



①

Quiz tomorrow : 12.1 - 12.3

12.4 Applications of \iint (I skip "moment of inertia").
Total mass, center of mass in 12.4

12.5 \iiint triple integrals.

$w = f(x, y, z)$ defined on some domain $E \subset \mathbb{R}^3$.

$$\iiint_E f(x, y, z) dV$$

$dxdydz$

volume of the tiny cube

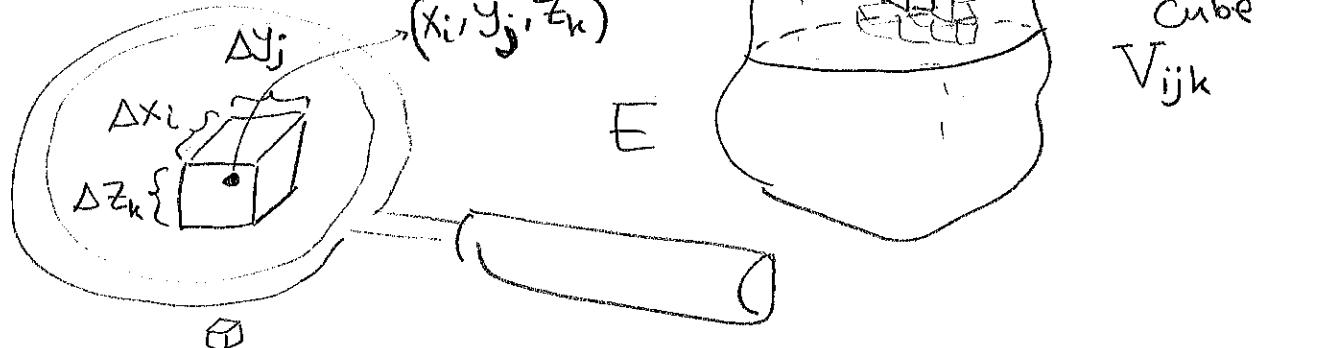
$$= \lim_{\substack{\text{Size of cubes} \rightarrow 0 \\ \Delta x_i, \Delta y_j, \Delta z_k \rightarrow 0}} \sum_{i,j,k} f(x_i, y_j, z_k) \Delta V_{ijk}$$

(sum over all the tiny cubes)

graph of f in 4 dim.
(x, y, z, w)

(Note: we cannot picture graph of f in 4 dim.)

You can think of $\iiint_E f(x, y, z) dV$ as 4-dim. vol.
under 3 dim. graph of f .)



Fubini theorem for triple integrals

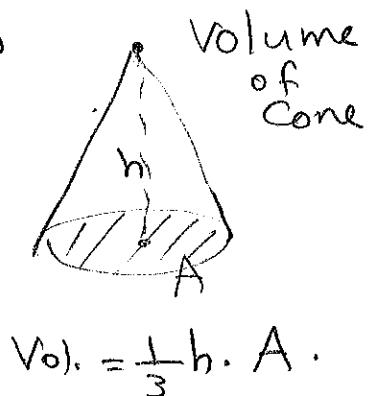
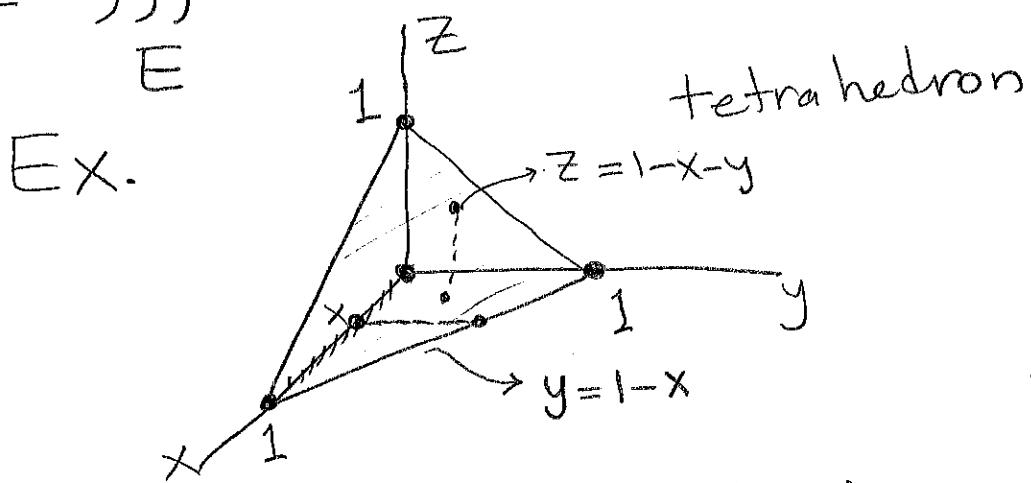
\iiint integrals are an iterated integral.

$$E = \underbrace{[a, b]}_x \times \underbrace{[c, d]}_y \times \underbrace{[e, h]}_z \quad \text{Cube}$$

$$\iiint_E f(x, y, z) dV = \int_e^h \left(\int_c^d \left(\int_a^b f(x, y, z) dx \right) dy \right) dz$$

Extends to regions E which are not cube.

