Outline

1. Logistics
2. Binary Relations
   1. Definition
   2. Properties
   3. Upper and Lower Contour Sets
3. Preferences
4. Choice Correspondences
5. Revealed Preferences
Logistics

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Office Hours: Wednesday 2pm.

Class Time and Location
Monday & Wednesday, 10:30am to 11:45am in WWPH 4916

Recitation Times
Friday, 11-Noon, in WWPH 4940.

Class Webpage http://www.pitt.edu/~luca/ECON2100/

Announcements
None for now.
Logistics

Mas-Colesl, Whinston and Green: Microeconomic Theory.
Other useful textbooks by Varian, Jehle and Reny, older Kreps.

Grading

- Problem Sets: 10%
  Weekly, they will appear during lectures.
  DO NOT look for answers... These are for you to learn what you can and cannot do; we can help you get better only if we know your limits.

- Midterm: 30%
  In class, 14 October (Wednesday), includes materials up to previous class.

- Final Exam: 60%
  Cumulative, 3 hours long, Wednesday, 16 December.

Working in teams strongly encouraged, but turn in problem sets individually.
Start working on exercises on your own, and then get together to discuss.
Typically, a Problem Set will be due at the beginning of Monday’s lecture.
Goals of the Micro Theory Sequence

- Learn and understand the microeconomic theory every academic economist needs to know.
- Stimulate interest in micro theory as a field some of you may want to pursue.
- Enable you to read papers that use theory and go to research seminars.
- This course covers the following standard topics:
  - consumers (preferences, choices, utility function representation, utility maximization, demand theory, aggregation, decision making under uncertainty)
  - firms (production, profit maximization, aggregation)
  - interaction between consumers and firms in competitive markets (Walrasian equilibrium, First and Second Welfare Theorems, existence of equilibrium, uniqueness of equilibrium, markets with uncertainty and time, Arrow-Debreu economies)
- In the Spring, Tymofiy Mylovanov will cover game theory and information economics.
- Even if you are not enrolled in it, attend ECON 2010 as long as you can (same place, T/Th at 9am); Roee Teper will start with constrained optimization, a fundamental tool for this class.
- Questions? Ready, Set, Go!
A binary relation on some set is a collection of ordered pairs of elements of that set.

**Definition**

$R \subseteq X \times Y$ is a binary relation from $X$ to $Y$. We write “$xRy$” if $(x, y) \in R$ and “not $xRy$” if $(x, y) \not\in R$. When $X = Y$ and $R \subseteq X \times X$, we say $R$ is a binary relation on $X$.

**Exercise**

Suppose $R, Q$ are two binary relations on $X$. Prove that, given our notation, the following are equivalent:

1. $R \subseteq Q$
2. For all $x, y \in X$, $xRy \Rightarrow xQy$.

**NOTE**

Items denoted as Exercises are meant as practice for you, distinct from graded problem sets.
Examples of Binary Relations

A Function Is a Binary Relation

Suppose \( f : X \rightarrow Y \) is a function from \( X \) to \( Y \).

- Then the binary relation \( R \subseteq X \times Y \) defined by
  \[
  xRy \iff f(x) = y
  \]
  is the graph of \( f \).

- One way to think of a function is as a binary relation \( R \) from \( X \) to \( Y \) such that: for each \( x \in X \), there exists exactly one \( y \in Y \) such that \((x, y) \in R\).
Examples of Binary Relations

**Weakly greater than**

Suppose \( X = \{1, 2, 3\} \)

- Consider the binary relation
  \[
  R \subseteq \{1, 2, 3\} \times \{1, 2, 3\}
  \]
  defined as follows
  \[
  R = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)\}.
  \]
- \( R \) is the binary relation “is weakly greater than,” or \( \geq \).
- We can represent it graphically as:

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Examples of Binary Relations

Equal to and Strictly less than

Suppose $X = \{1, 2, 3\}$.

