_o$$

while the new technology has state-dependent revenue given by

$$R_n(\theta, \delta) = \begin{cases} 21 - 4\delta_n & \text{if } \theta = H \\ 1 & \text{if } \theta = L \end{cases}$$

The new technology therefore fails in state L, with a scrap value of \$1. It succeeds in state H.

Imprecise risk is indexed by $\varepsilon \in [0, 1]$. The fundamental state space is $S = [0, 1]$, endowed with the Borel σ-algebra (Σ) and Lebesgue measure. Information about Θ is described by the correspondence:

$$\Gamma^{\varepsilon}(s) = \begin{cases} \{H\} & \text{if } s \in \left[0, \frac{3\varepsilon}{4}\right] \\ \{L\} & \text{if } s \in \left(\frac{3\varepsilon}{4}, \varepsilon\right) \\ \{H, L\} & \text{if } s \in [\varepsilon, 1] \end{cases}$$

Based on this information, the probability of state H is known to lie in the interval

$$\left[\frac{3\varepsilon}{4}, 1 - \frac{\varepsilon}{4}\right]$$

and the probability of state L to lie in the interval

$$\left[\frac{\varepsilon}{4}, 1 - \frac{3\varepsilon}{4}\right]$$

These intervals shrink—and therefore imprecision falls—as ε increases. If $\varepsilon = 0$, there is pure uncertainty: both intervals are $[0, 1]$. When $\varepsilon = 1$, there is no uncertainty (pure risk): states L and H have probabilities $\frac{1}{4}$ and $\frac{3}{4}$ respectively. In the latter case, the new technology is recognized as superior by Bulls and bears alike, since

$$\underline{\mathbb{E}}\left[R_n\left(\theta, \delta\right)\right] = \overline{\mathbb{E}}\left[R_n\left(\theta, \delta\right)\right] = 16 - 3\delta_n \;>\; 10 - 2\delta_o$$

for any δ_o and δ_n in $\left[0, \frac{1}{2}\right]$.

Finally, we assume Bullish optimism is comparatively rare ($\alpha < \frac{1}{2}$) and we let u be the identity function (risk neutrality) for convenience.

Thus, we define the following economy, indexed by parameter ε:

$$\mathcal{E}^{\varepsilon} \;=\; \left\{\left(S = [0, 1], \Sigma, \text{Leb}\right), \; \Theta = \{H, L\}, \; \Gamma^{\varepsilon}, \; \{R_o, R_n\}, \; \alpha \in \left(0, \frac{1}{2}\right), \; u \equiv \text{Id}\right\}$$

In the following, we use $\overline{\mathbb{E}}^{\varepsilon}$ and $\underline{\mathbb{E}}^{\varepsilon}$ to denote the upper and lower expectations with respect to the belief function $\underline{v}^{\varepsilon}$ induced on Θ by Γ^{ε}.

5.2 The innovation-proof equilibrium

To undertake a comparative static analysis, we need to establish the existence of a unique innovation-proof equilibrium of $\mathcal{E}^{\varepsilon}$ for each $\varepsilon \in [0, 1]$. Existence is guaranteed by Theorem 1. One cannot hope for uniqueness in the specification of the allocation functions, but we will show that δ and the wages in active industries are unique. In fact, unless $\varepsilon = 1$ (in which case Bulls and bears have identical preferences), the mass of each type of agent assigned to each occupation is also unique. Thus, all the relevant "macro" variables are uniquely determined.

Fix some $\varepsilon \in [0, 1]$ and let (w^*, δ^*) be the equilibrium wages and industry sizes in some innovation-proof equilibrium of $\mathcal{E}^{\varepsilon}$. We shall establish some properties of (w^*, δ^*).

First, unless the old industry is inactive, equilibrium wages in this sector are uniquely determined.

Proposition 3 *If $\delta_o^* > 0$, owners and workers share revenue equally in old industry firms. That is:*

$$w_o^* = \frac{1}{2} R_o \left(\theta, \delta^*\right) = 5 - \delta_o^* \tag{11}$$

Proof If this were not true, one of the old industry occupations would be strictly preferred to the other by all agents. □

Since the old industry exhibits no imprecision in returns, Proposition 3 establishes a common benchmark ($5 - \delta_o^*$) against which both types of agent evaluate the returns to occupations in the new sector (at least when $\delta_o^* > 0$). For Bulls, there will always be a more attractive option in the new industry.

Proposition 4 *In any equilibrium, all Bulls are allocated to roles in the new industry and hence* $\delta_n^* \geq \frac{\alpha}{2}$.

Proof This is obvious if $\delta_o^* = 0$, so suppose $\delta_o^* > 0$. Since

$$\overline{\mathbb{E}}^{\varepsilon} \left[R_n \left(\theta, \delta\right)\right] = 1 + \left(1 - \frac{\varepsilon}{4}\right) \left(20 - 4\delta_n\right) = 21 - 4\delta_n - \left(5 - \delta_n\right) \varepsilon \tag{12}$$

straightforward calculations show that $\frac{1}{2}\overline{\mathbb{E}}^{\varepsilon}[R_n(\theta, \delta)] > 7$, which strictly exceeds $5 - \delta_o^*$ for any δ. Therefore, recalling Proposition 3, Bulls can always find a role in the new industry that they strictly prefer to any old industry occupation. □

Proposition 4 establishes that the new industry must be active in equilibrium. Suppose that $\delta_o^* = 0$. Since $R_o(\theta, 0) = 10$, the most attractive old industry occupation (given $\delta_o^* = 0$ and w_o^*) gives a payoff at least 5 to either type of agent. Since $\delta_o^* = 0$, then both types must be getting payoff at least 5 in their current (new industry) occupations. It is obvious that there is no $w_o \neq w_o^*$ such that two agents can Pareto improve upon their current situation by forming an old industry firm. In other words:

Proposition 5 *Any equilibrium is innovation-proof. Moreover, we may assume that* $w_o^* = 5 - \delta_o^*$*, even if* $\delta_o^* = 0$.

It remains to prove that there is a unique equilibrium value for the pair (w_n^*, δ_n^*). Given this, we then obtain δ_o^* from the relation $\delta_o^* = \frac{1}{2} - \delta_n^*$ and w_o^* from (11). We proceed into two steps. First, we show that w_n^* is uniquely determined as a function of δ_n^*, and then we deduce that δ_n^* is unique.

5.2.1 Market clearing wages in the new industry

The objective of this section is to show that for given δ_n and ε there is a unique w_n that clears the new industry labor market (although it need not clear it at level δ_n). We begin by constructing bounds for this wage. To do so, we define three wage functions: $\overline{W}_n(\delta_n, \varepsilon)$ is the wage that makes Bulls indifferent between working in or owning firms in industry n; $\overline{w}_n(\delta_n, \varepsilon)$ is the wage that makes bears indifferent between working in or owning firms in industry n; and $\hat{w}_n(\delta_n, \varepsilon)$ is the wage that makes bears indifferent between any job in industry o and working in industry n.[28]

[28] See Table 1.

