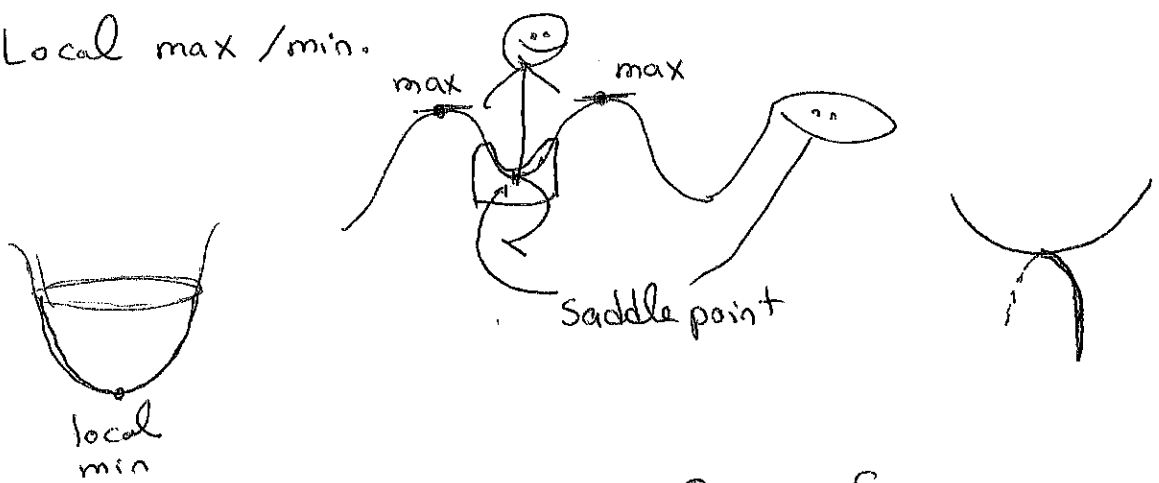
No quiz tomorrow ☺

- 11.7 $f(x,y)$ (or $f(x,y,z)$)

- Critical point
  (x_0, y_0) if $\frac{\partial f}{\partial x}(x_0,y_0)=0$ & $\frac{\partial f}{\partial y}(x_0,y_0)=0$
  (or at least one undef.).

- Local max / min.



- 2nd derivative test. $\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$

- Absolute max / min.
  ↳ max. / min. on some domain

- max/min with Presence of a Constraint
  (Lagrange's multipliers).

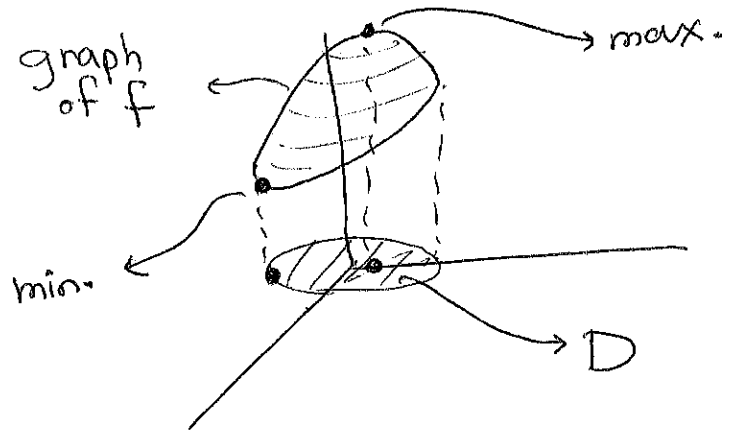
f defined on some domain D in $\mathbb{R}^2$.

Thm. If D is closed & bounded

Contains all its boundary point.

& f is Continuous.

Then f attains its max. (also its min.) on D.

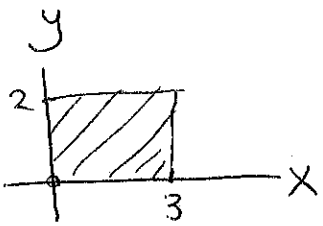


How to find abs. max./min. for a function f. (Continous)

- First find cri. points $f_x = f_y = 0$ (or undefined)
- Find max/min values of f on the boundary of D (∂D = boundary of D) (hopefully not too hard).
  we usually 1-var. methods. here
- Finally ~~[illegible]~~ compare values of f on the cri. points and the boundary of D.