- The following graphically represents the binary relation $\equiv$, or “is equal to:”

  $3 \llap{\bullet} \quad 2 \llap{\bullet} \quad 1 \llap{\bullet}$

  \[
  \begin{array}{c|c|c}
  \hline
  3 & & \llap{\bullet} \\
  2 & \llap{\bullet} & \\
  1 & \llap{\bullet} & \\
  \equiv & 1 & 2 & 3 \\
  \hline
  \end{array}
  \]

- The following would represent the binary relation $<$ or “is strictly less than:”

  $3 \llap{\bullet} \llap{\bullet} \quad 2 \llap{\bullet} \quad 1 \llap{\bullet}$

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**Definitions**

Given a binary relation $R$ on $X$.

- The **dual** of $R$, denoted $R'$, is defined by $xR'y$ if and only if $yRx$.
- The **asymmetric component** of $R$, denoted $P$, is defined by $xPy$ if and only if $xRy$ and not $yRx$.
- The **symmetric component** of $R$, denoted $I$, is defined by $xIy$ if and only if $xRy$ and $yRx$.

**Example**

Suppose $X = \mathbb{R}$ and $R$ is the binary relation $\geq$, or “weakly greater than.”

- The dual $R'$ is $\leq$ or “weakly less than,” because $x \geq y$ if and only if $y \leq x$.
- The asymmetric component $P$ is $>$ or “strictly greater than,” because $x > y$ if and only if $x \geq y$ and not $y \geq x$. (Verify this).
- The symmetric component of $I$ is $\equiv$ or “is equal to,” because $x \equiv y$ if and only if $x \geq y$ and $y \geq x$. 
**Example**

Suppose $X = \{1, 2, 3\}$ and consider the following binary relation $R$:

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Then the following represent $R'$, $P$, and $I$:

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## Properties of Binary Relations

### Definitions

A binary relation $R$ on $X$ is:

- **complete** if, for all $x, y \in X$, $xRy$ or $yRx$;
- **reflexive** if, for all $x \in X$, $xRx$;
- **irreflexive** if, for all $x \in X$, not $xRx$;
- **symmetric** if, for all $x, y \in X$, $xRy$ implies $yRx$;
- **asymmetric** if, for all $x, y \in X$, $xRy$ implies not $yRx$;
- **antisymmetric** if, for all $x, y \in X$, $xRy$ and $yRx$ imply $x = y$;
- **transitive** if, for all $x, y, z \in X$, $xRy$ and $yRz$ imply $xRz$;
- **negatively transitive** if, for all $x, y, z \in X$, not $xRy$ and not $yRz$ imply not $xRz$;
- **quasi-transitive** if, for all $x, y, z \in X$, $xPy$ and $yPz$ imply $xPz$;
- **acyclic** if, for all $x_1, x_2, \ldots, x_n \in X$, $x_1Px_2$, $x_2Px_3$, $\ldots$, and $x_{n-1}Px_n$ imply $x_1Rx_n$. 

Properties of Binary Relations

Exercise
Suppose $R$ is complete. Prove the following: If $R$ is transitive, then $R$ is quasi-transitive. If $R$ is quasi-transitive, then $R$ is acyclic.

Question 1, Problem Set 1 (due Wednesday 9 September)
Prove the following: if $R$ is asymmetric and negatively transitive, then $R$ is transitive.

Example
Suppose $X = \mathbb{R}$.
- The binary relation $\geq$ is reflexive, complete, antisymmetric, transitive, and negatively transitive; $\geq$ is not asymmetric.
- The binary relation $>$ is irreflexive, asymmetric, antisymmetric, transitive, negatively transitive, quasi-transitive, and acyclic; $>$ is not reflexive, not complete, and not symmetric.

NOTE
Items denoted as Problem Set Questions will be handed in individually on the due date, at the beginning of the lecture.
### Equivalence

**Definition**

A binary relation $R$ on $X$ is an **equivalence relation** if $R$ is reflexive, symmetric, and transitive.

**Definition**

The **equivalence class** of $x \in X$ is

$$[x] = \{y \in X : xRy\}.$$  

Let $X/R$ (the quotient of $X$ over $R$) denote the collection of all equivalence classes.

- The equivalence classes of an equivalence relation form a **partition** of $X$: every element of $X$ belongs to exactly one equivalence class.