Proposition 6 *The wage that clears the labor market in industry n cannot be higher than $\overline{W}_n(\delta_n, \varepsilon)$ and cannot be lower than $\overline{w}_n(\delta_n, \varepsilon)$.*

Proof We first construct the labor supply functions for Bulls. Recall that $\overline{W}_n(\delta_n, \varepsilon)$ is the wage which solves

$$\overline{\mathbb{E}}^{\varepsilon}\left[R_n(\theta,\delta) - \min\{w_n, R_n(\theta,\delta)\}\right] = \overline{\mathbb{E}}^{\varepsilon}\left[\min\{w_n, R_n(\theta,\delta)\}\right]$$

By straightforward calculations:

$$\overline{W}_n(\delta_n, \varepsilon) = 11 - 2\delta_n - \frac{2}{(4-\varepsilon)} \tag{13}$$

If w_n exceeds (13), all Bulls *strictly* prefer wage-labor to firm ownership. Proposition 2 then implies that no-one is willing to own a new industry firm, and this is inconsistent with equilibrium. For wage levels below $\overline{W}_n(\delta_n, \varepsilon)$, all Bulls own firms. If $w_n = \overline{W}_n(\delta_n, \varepsilon)$ the labor supply of Bulls (collectively) may take any value in $[0, \alpha]$. Thus, $\overline{W}_n(\delta_n, \varepsilon)$ provides an upper bound for the market clearing wage paid by firms.

Now consider the labor supply function for bears. It depends on *two* outside options. The wage at which bears are indifferent between owning a firm and being workers in n solves

$$\underline{\mathbb{E}}^{\varepsilon}\left[R_n(\theta,\delta) - \min\{w_n, R_n(\theta,\delta)\}\right] = \underline{\mathbb{E}}^{\varepsilon}\left[\min\{w_n, R_n(\theta,\delta)\}\right]$$

This gives:

$$\overline{w}_n(\delta_n, \varepsilon) = 11 - 2\delta_n - \frac{2}{3\varepsilon}$$

The wage that makes bears indifferent between being workers in n and an old industry occupation solves

$$\underline{\mathbb{E}}^{\varepsilon}\left[\min\{w_n, R_n(\theta,\delta)\}\right] = 5 - \delta_o$$

Hence

$$\hat{w}_n(\delta_n, \varepsilon) = 1 + \left(\frac{14 + 4\delta_n}{3\varepsilon}\right) \tag{14}$$

Using Eq. (14), one can derive a lower bound for the market-clearing w_n. The labor supply to industry n is non-zero only if $w_n \geq \overline{w}_n(\delta_n, \varepsilon)$. If $w_n < \overline{w}_n(\delta_n, \varepsilon)$, bears do not choose to be workers in new industry firms, and neither, by Proposition 2, do Bulls. Therefore, $\overline{w}_n(\delta_n, \varepsilon)$ provides a lower bound for the equilibrium wage paid by firms in industry n. □

Table 1 Reservation wage functions

Function	Value	Definition
$\overline{W}_n\,(\delta_n,\,\varepsilon)$	$11\,-\,2\delta_n\,-\,\frac{2}{(4-\varepsilon)}$	Bulls indifferent between owning and working in *n*
$\overline{w}_n\,(\delta_n,\,\varepsilon)$	$11\,-\,2\delta_n\,-\,\frac{2}{3\varepsilon}$	Bears indifferent between owning and working in *n*
$\hat{w}_n\,(\delta_n,\,\varepsilon)$	$1\,+\,\frac{(14+4\delta_n)}{3\varepsilon}$	Bears indifferent between working in *n* or *o*

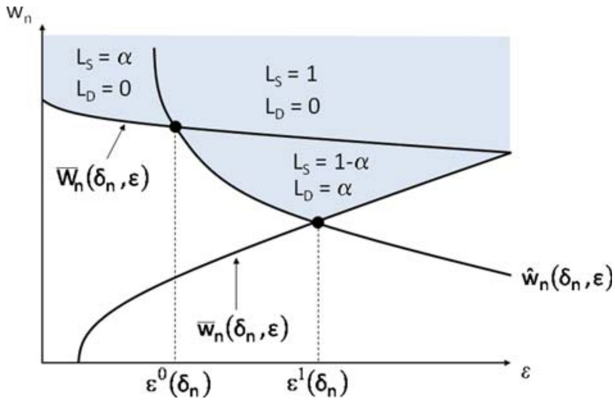

Fig. 1 New economy labor market conditions

For ease of reference, we summarize the definitions of these three reservation wage functions in Table 1.

Let $\varepsilon^0(\delta_n)$ be the value of ε such that $\overline{W}_n(\delta_n,\,\varepsilon) = \overline{w}_n(\delta_n,\,\varepsilon)$, and let $\varepsilon^1(\delta_n)$ be the value of ε such that $\overline{w}_n(\delta_n,\,\varepsilon) = \hat{w}_n(\delta_n,\,\varepsilon)$.[29] Figure 1, sketches the functions $\overline{W}_n(\delta_n,\,\varepsilon)$, $\overline{w}_n(\delta_n,\,\varepsilon)$ and $\hat{w}_n(\delta_n,\,\varepsilon)$ for a fixed δ_n. L_S and L_D denote labor supply and demand in the new industry.[30] The shaded area (boundary included) represents wage rates at which there is a positive labor supply. Since all Bulls are in industry n, w_n^* must lie in this area. As noted above, it must also lie between $\overline{W}_n(\delta_n,\,\varepsilon)$ and $\overline{w}_n(\delta_n,\,\varepsilon)$.

From Fig. 1, one can also deduce that for any ε, the labor market in industry n can only clear at the wage rate on the lower boundary of the shaded area. Above it, either labor demand is zero (in the regions with $L_S = \alpha$ or $L_S = 1$, including their mutual boundary), or labor demand is positive but falls short of supply (since $1 - \alpha > \frac{1}{2}$). The following Proposition uses Fig. 1 to summarize how the labor market behaves in the new industry industry.

Proposition 7 *For a given value of* δ_n, *the labor market in industry n clears as follows:*

[29] One can verify from Eqs. (13)–(14) that these values are uniquely defined, and $\varepsilon^1(\delta_n) > \varepsilon^0(\delta_n)$.

[30] Note that the indicated labor supply and demand levels only apply away from the boundaries of the various regions.

1. *If* $\varepsilon < \varepsilon^0(\delta_n)$ *then* $w_n = \overline{W}_n(\delta_n, \varepsilon)$*, all bears are in industry o and Bulls are indifferent between ownership and wage-labor in industry n. Hence, the market for new industry labor clears with mass* $\frac{\alpha}{2}$ *of firms.*
2. *If* $\varepsilon = \varepsilon^0(\delta_n)$ *the market clears at* $w_n = \overline{W}_n(\delta_n, \varepsilon)$ *with any mass in* $\left[\frac{\alpha}{2}, \alpha\right]$.
3. *If* $\varepsilon^0(\delta_n) < \varepsilon < \varepsilon^1(\delta_n)$ *then* $w_n = \hat{w}_n(\delta_n, \varepsilon)$*, all Bulls are firm-owners in n and employ mass* α *of the bears. The remaining bears are happy to work in the old industry.*
4. *If* $\varepsilon = \varepsilon^1(\delta_n)$ *the market clears at* $w_n = \hat{w}_n(\delta_n, \varepsilon)$ *with any mass in* $\left[\alpha, \frac{1}{2}\right]$.
5. *If* $\varepsilon^1(\delta_n) < \varepsilon < 1$ *then* $w_n = \overline{w}_n(\delta_n, \varepsilon)$*, all agents are in the new industry, so the mass of firms is* $\frac{1}{2}$*. All Bulls own firms while bears are divided between ownership and wage-labor.*
6. *If* $\varepsilon = 1$ *the market clears with* $w_n = \overline{w}_n(\delta_n, \varepsilon)$ *and mass* $\frac{1}{2}$*, but the allocation of bears and Bulls to occupations is otherwise indeterminate.*

5.2.2 Equilibrium wages and industry sizes

Since we have established that w_n^* is uniquely determined as a function of δ_n^*, all that remains is to show that δ_n^* is unique. Barring a few details, everything that follows is a direct consequence of Proposition 7.