Ex. $f(x,y) = x^4 + y^4 - 4xy + 2$.

$$D = \left\{ (x,y) \;\middle|\; \begin{array}{l} 0 \le x \le 3 \\ 0 \le y \le 2 \end{array} \right\} \longrightarrow$$



$$f_x = 4x^3 - 4y = 0 \Rightarrow x^3 = y$$
$$f_y = 4y^3 - 4x = 0 \Rightarrow y^3 = x$$

$$\Rightarrow (x^3)^3 = x \Rightarrow x^9 = x \Rightarrow x = 0$$
$$x = \pm 1 \quad (x^8 = 1)$$

$\Rightarrow y = 0, +1, -1$

$(0,0)$, $\boxed{(1,1)}$, $(-1,-1)$. Three cri. pts.

inside the region

- $(1,1)$ acceptable cri. pt. inside D.

$$f(1,1) = 1^4 + 1^4 - 4 + 2 = \boxed{0}$$

Boundary: $\longrightarrow$ Four pieces.

$$\begin{cases} x = 0 & 0 \le y \le 2 \\ x = 3 & 0 \le y \le 2 \\ y = 0 & 0 \le x \le 3 \\ y = 2 & 0 \le x \le 3 \end{cases}$$

$\longrightarrow$ $f(0,y) = y^4 + 2$

max $f(0,y)$ at $y = 2$

min $f(0,y)$ at $y = 0$

$\longrightarrow$ $f(3,y) = 3^4 + y^4 - 4 \cdot 3 \cdot y + 2$

$\rightsquigarrow$ we find max/min.

$y^4 - 12y + 83$

$4y^3 - 12 = 0 \Rightarrow y^3 = 3$

$0 \le y = \sqrt[3]{3} \le 2$

$\ldots$ $f(3, \sqrt[3]{3}) \approx 70 \rightarrow$ min.

$f(3,0) = 83 \rightarrow$ max.

Let me skip calculations for the othe two pieces of boundary...

$f(x,2)$ max $\rightarrow x = \sqrt[3]{2}$

min. $\rightarrow x = 3$

$f(x,0)$ max $x = 3$

min. $x = 0$ $\quad 0 \le y \le 2$

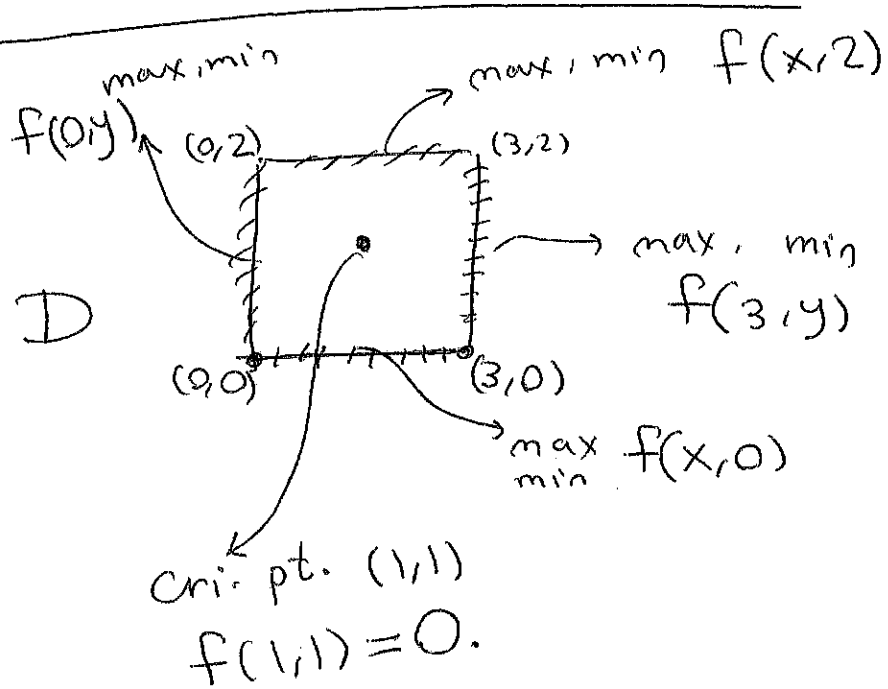
$\hookrightarrow x^4 + 2$

Then we compare all these values:

- Each piece of boundary $\longrightarrow$ max, min.
- Value at cri. pt. $(1,1)$. $\rightarrow f(1,1)=0$

$\rightarrow$ Abs. max. of f on D is $f(3,0)=83$

Abs. min. of f on D is $\boxed{f(1,1)=0}$.



~~(Easy)~~

Ex. $D=\{(x,y) \mid x^2+y^2 \leq 2\} \rightarrow$ Disk of radius 2.

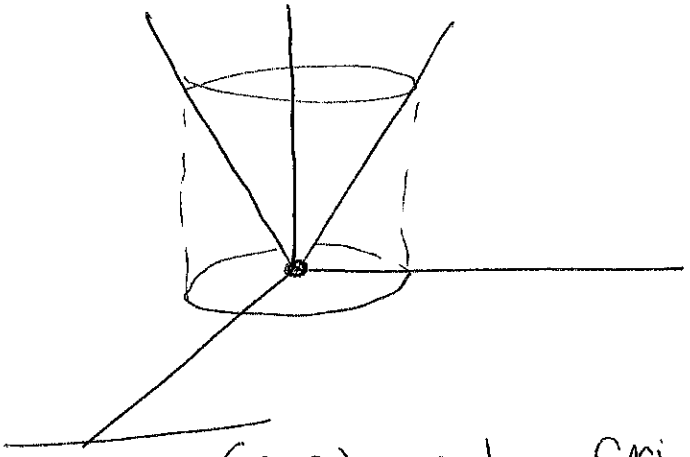
$$f(x,y)=\sqrt{x^2+y^2}.$$

$$f_x=\frac{2x}{2\sqrt{x^2+y^2}}=\frac{x}{\sqrt{x^2+y^2}}=0 \Longrightarrow x=0$$

$\longrightarrow$ undef. if $(x,y)=(0,0)$

$$f_y=\frac{y}{\sqrt{x^2+y^2}}=0 \longrightarrow y=0$$

$(x,y)=(0,0)$ undef.



$z = f(x,y) = \sqrt{x^2+y^2}$

$z^2 = x^2+y^2$

- $(0,0)$ only cri. pt. f_x & f_y undefined.

$f(0,0) = 0$.

- Boundary. $\boxed{x^2+y^2=2}$

$f(x,y) = \sqrt{2} \longrightarrow$ value of f on the boundary.

Abs. max. of $f(x,y) = \sqrt{x^2+y^2}$ on D is $\sqrt{2}$.

" min. " " " " " is $\underline{0}$.

Lagrange's multipliers 11.8

- $f(x,y,z) \longrightarrow$ makes more sense when we have 3 variable.

<u>Problem</u> Find max/min of f ~~[illegible]~~ under a constraint $g(x,y,z)=c$.

$\longrightarrow$ Find cri. pt. of f under a constraint $g(x,y,z)=c$.

Domain $= \{(x,y,z) \mid g(x,y,z) = \text{Const.}\}$

<u>Thm</u> (x_0, y_0, z_0) is a <u>cri. point</u> of f on this constrained domain $g = \text{Const.}$

if $\nabla f(x_0, y_0, z_0)$ & $\nabla g(x_0, y_0, z_0)$ are parallel. i.e. there exists λ scalar

$$\nabla f(x_0, y_0, z_0) = \underbrace{\lambda}_{\text{scalar}} \nabla g(x_0, y_0, z_0).$$

$$\begin{cases} f_x = \lambda g_x \\ f_y = \lambda g_y \\ f_z = \lambda g_z \\ g(x,y,z) = \text{Const.} \end{cases}$$

4 equ. in x, y, z & λ
4 unknowns.

Solve this to find $(\boxed{x, y, z}, \lambda)$

cri. pts. of f subject to $g = \text{Const.}$