**Theorem**

*Let $R$ be an equivalence relation on $X$. Then $\forall x \in X$, $x \in [x]$. Given $x, y \in X$, either $[x] = [y]$ or $[x] \cap [y] = \emptyset$.***

**Proof.**

Question 2, Problem Set 1.
Equivalence: Examples

Equivalence Relation and Equivalence Class

The following is an equivalence relation on \( X = \{a, b, c, d\} \):

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- The equivalence classes of \( R \) are \( \{a, b\} \) and \( \{c, d\} \), so the collection is \( X/R = \{\{a, b\}; \{c, d\}\} \).

Exercise

Let \( X \) be the set of all living people. Are the following relations on \( X \) reflexive? symmetric? transitive? complete?

- “is married to” (assuming monogamy)
- “is the son or daughter of”
- “is an ancestor or descendant of”
- “is taller than”

For those that are equivalence relations, interpret \( X/R \).
Orders

Definition
A binary relation $R$ on $X$ is a **preorder** if $R$ is reflexive and transitive.

Definition
A binary relation $R$ on $X$ is a **weak order** if $R$ is complete and transitive.

Definition
A binary relation $R$ on $X$ is a **linear order** if $R$ is complete, transitive, and antisymmetric.

Exercise
Define the binary relation $\geq$ on $\mathbb{R}^2$ by: $x \geq y \iff x_1 \geq y_1$ and $x_2 \geq y_2$. Verify that $\geq$ is a preorder on $\mathbb{R}^2$. Verify that $\geq$ is not a weak order on $\mathbb{R}^2$.

Exercise
Define $\geq^\dagger$ on $\mathbb{R}^2$ by: $x \geq^\dagger y \iff x_1 > y_1$ or, $x_1 = y_1$ and $x_2 \geq y_2$. Verify that $\geq^\dagger$ is a linear order on $\mathbb{R}^2$ (this is sometimes called lexicographic ordering).
**Definitions**

Given a binary relation $R$ on $X$, the **upper contour set of** $x \in X$ is
\[
\{ y \in X : yRx \}.
\]

Given a binary relation $R$ on $X$, the **lower contour set of** $x \in X$ is
\[
\{ y \in X : xRy \}.
\]

**Exercise**

Suppose $R$ is a preorder on $X$. Prove that if $xRy$, then the lower contour set of $y$ is a subset of the lower contour set of $x$; that is, prove that
\[
xRy \implies \{ z \in X : yRz \} \subseteq \{ z \in X : xRz \}.
\]
In order to have a good model of how consumers behave we need to define:

- The set of options that are **possible** objects of choice
- Among those, a subset that describes what are the **feasible** objects of choice.
- A theory that gives us the way a consumer chooses among the feasible items.

We start by trying to build the latter starting from a description of how a consumer would rank any two elements that belong to the set of possible objects of choice. These are hypothetical rankings.

We then try to see what these hypothetical rankings imply about actual choices:

- Are choices rational in the sense of being consistent with those rankings?

Finally, we ask whether rationality can be tested by an outside observer (the economist) who does not know what is inside the consumer’s head:

- Can we say whether or not a consumer is rational without knowing her hypothetical rankings?
Preference Relations

**Definition**
A binary relation $\succsim$ on $X$ is a preference relation if it is a weak order, i.e. complete and transitive.

- $x \succsim y$ reads: the decision maker (DM) weakly prefers $x$ to $y$; we also say $x$ is at least as good as $y$.
- Kreps: “A preference relation expresses the consumer’s feelings between pair of objects in $X$”.
- These are not choices. Later, we will talk about choices the consumer would make given some set of options to choose from, and how these choices are related to her preference relation.

**Definitions**
The upper contour set of $x$ (denoted $\succsim^+(x)$) consists of the elements of $X$ that are weakly preferred to $x$ according to $\succsim$:

$$\succsim^+(x) = \{ y \in X : y \succeq x \}$$

The lower contour set of $x$ (denoted $\succsim^-(x)$) consists of the elements of $X$ that $x$ is weakly preferred to according to $\succsim$:

$$\succsim^-(x) = \{ y \in X : x \succeq y \}$$
Preference Relations

**Definitions**

For any preference relation \( \succeq \) on \( X \),

- \( \succeq \) denotes the **dual** of \( \succeq \), defined by
  \[
  x \succeq y \iff y \succeq x;
  \]