If $\varepsilon < \varepsilon^0\left(\frac{\alpha}{2}\right)$, Proposition 7(1) implies equilibrium values $w_n^* = \overline{W}_n\left(\frac{\alpha}{2}, \varepsilon\right)$ and $\delta_n^* = \frac{\alpha}{2}$. Notice that all new industry firms are of the *entrepreneurial* type in this equilibrium: Bulls employ Bulls. The high level of imprecision keeps bears in the old industry, so Bull employers must pay wages that just keep Bullish employees from starting their own firms.

If $\varepsilon \in \left[\varepsilon^0\left(\frac{\alpha}{2}\right), \varepsilon^0(\alpha)\right]$ then $\varepsilon^0(\delta_n^*) = \varepsilon$, which implies $\delta_n^* \in \left[\frac{\alpha}{2}, \alpha\right]$.[31] Since ε^0 is strictly increasing in δ_n, the equation $\varepsilon^0(\delta_n^*) = \varepsilon$ uniquely determines δ_n^*, which is increasing in ε. Proposition 7(2) implies $w_n^* = \overline{W}_n(\delta_n, \varepsilon)$ in this case. As ε increases over the range $\left[\varepsilon^0\left(\frac{\alpha}{2}\right), \varepsilon^0(\alpha)\right]$, bearish workers coming from the old industry gradually replace Bull workers, the latter starting up their own (new technology) firms instead. As this transition takes place, δ_n rises. For Bull owners, the rise in ε and δ_n both exert downward pressure on expected profit—the former by reducing the scope for optimism, the latter by increasing competition. Bull workers also become less optimistic about avoiding wage default. One can show that the first effect dominates, and w_n^* falls in order to maintain Bulls' indifference between ownership and labor. This fall also keeps bears indifferent between jobs in the old or new industry: as δ_n rises, δ_o decreases and therefore returns in the old industry go up, while the rise in ε lowers perceived default risks in the new industry.

If $\varepsilon \in \left(\varepsilon^0(\alpha), \varepsilon^1(\alpha)\right)$, Proposition 7(3) implies $w_n^* = \hat{w}_n(\alpha, \varepsilon)$ and $\delta_n^* = \alpha$. New industry firms are *traditional* in structure: Bulls employ bears. Bulls strictly prefer ownership, while ambiguity is low enough for bears to be hired as workers. The Bull owners of such firms perceive an expected surplus from ownership.

[31] If $\varepsilon < \varepsilon^0(\delta_n^*)$, then $\delta_n^* = \alpha/2$, which yields a contradiction. Similarly, if $\varepsilon > \varepsilon^0(\delta_n^*)$, then $\delta_n^* \geq \alpha$, which again yields a contradiction since ε^0 is strictly increasing in δ_n. Therefore $\varepsilon = \varepsilon^0(\delta_n^*)$.

If $\varepsilon \in \left[\varepsilon^1(\alpha), \varepsilon^1\left(\frac{1}{2}\right)\right]$, an argument analogous to the one in footnote 31 shows that $\varepsilon = \varepsilon^1(\delta_n^*)$. This equation can be solved for δ_n^*, which is increasing in ε over the range $\left[\alpha, \frac{1}{2}\right]$. Proposition 7(4) implies $w_n^* = \hat{w}_n(\delta_n, \varepsilon)$. In this case, bears are making the transition to ownership roles in the new industry, while the old industry is gradually vanishing under pressure from the successful innovation. During this phase, w_n^* continues to fall.

If $\varepsilon > \varepsilon^1\left(\frac{1}{2}\right)$, Proposition 7(5) implies $w_n^* = \overline{w}_n\left(\frac{1}{2}, \varepsilon\right)$ and $\delta_n^* = \frac{1}{2}$. We observe a mixture of *traditional* and *bureaucratic* firms in the new industry.

Finally, if $\varepsilon = 1$, Proposition 7(6) implies $w_n^* = \overline{w}_n(\delta_n, \varepsilon)$ and density $\delta_n^* = \frac{1}{2}$.

Since $\alpha \in \left(0, \frac{1}{2}\right)$, one can verify that

$$\varepsilon^0\left(\frac{\alpha}{2}\right) < \varepsilon^0(\alpha) < \varepsilon^1(\alpha) < \varepsilon^1\left(\frac{1}{2}\right)$$

Therefore, we have fully described the equilibrium and shown, by construction, that all "macro" variables are uniquely determined.

In the next section, we examine the behavior of equilibrium as ε varies.

5.3 Comparative static results

In any equilibrium, the old industry firms are always bureaucratic, sharing revenue equally between owner and worker. The interesting effects of imprecise risk are on the new industry, so we focus on this sector in what follows.

5.3.1 Wages and firm structure

Figure 2 describes how the equilibrium wage level is affected by different levels of precision. When imprecision is very high (for $\varepsilon \leq \varepsilon^0\left(\frac{\alpha}{2}\right)$), bears are unwilling to enter the new industry. Innovation is left to Bulls, who share expected revenue equally in

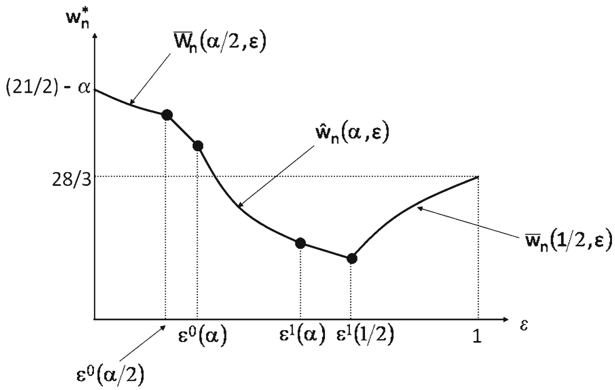

Fig. 2 Equilibrium comparative statics for w_n^*

entrepreneurial firms. As ε rises, Bullish optimism about the returns to the innovative technology is confined within progressively tighter bounds. This results in a downward trend in wages. This feature of equilibrium appears realistic. Start-ups are often observed to employ workers who share the sentiments of the owners regarding the prospects of the industry. Therefore, innovation is constrained not only by the number of adventurous firm-owners in the economy, but also the availability of an equally adventurous workforce.

At somewhat lower levels of imprecision—for $\varepsilon \in \left(\varepsilon^0\left(\frac{\alpha}{2}\right), \varepsilon^1(\alpha)\right]$—bears are attracted into the new industry as workers. Bulls become an expensive labor input because they perceive a lucrative outside opportunity as owners of rival firms. Since bears are less optimistic about the returns to firm ownership, they have a lower reservation wage (tied to returns in the old industry). Thus, wages drop more sharply than before. This downward trend reflects the falling default risk perceived by bears, rather than Bulls' reduced optimism about the innovative technology.

