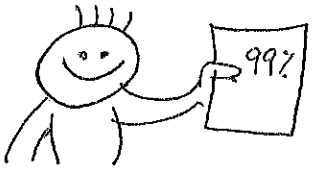
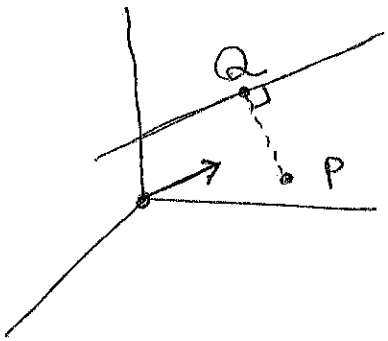


Ave. ≈ 68

#1 MT 1



$\underline{r(t)} = (\underline{t}, 1-\underline{t}, 2+\underline{t})$

$P = (1,0,2)$

min. distance from P to the line.

Solution I had in mind:

direction vec. $= \langle 1, -1, 1 \rangle$

$\overrightarrow{PQ} \perp \langle 1,-1,1 \rangle$.

$(t, 1-t, 2+t) - (1,0,2) = \boxed{(t-1, 1-t, t)}$

$(t-1, 1-t, t) \perp (1,-1,1) \Rightarrow$

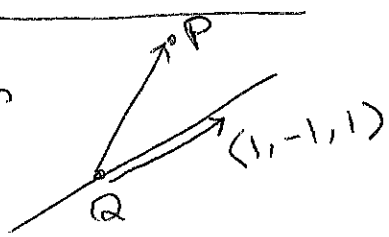
$(t-1, 1-t, t) \cdot (1,-1,1) = (t-1)+(t-1)+t$

$= 3t - 2 = 0$

$\Rightarrow t = \frac{2}{3} \Rightarrow \left(\frac{2}{3}-1, 1-\frac{2}{3}, \frac{2}{3}\right) = \left(\frac{-1}{3}, \frac{1}{3}, \frac{2}{3}\right)$

Find length of $= \sqrt{\frac{1}{9}+\frac{1}{9}+\frac{4}{9}} = \sqrt{\frac{6}{9}} = \sqrt{\frac{2}{3}}$.

Alt. Solution



$\longrightarrow$ find dist. of P to the line using formula for orthogonal proj.

Another Sol.

of course one can find length of $\overrightarrow{PQ} = (t-1, 1-t, t)$

& finding min. by taking derivative.

$\vec{u} = \frac{1}{\sqrt{3}}(1,-1,1)$

5 (a) $f(x,y,z) = y\ln(z) + x^2$ $P=(2,1,1)$

$$\nabla f(2,1,1) = (4,0,1)$$

$$Df_{\vec{u}}(2,1,1) = \underbrace{\nabla f(2,1,1)} \cdot \underbrace{\vec{u}}$$

$$= \left(4\tfrac{1}{\sqrt{3}}, \cancel{0\tfrac{-1}{\sqrt{3}}}, \tfrac{1}{\sqrt{3}}\right)$$

$$= \frac{4}{\sqrt{3}} + 0 + \frac{1}{\sqrt{3}} = \frac{5}{\sqrt{3}}$$

---

5 hyperboloid of one sheet $\rightsquigarrow$ $x^2+y^2-z^2=1$

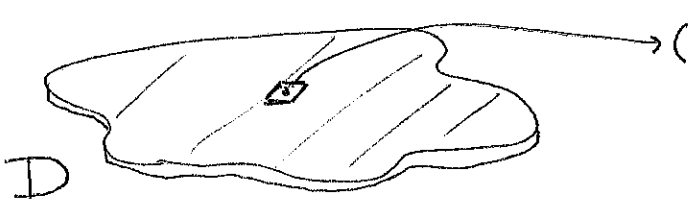
similar to

Back to integrals!

12.4 Finding total mass & center of mass.

$\rho(x,y)$ = mass density at (x,y).

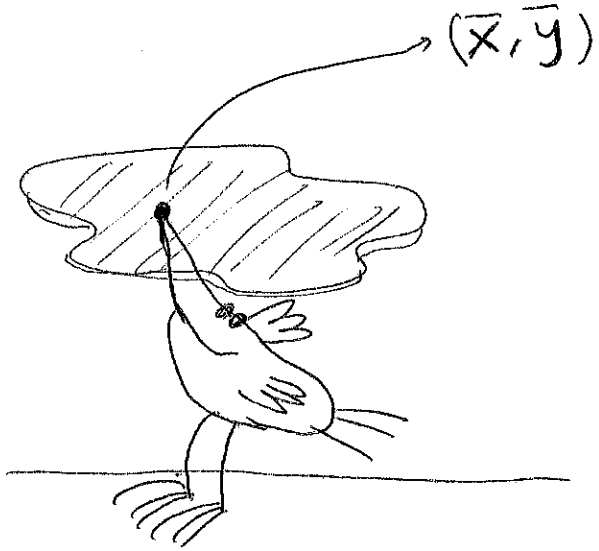
resembles def. of derivative

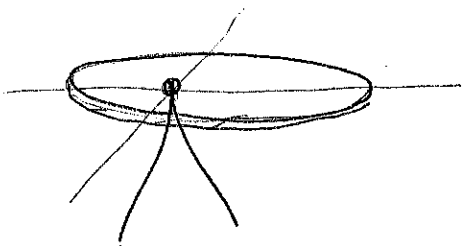


$$\rho(x,y) \overset{\text{def.}}{=} \lim_{\substack{\text{size of rect.} \\ \to 0}} \left\{ \frac{\text{mass of tiny rect.}}{\text{area of tiny rect.}} \right\}$$