- \( \succ \) denote the **asymmetric component** of \( \succeq \), defined by
  \[
  x \succ y \iff [x \succeq y \text{ and not } y \succeq x];
  \]

- \( \sim \) denote the **symmetric component** of \( \succeq \), defined by
  \[
  x \sim y \iff [x \succeq y \text{ and } y \succeq x].
  \]

- \( x \succ y \) reads: DM *strictly prefers* \( x \) to \( y \);

- \( x \sim y \) reads: DM *is indifferent* between \( x \) and \( y \).
Exercise

Let $X = \{a, b, c\}$. Determine if the following binary relations are complete and/or transitive:

1. $\preceq = X \times X$;
2. $\preceq = \emptyset$;
3. $\preceq = \{(a, a), (b, b), (c, c), (a, b), (b, a), (a, c), (b, c)\}$;
4. $\preceq = \{(a, a), (b, b), (c, c), (a, b), (b, c), (c, a)\}$;
5. $\preceq = \{(a, b), (b, c), (a, c)\}$.

Question 3, Problem Set 1.

Prove that if $\succeq$ is a preference relation (i.e. it is complete and transitive), then:

1. $\succeq$ is a preference relation;
2. $\succ$ is asymmetric and transitive;
3. $\sim$ is an equivalence relation.
4. $x \succeq y$ and $y \sim z$ imply $x \succeq z$;
5. $x \succeq y$ and $y \succ z$ imply $x \succ z$. 
A preference relation describes the decision maker’s taste among any two hypothetical pairs.

Given this preference, the decision makers will make choices whenever she is offered some set of alternatives to choose from.

This defines a “choice procedure”: a way to select elements out of a particular set.

- For example, in intermediate microeconomics we talk about a consumer maximizing her utility given a set of affordable consumption bundles.
- Choosing something that maximizes utility given the budget constraint is a choice procedure.

The big picture questions then are the following:

- when is some choice procedure the by-product of a preference relation?
- can we use observable choices to deduce what is the preference relation guiding those choices and whether that preference relation satisfies some rationality criteria?
We model choice formally by saying that some objects could be selected from the set of available objects.

**Definition**

A correspondence $\varphi$ from $X$ to $Y$ is a mapping from $X$ to $2^Y$; that is, $\varphi(x) \subseteq Y$ for every $x \in X$.

**Definition**

A choice rule for $X$ is a correspondence

$$C : 2^X \setminus \{\emptyset\} \to X$$

such that $C(A) \subseteq A$ for all $A \subseteq X$.

**Interpretation**

- Subsets of $2^X \setminus \{\emptyset\}$ are “menus” or “budgets”.
  - A particular $A \in 2^X \setminus \{\emptyset\}$ is interpreted as the set of available options (for example, affordable consumption).
- Given a budget, $C(A)$ is the set of options DM might choose from it.
- If $C(A)$ has more than one element, she could choose any of them (but not all of them at once).
Choice Rules: An Example

Example

Let $X = \{\text{apple, banana, carrot, dessert, elephant}\}$.

- If $C(\{a, b, c\}) = \{a, b\}$, DM could choose either the apple or the banana from a basket containing an apple, a banana, and a carrot;
- This is **not** interpreted as meaning the decision maker will consume both the apple and the banana.
- By definition, this means she will consume only one between the apple and the banana, but we do not know which one.
**Induced Choice Rules**

**Definition**

Given a binary relation \( \succeq \), the induced choice rule \( C_{\succeq} \) is defined by

\[
C_{\succeq}(A) = \{ x \in A : x \succeq y \text{ for all } y \in A \}.
\]

- This is a natural method to construct a choice rule from a binary relation: DM chooses something she prefers to all other available alternatives.
- This definition answers our first big question: the induced choice rule gives a choice procedure that is consistent with a given preference relation.

**Example**

Let \( X = \{ a, b, c \} \) and let \( \succeq = \{ (a, a), (b, b), (c, c), (a, b), (b, c), (c, a) \} \). What is \( C_{\succeq}(\{a, b, c\}) \)?