After an initial transition phase, all firms in the new industry have a *traditional* structure: Bulls hire bears. From Fig. 1 we observe that $w_r^* < \overline{W}_r(\delta_r, \varepsilon)$, so Bulls earn an expected surplus from firm ownership. In other words, the new industry attracts Bulls even at very high levels of ambiguity, but ownership yields an expected surplus only when ambiguity—and hence default risk—is low enough for pessimistic workers to be employed. The advantage of the traditional structure is that it exploits the different attitudes to ambiguity of owner and worker.

Finally, when $\varepsilon > \varepsilon^1(\alpha)$, imprecision is sufficiently low that bears are willing to own new industry firms. This represents the new industry's mature phase, as its comparative advantage over the old is perceived by everyone: when $\varepsilon > \varepsilon^1\left(\frac{1}{2}\right)$ all agents are in the new industry. Since Bulls are relatively scarce ($\alpha < \frac{1}{2}$) some bears must own firms that employ other bears and therefore $w_r = \overline{w}_r(\delta_r, \varepsilon)$. As ambiguity goes down, bears perceive the profitability of the innovative technology to rise. Hence, wages rise as ambiguity decreases. Meanwhile, the expected surplus to ownership as perceived by the Bulls is being eroded; ultimately vanishing when $\varepsilon = 1$.

Table 2 summarizes the previous discussion. In our model, only two features of the economy are exogenous: the limited liability structure of the wage contract and the number of technologies. Firm formation and occupational choice, on the other hand, are fully determined by the forces of equilibrium. The resulting relationship between imprecise risk and firm culture is very natural (see Table 2), and it suggests that this simple model captures an important force in the emergence of innovative firms.

Table 2 Structure of firms in equilibrium

Precision of information	Firm structures
$0 \le \varepsilon \le \varepsilon^0(\alpha/2)$	Entrepreneurial (Bulls hire Bulls)
$\varepsilon^0(\alpha/2) < \varepsilon < \varepsilon^0(\alpha)$	Entrepreneurial and Traditional (Bulls hire bears)
$\varepsilon^0(\alpha) \le \varepsilon \le \varepsilon^1(\alpha)$	Traditional
$\varepsilon^1(\alpha) < \varepsilon \le 1$	Traditional and Bureaucratic (bears hire bears)

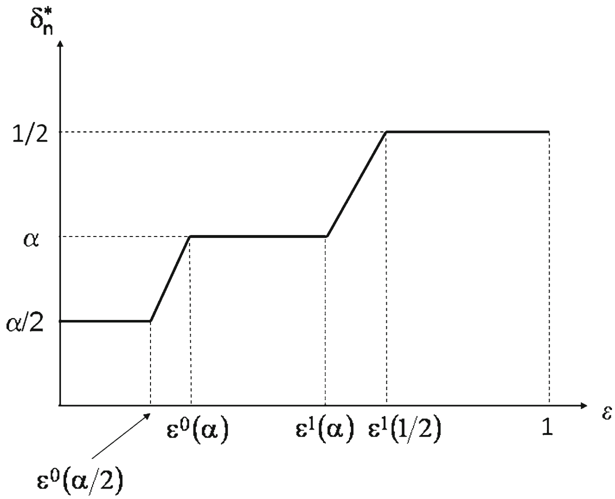

Fig. 3 Equilibrium comparative statics in δ_n^*

5.3.2 Diffusion of innovations

As firms innovate they release information about their product or technology. This allows the calculation of more precise odds about the innovation's value and may promote or discourage further innovators. Although our model is static, one can think of ε as representing not only the precision of information about the innovative technology but also (and simultaneously) the passage of time. Time $\varepsilon = 0$ denotes the point at which the new technology is discovered. This interpretation of ε stems from assuming that the use of a new technology gradually releases information to the market; it thus allows more accurate assessments of the technology's revenue-generating properties.[32]

As time goes by, there is increasing precision of risk information and Table 2 reveals that significant structural change occurs over the diffusion path of innovations. After time $\varepsilon = 0$, and at least until time $\varepsilon = \varepsilon^1(\alpha)$, all firms in the old industry are bureaucratic while the firms forming in the new industry are entrepreneurial or traditional in structure. Thus, no firm moves intact from the old to the new sector over this period: innovation occurs because new firms are formed.

Figure 3 represents the equilibrium mass of firms in the new industry, and suggests an interesting factor contributing to the commonly observed S-shaped diffusion profile. Diffusion is initially slow because both owners and workers are optimistic types. All members of the firm must share an optimistic vision when ambiguity is high, and such optimists are not abundant ($\alpha < \frac{1}{2}$). Also, as entrepreneurial firms share (upper) expected revenues equally, if wages are too low, Bullish workers will quit to start their own firms. Therefore, entrepreneurial firms are relatively expensive to run. Diffusion accelerates once imprecision becomes low enough that bears perceive wages in the new industry to be sufficiently free of default risk. At $\varepsilon^0\ (\alpha/2)$ the possibility of

[32] Recall from Proposition 4 that the new sector is active at all levels of ambiguity.

operating new industry firms with a traditional structure emerges. Firm owners now have access to a large and relatively cheap workforce. Eventually, the riskiness of the new technology becomes so precisely known that some firms become bureaucratic, with bears employing bears.

5.3.3 Robustness

Our example is suggestive but also quite specific in its details. It is reasonable to wonder about the robustness of our conclusions.

One can easily see that the restriction to linear utility is inconsequential. Provided all agents share the same u, we could introduce some non-linearity without affecting the qualitative results.

The restriction to common u across agents can also be relaxed. We could allow agents to have heterogeneous utility functions, provided all were sufficiently close to linearity, without affecting our results in any substantial manner. Some details might change, but the broad picture would be unaffected. The example's key features derive from the imprecise riskiness inherent in the environment and the difference between Bullish optimism and bearish pessimism in this context. Of course, if risk attitudes differed substantially across individuals, and especially if Bulls were substantially more risk-averse than bears, results may change more significantly. Occupational choice depends on risk attitude as well as attitude to imprecision, so the characterization of equilibrium for any given ε value could be affected. There will still a tendency for reductions in imprecision to encourage bears into the new technology sector, but the details of the dynamic process might be different. If, on the other hand, all agents are risk-averse to some extent, but there is no correlation between risk aversion and attitude to ambiguity, then we expect broadly similar results. Risk aversion will retard innovation, but the dynamics should conform to a "noisy" version of those described above.

The restriction to two industries is also innocuous, at least for the points we wish to make. Our purpose is to examine the role of the imprecise risk surrounding a new innovation. It is natural to treat the other sectors as well-established, however many there might be. The main feature of the example is that each type of agent has a stable outside option whose value is unaffected by the imprecision of risk surrounding the new innovation.

However, some features of the example are important to the results. One is the that the new innovation attracts Bulls even at maximal levels of imprecision. If this were not so, the new industry could never get under way. Another is that workers in the new sector must be exposed to sufficient ambiguity in the early phases so as to keep bears out of the industry. Wage default must be a real, but imprecisely quantified, possibility.