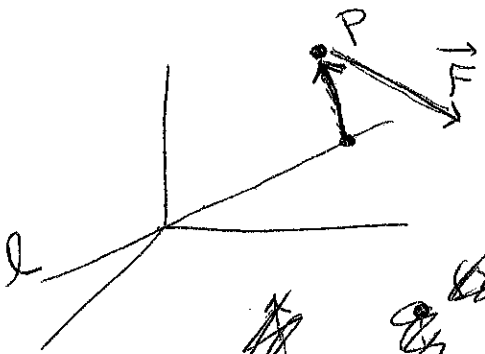
$$\boxed{\text{Total mass} = \iint_D \rho(x,y)\,\overbrace{dx\,dy}^{dA}}$$

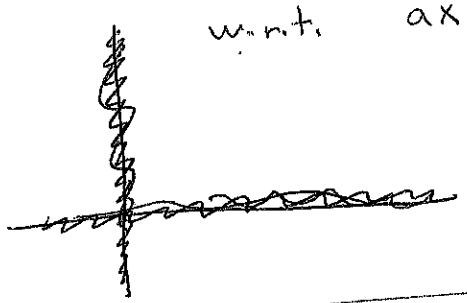
Center of mass





Angular Momentum of a force $\vec{F}$ applied to a point P w.r.t. axis ℓ.



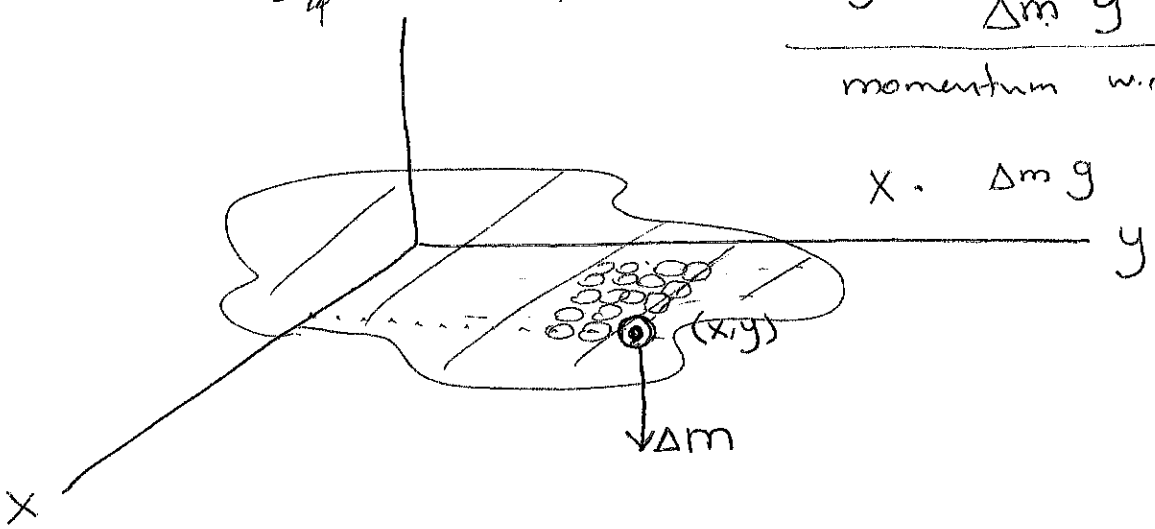


momentum w.r.t. x-axis:

$y \cdot \Delta m\, g$

momentum w.r.t. y-axis:

$x \cdot \Delta m\, g$



Total momentum w.r.t. x-axis :

$$\iint_D y\,\rho(x,y)\,dA = \bar{y}\cdot M$$

total mass

" " " y-axis :

$$\iint_D x\,\rho(x,y)\,dA = \bar{x}\cdot M$$

$(\bar{x},\bar{y})$ Coor. of center of mass:

$$\bar{x} = \frac{\iint_D x\rho(x,y)\,dA}{\iint_D \rho(x,y)\,dA}$$

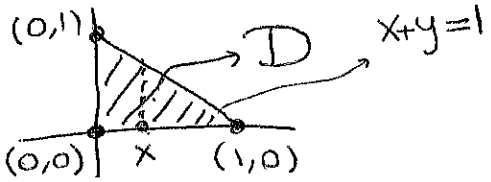
$$\bar{y} = \frac{\iint_D y\,\rho(x,y)\,dA}{\iint_D \rho(x,y)\,dA}$$

$dA \to dxdy$

remember.

(very similar to notion of averaging)

Ex. 2



$\rho(x,y) = 2x+y$.
mass density.

$$M = \text{total mass} = \iint_D 2x+y\,dxdy$$

$$\int_0^1 \left(\int_0^{1-x} 2x+y \; dy \right) dx$$

$$\frac{M_y}{M} = \left(\int_0^1 \int_0^{1-x} x\,(2x+y)\; dy\,dx \right) \Big/ M = \bar{x}$$

$$\frac{M_x}{M} = \left(\int_0^1 \int_0^{1-x} y\,(2x+y)\; dy\,dx \right) \Big/ M = \bar{y}$$