**Example**

Suppose \( X = \{1, 2, 3, \ldots\} \) and consider \( \geq, \leq, \) and \( > \). Then:

- \( C_{\geq}(A) = \max A \) if \( A \) is finite and \( C_{\geq} = \emptyset \) if \( A \) is infinite.
- \( C_{\leq}(A) = \min A \) for all sets \( A \).
- \( C_{>} (A) = \emptyset \) for all \( A \).
Non Empty Choice Rules

**Definition**

The choice rule $C$ is **non-empty** if $C(A) \neq \emptyset$ for all non-empty $A \subseteq X$.

**Question 4, Problem Set 1.**

Prove that if $\succsim$ is a preference relation, then $C_{\succsim}(A) \neq \emptyset$ whenever $A$ is finite.

- Given a preference relation, the corresponding induced choice rule is non-empty on finite menus.

**Question 5, Problem Set 1.**

Prove that $\succsim$ is complete and acyclic (but not necessarily transitive) if and only if $C_{\succsim}(A) \neq \emptyset$ whenever $A$ is finite.
Observable Choices

**Definition**

Given a binary relation $\succeq$, the **induced choice rule** $C_{\succeq}$ is defined by

$$C_{\succeq}(A) = \{x \in A : x \succeq y \text{ for all } y \in A\}.$$ 

- This definition starts with a binary relation and derives a choice set.
- A preference relation describes the decision maker’s taste. It is **not** observable.
- Therefore, the induced choice rule is also unobservable: the only way an observer can know $C_{\succeq}$ is to know $\succeq$.

**Definition**

A **choice rule** for $X$ is a correspondence $C : 2^X \setminus \{\emptyset\} \rightarrow X$ such that $C(A) \subseteq A$ for all $A \subseteq X$.

**Remark**

- It is unclear whether $C(A)$ is actually observable. At best, one observes the decision maker choose an element of $C(A)$.
- So, one can **include** elements in the choice set from observation, but one cannot **exclude** them without making additional assumptions on $C(A)$. 
Deduce Preferences From Choice Rules

**GOAL**

Observe DM's choices and use these observations to learn about her preferences.

- In some sense, we want to go in the opposite direction relative to induced choice.
  - Deduce whether or not a particular choice rule could have been induced by some unobserved preference relation.
  - We want to know the rationale behind some particular choices.

- Naturally, this is not possible for any arbitrary choice rule.
### Rationalizable Choice Rules

**Definition**

A choice rule $C$ is rationalized by $\succeq$ if $C = C_{\succeq}$ and $\succeq$ is a preference relation.

- This is a choice rule that behaves as if it maximizes some complete and transitive binary relation among the available alternatives.

**Definition**

A choice rule $C$ is **rationalizable** if there exists a preference relation $\succeq$ such that $C = C_{\succeq}$.

- If a choice rule is not rationalizable, there is no hope of learning whether or not choices are consistent with some preference.
- If a choice rule is rationalizable, one may still not be able to identify the preferences behind it because there may be many possible $\succeq$ that are consistent with it.
- Next, define the preference relation behind a given choice rule.
**Revealed Preferences**

**Definition**

Given a choice rule $C$, its revealed preference relation $\succsim_C$ is defined by

$x \succsim_C y$ if there exists some $A$ such that $x, y \in A$ and $x \in C(A)$.

- $x \succsim_C y$ reads “$x$ is revealed preferred to $y$”.
- The idea is that if DM chooses $x$ when $y$ is available one can say that $x$ is revealed to be preferred to $y$.
- Observing choices and the menus they come from one builds a preference relation $\succsim_C$.
  - These are the preferences consistent with particular choice behavior.

**Remark**

The says that there exists some menu including $x$ and $y$ where $x$ is chosen, not that $x$ is chosen in all menus including $x$ and $y$.

- The “revealed” preferences do not necessarily reflect DM’s levels of happiness or well-being; they only reflect what DM decided to do.
  - If we observe her order a burger at a restaurant, we say she prefers the burger to a salad, but that is not necessarily the same as saying she thinks the burger tastes better.
Proposition

If $C$ is rationalized by a complete and transitive $\succsim$, then $\succsim = \succsim_C$.

Proof.