Nevertheless, it is also important that there is a preponderance of upside ambiguity in the new sector. We used Proposition 2 extensively to characterize the equilibria. If the new technology generates more than two possible revenue levels for some δ, then the nature of the imprecision should be such that the conclusion of Proposition 2 still obtains. In particular, Bulls should have a relatively greater affinity than bears for ownership over wage-laboring in the new sector. If workers experience greater income ambiguity than owners, then we might expect different comparative static results. However, for evaluating the effects of imprecise risk on innovations which are ultimately successful, it is more natural to suppose that owners will bear the greater burden of ambiguity.

6 Concluding remarks

The motivation for this paper is the recognition that an "entrepreneurial personality" has a high tolerance for ambiguity (rather than risk). In a multi-technology setting, we have shown that entrepreneurial personalities may be found working in, as well as owning, infant industry start-ups. Acknowledging this possibility has important implications for empirical research on entrepreneurship. According to our model, individuals who start their own businesses in mature industries are fueled by different motives relative to owners and workers in ambiguous, emerging industries. The firm is more than simply a production technology: it is also a mechanism for resolving differences in attitudes to common information about risk. When an industry is very new and information is vague, both workers and owners share the same optimistic attitude. As information increases, there are gains from trading across differences in attitude, and the nature of the firm changes.

Our results also address a question in the management literature which asks whether large and bureaucratic firms operating in mature industries lose their entrepreneurial edge. The comparative static results suggest that the disappearance of Bull workers from innovative firms and the increasing bureaucratization may be natural consequences of increasing precision. As information becomes more precise, replacing Bull workers with bears is efficient since the latter value the insurance aspect of the role. This process will give rise to "the perfectly bureaucratized great industrial unit" (Schumpeter 1942, p. 134). Contrary to the Schumpterian view, we conclude that this process is not necessarily bad, since entrepreneurial spirit resides in the individual, not the firm. When firms become conservative, Bulls will depart. In this sense, conservative firms promote innovation by spurring potential entrepreneurs to leave and seek a new innovative technology. As a successful entrepreneur remarked when asked about his prognosis for the *intrapreneur*, the dynamic worker who maintains an innovative culture within a large firm: "The only successful intrapreneurs are ones who leave to become entrepreneurs".[33]

Proof of Theorem 1 We shall first establish the existence of an equilibrium in the sense of Definition 5.

Lemma 1 *Under Assumptions 1 and 2 , every economy $\mathcal{E}$ possesses an equilibrium.*

Proof Given $\mathcal{E}$ and (w, δ), let $\Phi(w, \delta)$ denote the set of allocation functions satisfying conditions (i) and (ii) of Definition 5 and let

$$Z(w, \delta) = \left\{ \left(\text{Leb}\left[\phi^{-1}(i)\right]\right)_{i=1}^{2I} \middle| \phi \in \Phi(w, \delta) \right\}$$

Notice that elements of $Z(w, \delta)$ belong to the unit simplex $\Delta^{2I-1} \subseteq \mathbb{R}_+^{2I}$. Now define the $2I \times 2I$ non-singular matrix $C = \left[c_{ij}\right]$ as follows:

[33] "Fear of the Unknown", *The Economist* , 4 December 1999, p. 64.

$$c_{ij} = \begin{cases} 1 & \text{if } \ j = 2i - 1 \quad \text{or} \quad j = 2\,(i - I) - 1 \\ -1 & \text{if } \ j = 2i \quad \text{and} \quad i \le 2I \\ 0 & \text{otherwise} \end{cases}$$

Let $X\,(w, \delta) = CZ\,(w, \delta)$, so that $X\,(w, \delta)$ is a subset of $B := C\Delta^{2I-1}$.

Each $x \in X\,(w, \delta)$ is a $2I$-vector whose first I components give the *net excess measure of owners* in each industry for some allocation in $\Phi\,(w, \delta)$, and whose second I components give the *total measure of owners* in each industry. Therefore, we need to show the existence of some (w, δ) such that $X\,(w, \delta)$ contains a vector whose first I elements are zeroes and whose last I elements are δ.

For any $y \in \mathbb{R}^{2I}$, let $\mathrm{proj}_I\, y$ denote the projection of y onto its first I components; and $\mathrm{proj}_{-I}\, y$ the projection of y onto its second I components. Observe that

$$\mathrm{proj}_{-I}\, B = \left\{ y \in \mathbb{R}^I_+ \,\middle|\, \sum_{k=1}^{I} y_k \le 1 \right\} \subseteq [0, 1]^I$$

It will therefore prove convenient to extend each R_i to the domain $\Theta \times [0, 1]^I$ as follows:

$$R_i\,(\theta, \delta) \;:=\; R_i \left(\theta,\, \left(\min\left\{ \frac{1}{2}, \delta_i \right\} \right)_{i=1}^{I} \right)$$

These extended R_i functions are still strictly positive and continuous. In particular, we may define

$$\overline{W} = \max_i \sup_{(\theta,\delta) \in \Theta \times [0,1]^I} R_i\,(\theta, \delta)$$

The foregoing observations and definitions confirm that $D := \left[0, \overline{W}\right]^I \times [0, 1]^I \times B$ is a compact and convex subset of Euclidean space. Let the correspondence $\xi : D \twoheadrightarrow D$ be defined as follows:

$$\xi\,(w, \delta, x) = \left\{ \arg \max_{\tilde{w} \in \left[0, \overline{W}\right]^I} \tilde{w} \cdot \mathrm{proj}_I\, x \right\} \times \left\{ \mathrm{proj}_{-I}\, x \right\} \times X\,(w, \delta)$$

One can easily verify that ξ is well-defined on its domain, and that $\xi\,(w, \delta, x) \subseteq D$ for any (w, δ, x) in D.

To see that any fixed point of ξ determines an equilibrium of $\mathcal{E}$, suppose that $(w, \delta, x) \in \xi\,(w, \delta, x)$. Then

$$\delta \;=\; \mathrm{proj}_{-I}\, x \tag{15}$$

and it suffices to show that

$$x^{(I)} := \text{proj}_I \, x = 0 \tag{16}$$

(Note that (16) and (15) imply $\delta \in \frac{1}{2}\Delta^{I-1}$)

We now verify that (16) holds at a fixed point of $\xi\,(w, \delta, x)$. Suppose instead that $x_i^{(I)} > 0$ for some i. The definition of ξ and the fact that (w, δ, x) is a fixed point imply $w_i = \overline{W}$. But then all agents *strictly* prefer being wage-earners in industry i than being firm owners in industry i. This follows because R_i is strictly positive by assumption: no matter how pessimistic agents' beliefs, wage-earners in industry i must expect a strictly positive income when $w_i = \overline{W}$. Since $x \in X\,(w, \delta)$, this means $x_i^{(I)} \leq 0$, which is a contradiction. Assuming $x_i^{(I)} < 0$ leads to a contradiction by a symmetric argument. Equation (16) is therefore confirmed.

Summarizing, we defined the correspondence $\xi : D \twoheadrightarrow D$, and deduced that for any fixed point (w, δ, x), there is a ϕ such that (w, δ, ϕ) is an equilibrium of $\mathcal{E}$. The final step is to show that ξ does indeed have a fixed point. But ξ satisfies all the conditions of Kakutani's Fixed Point Theorem. The arguments are somewhat lengthy and uninspiring, and are therefore omitted. This completes the proof of Lemma 1. $\square$

We next establish the existence of equilibria that satisfy the innovation-proofness requirement.