Note $\iiint 1 dV = \text{vol}(E)$



(Cone with triangular base).

$$\text{Vol.} = \iiint_E 1 \, dV = \iiint_{\substack{1 \\ x=0 \\ y=0 \\ z=0}}^{1-x-y} 1 \, dz \Big) dy \Big) dx$$

(3)

$$\int_0^{1-x-y} 1 \, dz = \left[z \right]_0^{1-x-y} = 1-x-y.$$

$$\int_0^{1-x} (1-x-y) \, dy = \left[y - xy - \frac{y^2}{2} \right]_0^{1-x}$$

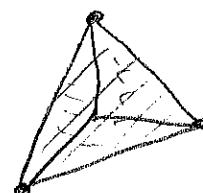
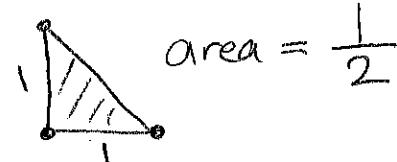
$$= (1-x) - x(1-x) - \frac{(1-x)^2}{2}$$

$$= (1-x)^2 - \frac{(1-x)^2}{2} = \frac{(1-x)^2}{2}$$

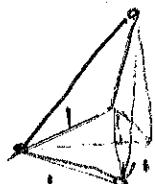
$$\int_0^1 \frac{(1-x)^2}{2} \, dx = \int_{u=1-x}^{u=0} \frac{u^2}{2} (-du)$$

$$= \left[-\frac{u^3}{6} \right]_1^0 = \frac{1}{6} \quad \text{smiley face}$$

2.3



$$\text{Vol.} = \frac{1}{6} =$$



4-dim

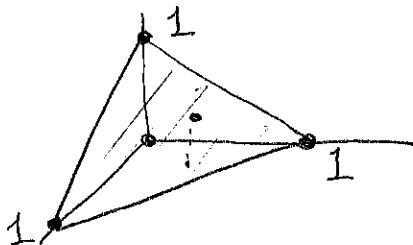
$$\text{Vol.} = \frac{1}{2 \cdot 3 \cdot 4} = \frac{1}{24}$$

$$n\text{-dim} \rightarrow \frac{1}{n!} \quad \text{smiley face}$$

String theory \rightarrow 10-dim. !!
 Universe is

(4)

Ex.



tetrahedron E

Mass & center of mass,

$$\begin{aligned} \rho(x, y, z) &= \text{mass density at } (x, y, z) \\ &= [2 - z] \cdot \text{density} \end{aligned}$$

$$M = \text{total mass} = \iiint_E \rho(x, y, z) dx dy dz$$

$$= \iiint_E (2 - z) dz dy dx$$

← M

$$\bar{x} = \iiint_E x \rho(x, y, z) dx dy dz$$

$$\bar{x} = \iiint_E x \rho(x, y, z) dx dy dz / M$$

Coor.
of
Center
of
mass

$$\bar{y} = \iiint_E y \rho(x, y, z) dx dy dz / M$$

$$\bar{z} = \iiint_E z \rho(x, y, z) dx dy dz / M$$

(5)

$$\bar{x} = \left(\iiint_{0 \ 0 \ 0}^{1-x \ 1-x-y} x (2-z) dz dy dx \right) / M$$

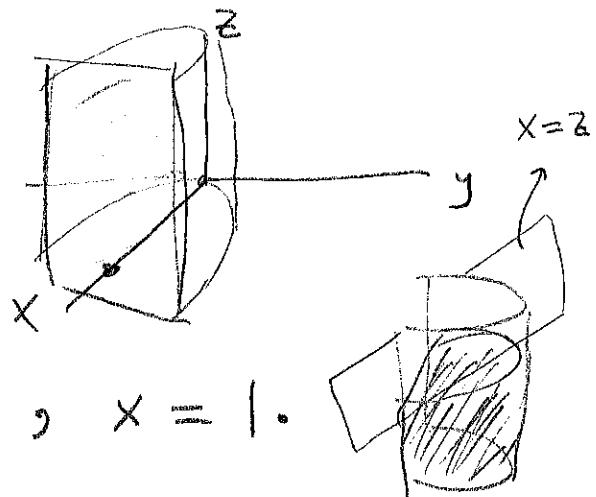
Similar formulae for \bar{y} & \bar{z} .

Ex. Center of mass & total mass

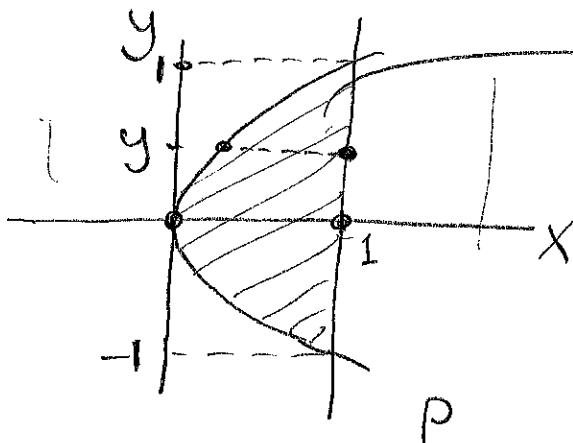
of the solid E of constant density 1
bounded by :

$$x = y^2$$

parabolic cylinder



& planes $x = z$, $z = 0$, $x = 1$.



→ this is the proj. of E
onto xy -plane

$$-1 \leq y \leq 1$$

$$y^2 \leq x \leq 1$$

$$0 \leq z \leq x$$

$$M = \iiint_E 1 dV = \iint_{y=-1}^1 \iint_{x=y^2}^x 1 dz dx dy$$