Let $\succsim$ be a preference relation which rationalizes $C$; that is, for all $A \subseteq X$:

$C(A) = C_{\succsim}(A) = \{x \in A : x \succsim y \text{ for all } y \in A\}$

- Suppose $x \succsim y$. We need to show $x \succsim_C y$.
  - Since $\succsim$ is complete and transitive, we have $x \succsim x$.
  - Thus $x \succsim z$ for all $z \in \{x, y\}$, so $x \in C_{\succsim}(\{x, y\})$.
  - Since $C = C_{\succsim}$, this implies $x \in C(\{x, y\})$.
  - Therefore $x$ is revealed to be preferred to $y$ and $x \succsim_C y$.

- Suppose $x \succsim_C y$. We need to show $x \succsim y$.
  - By definition, there exists some set $A$ with $x, y \in A$ and $x \in C(A)$.
  - Then $x \in C_{\succsim}(A)$ because $C = C_{\succsim}$.
  - By definition, $x \succsim z$ for all $z \in A$.
  - But $y \in A$, so $x \succsim y$. 

□
Proposition

If $C$ is rationalized by $\succsim$, then $\succsim = \succsim_C$.

Now the proof is done, what does this mean?

Remark

The only preference relations that can rationalize $C$ are revealed preference relations.

• To check whether or not a choice rule is rational, all one needs to do is check whether or not it acts as if it were “maximizing” its revealed preference relation.

• Contrapositively, if $\succsim_C$ does not rationalize $C$, then no other preference relation will rationalize $C$.

• This imposes restrictions on DM’s behavior: from some observed choices we can deduce what other (unobserved) choices will have to be.
Restrictions on Choice Rules

Example

Let $X = \{a, b, c\}$ and let $C$ be a rationalizable choice rule such that

\[
C(\{a\}) = \{a\}, \quad C(\{b\}) = \{b\}, \quad C(\{c\}) = \{c\}, \quad \text{and} \quad C(\{b, c\}) = \{b\}.
\]

Can one predict what $C(\{a, b, c\})$ looks like with the help of the previous result?

Claim: $C(\{a, b, c\}) = \{a\}$.

- Note that $a \succ_C a$, $a \succ_C b$, and $a \succ_C c$, so $a \in C_{\succ_C}(\{a, b, c\}) = C(\{a, b, c\})$.

- Since $b \notin C(\{a, b\})$, $b$ must fail to be preferred to some element of $\{a, b\}$, either $\neg (b \succeq a)$ or $\neg (b \succeq b)$.

  - But we have $b \succeq b$ by completeness, therefore we must have $\neg (b \succeq a)$.

- Similarly, we know $\neg (b \succeq_C a)$ because $b \notin C(\{a, b\})$, so $b \notin C_{\succ_C}(\{a, b, c\})$.

- Finally, we know $\neg (c \succeq_C a)$ because $c \notin C(\{a, c\})$, so $c \notin C_{\succ_C}(\{a, b, c\})$.

- Collecting these findings, $C_{\succ_C}(\{a, b, c\}) = \{a\}$.

- Since $C$ is rationalizable, $C(A) = C_{\succ_C}(A)$ for all $A \subseteq X$.

- Therefore $C(\{a, b, c\}) = \{a\}$.
1. Prove the following: if a binary relation $R$ is asymmetric and negatively transitive, then it is transitive.

2. Let $R$ be an equivalence relation on $X$. Then $\forall x \in X, x \in [x]$. Given $x, y \in X$, either $[x] = [y]$ or $[x] \cap [y] = \emptyset$.

3. Prove that if $\succeq$ is a preference relation (i.e. it is complete and transitive), then: $\succeq$ is a preference relation; $\succ$ is asymmetric and transitive; $\sim$ is an equivalence relation; $x \succeq y$ and $y \sim z$ imply $x \succeq z$; and $x \succeq y$ and $y \succ z$ imply $x \succ z$.

4. Prove that if $\succeq$ is a preference relation, then $C_{\succeq}(A) \neq \emptyset$ whenever $A$ is finite.

5. Prove that $\succeq$ is complete and acyclic (but not necessarily transitive) if and only if $C_{\succeq}(A) \neq \emptyset$ whenever $A$ is finite.