First, we introduce a definition. Belief functions may be described by their *Möbius inverse* (Shafer 1976; Chateauneuf and Jaffray 1989). This inverse is the mapping $m : 2^{\Theta} \to [0, 1]$ defined from Γ as follows:

$$m(E) = p\left(\{s \in S \mid \Gamma(s) = E\}\right) \quad \forall E \subseteq \Theta$$

Thus, m is the probability induced by p and Γ on the power set 2^{Θ}. Furthermore:

$$\underline{v}(E) = \sum_{A \subseteq E} m(A)$$

One may think of $m(E)$ as the quantity of probability necessarily attached to event E that is not necessarily attached to any of its subsets.

The following is useful.[34]

[34] Note that for any $\lambda^B \in \left(\frac{1}{2}, 1\right)$:

$$\lambda^B \overline{v}(E) + \left(1 - \lambda^B\right) \underline{v}(E) = 0$$

if and only if

$$\lambda^b \overline{v}(E) + \left(1 - \lambda^b\right) \underline{v}(E) = 0$$

when $\lambda^b = 1 - \lambda^B$. Therefore, under the alternative behavioral specification discussed in Sect. 2.4, Lemma 2 automatically guarantees the existence of an innovation-proof equilibrium.

Lemma 2 *Suppose that the information correspondence* Γ *for economy* $\mathcal{E}$ *is such that, for any* $E \in \mathcal{P}^*(\Theta) := 2^{\Theta} \setminus \{\emptyset\}$

$$\overline{v}(E) > 0 \;\Rightarrow\; \underline{v}(E) > 0 \tag{17}$$

Then any equilibrium of $\mathcal{E}$ *is innovation-proof.*

Proof Recall from the discussion following Theorem 1 that innovation-proofness fails only if a potential Bull entrepreneur and potential bear worker disagree about zero probability revenue contingencies. Condition (17) rules out this sort of disagreement.

Suppose that (w, δ, ϕ) is an equilibrium of $\mathcal{E}$. Suppose further that there exists an industry i with $\delta_i = 0$, wage $\hat{w}_i \geq 0$ and agents $k, k' \in [0, 1]$, $k \neq k'$, satisfying

$$\mathbb{E}_{v_k}\left[u\left(R_i(\theta, \delta) - \min\{\hat{w}_i, R_i(\theta, \delta)\}\right)\right] > u^{\mathcal{E},\beta_k}(w, \delta) \tag{18}$$

and

$$\mathbb{E}_{v_{k'}}\left[u\left(\min\{\hat{w}_i, R_i(\theta, \delta)\}\right)\right] \geq u^{\mathcal{E},\beta_{k'}}(w, \delta) \tag{19}$$

Note first that $\hat{w}_i < w_i$. If this were not the case, (18) would fail. Owning a firm in industry i and paying wage w_i is weakly less desirable to agent k than $\phi(k)$, so owning an industry i firm and paying an even higher wage cannot make k strictly better off than occupation $\phi(k)$.

It follows that:

$$\mathbb{E}_{v_{k'}}\left[u\left(\min\{\hat{w}_i,\; R_i(\theta, \delta)\}\right)\right] \;\leq\; \mathbb{E}_{v_{k'}}\left[u\left(\min\{w_i,\; R_i(\theta, \delta)\}\right)\right] \;\leq\; u^{\mathcal{E},\beta_{k'}}(w, \delta)$$

So Eq. (19) implies

$$\mathbb{E}_{v_{k'}}\left[u\left(\min\{\hat{w}_i,\; R_i(\theta, \delta)\}\right)\right] \;=\; \mathbb{E}_{v_{k'}}\left[u\left(\min\{w_i, R_i(\theta, \delta)\}\right)\right]$$

Hence, we conclude that

$$v_{k'}\left(\left\{\theta \in \Theta \mid R_i(\theta, \delta) > \hat{w}_i\right\}\right) = 0 \tag{20}$$

But, Eq. (18) and

$$u^{\mathcal{E},\beta_k}(w, \delta) \geq \mathbb{E}_{v_k}\left[u\left(R_i(\theta, \delta) - \min\{w_i, R_i(\theta, \delta)\}\right)\right]$$

imply

$$v_k\left(\left\{\theta \in \Theta \mid R_i(\theta, \delta) > \hat{w}_i\right\}\right) > 0 \tag{21}$$

In other words, the potential worker—agent k'—assigns zero weight to contingencies in which revenue exceeds $\hat{w}_i$, while the potential entrepreneur—agent k—assigns positive weight to such contingencies.

Observe that Eqs. (20) and (21) imply $v_k = \overline{v}$ and $v_{k'} = \underline{v}$. This contradicts the assumption of the Lemma. □

Corollary 1 *Suppose that the information correspondence* Γ *for economy* $\mathcal{E}$ *is such that the associated Möbius inverse satisfies*

$$m(\{\theta\}) > 0 \quad \forall\theta \in \Theta$$

Then, any equilibrium of $\mathcal{E}$ *is innovation-proof.*

Proof The assumption on m implies $\underline{v}(E) > 0$ for all $E \in \mathcal{P}^*(\Theta)$ and the result follows from Lemma 2. □

We are now ready to prove Theorem 1.

Let $\overline{v}$ and $\underline{v}$ denote the upper and lower probabilities induced by Γ. For the purposes of the present argument it is convenient to make the assumption that all redundant elements have been removed from Θ; that is, all θ such that

$$\underline{v}(\{\theta\}) = \overline{v}(\{\theta\}) = 0$$

If m, the Möbius inverse of $\underline{v}$, satisfies the condition in Corollary 1 we are done. Suppose not; that is, $m(\{\theta\}) = 0$ for some $\theta \in \Theta$.

Consider the space of functions

$$\mathcal{M} = \left\{ f : 2^{\Theta} \to [0, 1] \;\middle|\; f(\emptyset) = 0, \quad \sum_{E \in 2^{\Theta}} f(E) = 1 \right\}$$

Each $f \in \mathcal{M}$ is the Möbius inverse of some belief function on Θ; conversely, each belief function on Θ has a Möbius inverse in $\mathcal{M}$ (Shafer 1976). We may identify $\mathcal{M}$ with the unit simplex $\Delta^{|\mathcal{P}^*(\Theta)|-1}$. In particular, $\mathcal{M}$ is compact and convex.

Choose some $\tilde{\pi} \in \text{ri}[\text{core}(\underline{v})]$, where $\text{ri}\,(\cdot)$ denotes the relative interior. Then, $\tilde{\pi}(E) = \underline{v}(E)$ if and only if $\overline{v}(E) = \underline{v}(E)$, and $\tilde{\pi}(E) > \underline{v}(E)$ otherwise. In particular, letting $\tilde{m} \in \mathcal{M}$ denote the Möbius inverse of $\tilde{\pi}$, we must have $\tilde{m}(\{\theta\}) > 0$ for every $\theta \in \Theta$ by our non-redundancy assumption.

Now let us define

$$m^s = \frac{1}{s}\tilde{m} + \left(1 - \frac{1}{s}\right) m$$

The sequence $\{m^s\}_{s=1}^{\infty}$ in $\mathcal{M}$ clearly converges to m and satisfies $m^s(\{\theta\}) > 0$ for every $\theta \in \Theta$. If $\left\{\underline{v}^s\right\}_{s=1}^{\infty}$ is the associated sequence of belief functions, then $\underline{v}^s \to \underline{v}$ as $s \to \infty$, and for each s and each $E \subseteq \Theta$:

$$\underline{v}^s(E) = \underline{v}(E) + \frac{1}{s}\left[\tilde{\pi}(E) - \underline{v}(E)\right] \geq \underline{v}(E)$$

From $\underline{v}^s$ we may construct the associated upper probability $\overline{v}^s$ as follows:

$$\overline{v}^s(E) = 1 - \underline{v}^s(E^c) \quad \forall E \subseteq \Theta$$

For each s, define $\mathcal{E}^s$ to be the economy with identical α, u and revenue functions to $\mathcal{E}$, but in which Bulls evaluate their employment options using $\overline{v}^s$, and bears evaluate their options using $\underline{v}^s$. For the purposes of the following argument it is unnecessary to define a sequence of information correspondences generating these beliefs. By Corollary 1, each $\mathcal{E}^s$ has an innovation-proof equilibrium (w^s, δ^s, ϕ_s).

The next step is to extract a convergent subsequence of $\{(w^s, \delta^s, \phi_s)\}_{s=1}^{\infty}$ and to verify that its limit is an equilibrium. This is conceptually straightforward, but since there are some technicalities in dealing with limits of the allocation functions ϕ_s, we omit the details. Letting (w, δ, ϕ) denote this limit equilibrium, it remains to verify innovation-proofness.

Suppose that (w, δ, ϕ) is *not* innovation-proof. That is, there exist agents k and k', an industry i with $\delta_i = 0$, and a wage $\hat{w}_i$ such that if k owns a firm in industry i and employs k' as a worker at $\hat{w}_i$, both do as well as under the candidate equilibrium, and

$$\mathbb{E}_{v_k}\left[u\left(R_i(\theta,\delta) - \min\left\{\hat{w}_i, R_i(\theta,\delta)\right\}\right)\right] > u^{\mathcal{E},\beta_k}(w,\delta) \tag{22}$$

Clearly, it must be the case that $w_i > \hat{w}_i$. Furthermore, Eq. (22) and the fact that

$$v_{k'}^s(E) > 0 \quad \forall E \in \mathcal{P}^*(\Theta)$$

imply the existence of some $\eta > 0$ such that, for every s,

$$\mathbb{E}_{v_{k'}^s}\left[u\left(\min\left\{c,\ R_i(\theta,\delta)\right\}\right)\right]$$

is *strictly* increasing in c when $c \in \left[\hat{w}_i, \hat{w}_i + \eta\right]$. On the other hand, since

$$\mathbb{E}_{v_{k'}}\left[u\left(\min\left\{\hat{w}_i,\ R_i(\theta,\delta)\right\}\right)\right] \geq u^{\mathcal{E},\beta_{k'}}(w,\delta) \geq \mathbb{E}_{v_{k'}}\left[u\left(\min\left\{w_i, R_i(\theta,\delta)\right\}\right)\right]$$

we see that

$$\mathbb{E}_{v_{k'}}\left[u\left(\min\left\{c, R_i(\theta,\delta)\right\}\right)\right] = u^{\mathcal{E},\beta_{k'}}(w,\delta) \quad \forall c \in \left[\hat{w}_i, w_i\right]$$

In particular, we must have $v_{k'} \equiv \underline{v}$ (the worker is a bear) and

$$m(B) = 0 \quad \forall B \subseteq \left\{\theta \in \Theta \;\middle|\; R_i(\theta,\delta) > \hat{w}_i\right\} \neq \emptyset$$

Recalling that $\underline{v}^s \geq \underline{v}$, we deduce

$$\mathbb{E}_{v_{k'}^s}\left[u\left(\min\left\{\hat{w}_i, R_i(\theta,\delta)\right\}\right)\right] \geq \mathbb{E}_{v_{k'}}\left[u\left(\min\left\{\hat{w}_i, R_i(\theta,\delta)\right\}\right)\right]$$

for each s. Letting $\tilde{w}_i \in (\hat{w}_i, \hat{w}_i + \eta)$, we therefore have

$$\mathbb{E}_{v^s_{k'}}\left[u\left(\min\left\{\tilde{w}_i, R_i(\theta,\delta)\right\}\right)\right] - u^{\mathcal{E},\beta_{k'}}(w,\delta) > \mathbb{E}_{v_{k'}}\left[\min\left\{\tilde{w}_i, R_i(\theta,\delta)\right\}\right] - u^{\mathcal{E},\beta_{k'}}(w,\delta)$$

for all s. Hence

$$\mathbb{E}_{v^s_{k'}}\left[u\left(\min\left\{\tilde{w}_i, R_i\left(\theta,\delta^n\right)\right\}\right)\right] - u^{\mathcal{E}^s,\beta_{k'}}\left(w^s,\delta^s\right) \geq \mathbb{E}_{v_{k'}}\left[\min\left\{\tilde{w}_i, R_i(\theta,\delta)\right\}\right] - y^{\mathcal{E},\beta_{k'}}(w,\delta)$$

for all sufficiently large s, using the continuity of the functions on the left-hand side of the inequality. Therefore, for all large s,

$$\mathbb{E}_{v^s_{k'}}\left[u\left(\min\left\{w^s_i, R_i\left(\theta,\delta^s\right)\right\}\right)\right] - u^{\mathcal{E}^s,\beta_{k'}}\left(w^s,\delta^s\right) > \mathbb{E}_{v_{k'}}\left[\min\left\{\tilde{w}_i, R_i(\theta,\delta)\right\}\right] - y^{\mathcal{E},\beta_{k'}}(w,\delta)$$

since $\mathbb{E}_{v^s_{k'}}\left[u\left(\min\left\{c, R_i\left(\theta,\delta^n\right)\right\}\right)\right]$ is strictly increasing in c at $c = \tilde{w}_i$ when s is sufficiently large. But recall that the right-hand side of this inequality is equal to zero, so

$$\mathbb{E}_{v^s_{k'}}\left[\min\left\{w^s_i, R_i\left(\theta,\delta^s\right)\right\}\right] > u^{\mathcal{E}^s,\beta_{k'}}\left(w^s,\delta^s\right)$$

which is a contradiction. This completes the proof.

References

Anscombe, F.J., Aumann, R.J.: A definition of subjective probability. Ann Math Stat **34**, 199–205 (1963)

Arabsheibani, G., de Meza, D., Maloney, J., Pearson, B.: And a vision appeared unto them of a great profit: evidence of self-deception among the self-employed. Econ Lett **67**, 35–41 (2000)

Arrow, K.J., Hurwicz, L.: An optimality criterion for decision making under ignorance. In: Carter, C.F., Ford, J.F. (eds.) Uncertainty and Expectations in Economics, pp. 1–11. Oxford: Basil Blackwell & Mott (1972)

Bernardo, A.E., Welch, I.: On the evolution of overconfidence and entrepreneurs. J Econ Manag Strateg **10**, 301–330 (2001)

Bewley, T.: Market innovation and entrepreneurship: a Knightian view. Discussion Paper Serie A 233, University of Bonn (1989)

Bhidé, A.H.: The Origins and Evolution New Businesses. Oxford: Oxford University Press (2000)

Busenitz, L.W.: Entrepreneurial risk and strategic decision making. J Appl Behav Sci **35**, 325–340 (1999)

Calvo, G.A., Wellisz, S.: Technology, entrepreneurs, and firm size. Q J Econ **95**, 663–677 (1980)

Chateauneuf, A., Jaffray, J.-Y.: Some characterizations of lower probabilities and other monotone capacities through the use of Möbius inversion. Math Soc Sci **17**, 263–283 (1989)

Cooper, A.C., Woo, C.Y., Dunkelberg, W.C.: Entrepreneurs' perceived chances for success. J Bus Ventur **3**, 97–108 (1988)

de Meza, D., Southey, C.: The borrower's curse: optimism, finance and entreprenurship. Econ J **106**, 375–386 (1996)

Dempster, A.P.: Upper and lower probabilities induced by a multivalued mapping. Ann Math Stat **38**, 325–339 (1967)

Dubey, P., Geanakoplos, J., Shubik, M.: Default and punishment in general equilibrium. Econometrica **73**(1), 1–37 (2005)
Ellsberg, D.: Risk, ambiguity, and the Savage axioms. Q J Econ **75**, 643–669 (1961)
Evans, D.S., Leighton, L.S.: Some empirical aspects of entrepreneurship. Am Econ Rev **79**, 519–535 (1989)
Gajdos, T., Hayashi, T., Tallon, J.-M., Vernaud, J.-C.: Attitude toward imprecise information. J Econ Theory **140**, 27–65 (2008)
García, D., Sangiorgi, F., Urosěvić, B.: Overconfidence and market efficiency with heterogeneous agents. Econ Theory **30**, 313–336 (2007)
Ghirardato, P.: Coping with ignorance: unforeseen contingencies and non-additive uncertainty. Econ Theory **17**, 247–276 (2001)
Gilboa, I., Schmeidler, D.: Maxmin expected utility with non-unique prior. J Math Econ **18**(2), 141–153 (1989)
Grüner, H.P.: Redistribution as a selection device. J Econ Theory **108**, 194–216 (2003)
Hansemark, O.C.: Need for achievement, locus of control and the prediction of business start-ups: a longitudinal study. J Econ Psychol **24**, 301–319 (2003)
Hart, O.D.: Perfect competition and optimal product differentiation. J Econ Theory **22**, 279–312 (1980)
Heath, C., Tversky, A.: Preference and belief: ambiguity and competence in choice under uncertainty. J Risk Uncertain **4**, 5–29 (1991)
Hendon, E., Jacobsen, H.J., Sloth, B., Tranæs, T.: Expected utility with lower probabilities. J Risk Uncertain **8**, 197–216 (1994)
Holmes, T.J., Schmitz, J.A.: A theory of entrepreneurship and its application to the study of business transfers. J Polit Econ **98**, 265–294 (1990)
Jaffray, J.-Y.: Linear utility theory for belief functions. Oper Res Lett **8**, 107–112 (1989)
Jaffray, J.-Y.: Linear utility theory and belief functions: a discussion. In: Chikán, A. (ed.) Progress in Decision, Utility and Risk Theory, pp. 221–229. Dordrecht: Kluwer (1991)
Jaffray, J.-Y.: Dynamic decision-making with belief functions. In: Yager, R.R., Kacprzyk, J., Fedrizzi, M. (eds.) Advances in Dempster–Shafer Theory of Evidence, pp. 331–352. New York: Wiley (1994)
Jaffray, J.-Y., Phillipe, F.: On the existence of subjective upper and lower probabilities. Math Oper Res **22**, 165–185 (1997)
Jaffray, J.-Y., Wakker, P.: Decision making with belief functions: compatibility and incompatibility with the sure-thing principle. J Risk Uncertain **8**, 255–271 (1994)
Keenan, A., McBain, G.D.M.: Effects of type a behavior, intolerance of ambiguity, and locus of control on the relationship between role stress and work-related outcomes. J Occup Psychol **52**, 277–285 (1979)
Kihlstrom, R.E., Laffont, J.-J.: A general equilibrium entrepreneurial theory of firm formation based on risk aversion. J Political Econ **87**, 719–748 (1979)
Kirzner, I.M.: Entrepreneurial discovery and the competitive market process. J Econ Lit **35**, 60–85 (1997)
Knight, F.: Risk, Uncertainty and Profit. Cambridge: Houghton Mifflin (1921)
Lazear, E.: Entrepreneurship. J Labor Econ **23**(4), 649–680 (2004)
Laussel, D., Le Breton, M.: A general equilibrium theory of firm formation. Eur Econ Rev **39**, 1303–1319 (1995)
Lucas, R.E. Jr.: On the size distribution of business firms. Bell J Econ **9**, 508–523 (1978)
Luce, R.D., Raiffa, H.: Games and Decisions: Introduction and Critical Survey. New York: Wiley (1957)
Makowski, L.: Perfect competition, the profit criterion, and the organization of economic activity. J Econ Theory **22**, 222–242 (1980)
Manove, M.: Entrepreneurs, Optimism and the Competitive Edge. Mimeo: Boston University (2000)
Moskowitz, T.J., Vissing-Jørgensen, A.: The returns to entrepreneurial investment: a private equity premium puzzle? Am Econ Rev **92**, 745–778 (2002)
Mukerji, S.: Understanding the non-additive probability model. Econ Theory **9**, 23–46 (1997)
Olszewski, W.: Preferences over sets of lotteries. Rev Econ Stud **74**, 567–595 (2007)
Philippe, F., Debs, G., Jaffray, J.-Y.: Decision making with monotone lower probabilities of infinite order. Math Oper Res **24**, 767–784 (1999)
Rigotti, L.: Decisive Entrepreneurs and Cautious Investors. Mimeo: CentER (2000)
Rigotti, L., Ryan, M.J., Vaithianathan, R.: Tolerance of Ambiguity and Entrepreneurial Innovation. Mimeo: Fuqua School of Business, Duke University (2008)
Rigotti, L., Shannon, C., Strzalecki, T.: Subjective beliefs and ex-ante trade. Econometrica **76**, 1167–1190 (2008)
Schmeidler, D.: Integral representation without additivity. Proc Am Math Soc **97**, 255–261 (1986)

Schmeidler, D.: Subjective probability and expected utility without additivity. Econometrica **57**, 571–587 (1989)

Schumpeter, J.A.: The Theory of Economic Development. Cambridge: Harvard University Press (1934)

Schumpeter, J.A.: Capitalism, Socialism and Democracy. New York: Harper and Row (1942)

Shafer, G.: A Mathematical Theory of Evidence. Princeton: Princeton University Press (1976)

Srivastava, R.P., Mock, T.J.: Belief Functions in Business Decisions. New York: Physica-Verlag (2002)

Vierø, M.-L.: Exactly what happens after the Anscombe–Aumann race? Representing preferences in vague environments. Econ Theory **41**, 175–212 (2009)

Wakker, P.P.: Testing and characterizing properties of nonadditive measures through violations of the sure-thing principle. Econometrica **69**, 1039–1059 (2001)

Wärneryd, K.-E.: The psychology of innovative entrepreneurship. In: van Raaij, W.F., van Veldhoven, G.M., Wärneryd, K.-E. (eds.) Handbook of Economic Psychology, pp. 404–447. Dordrecht: Kluwer (1988)