

9-9:50am

Exam 1

Fall 2011

Math 0240

100 points total

Your name: Solutions

No calculators. Show all your work (no work = no credit). Explain every step. Write neatly.

1. [10 points] Find a vector equation and parametric equations for the line segment that joints $P(-2, 4, 0)$ to $Q(6, -1, 2)$.

Vector eq: $\bar{r}(t) = (1-t)\bar{r}_0 + t\bar{r}_1$,

where $0 \leq t \leq 1$, $\bar{r}_0 = \langle -2, 4, 0 \rangle$

$$\bar{r}_1 = \langle 6, -1, 2 \rangle$$

$$\bar{r}(t) = \langle -2+2t+6t, 4-4t-t, 0+2t \rangle$$

$$\boxed{\bar{r}(t) = \langle -2+8t, 4-5t, 2t \rangle, 0 \leq t \leq 1}$$

parametric

eq's:

$$\boxed{\begin{aligned} x &= -2+8t \\ y &= 4-5t \\ z &= 2t \\ 0 &\leq t \leq 1 \end{aligned}}$$

(w/out the condition $0 \leq t \leq 1$ it is equation of
a line, not a line segment)

2. [10 points] Explain why the function $f(x, y) = x/y$ is differentiable at the point $(6, 3)$. Find the linearization $L(x, y)$ of the function at that point.

$$f_x = \frac{1}{y}, f_y = -\frac{x}{y^2}$$

f_x and f_y exist and are continuous near $(6, 3)$. Therefore $f(x, y)$ is differentiable at $(6, 3)$.

$$f_x(6, 3) = \frac{1}{3}, f_y(6, 3) = -\frac{6}{9} = -\frac{2}{3}$$

$$f(6, 3) = \frac{6}{3} = 2$$

$$L(x, y) = f(6, 3) + f_x(6, 3)(x-6) + f_y(6, 3)(y-3)$$

$$L(x, y) = 2 + \frac{1}{3}(x-6) - \frac{2}{3}(y-3)$$

$$L(x, y) = \frac{1}{3}x - \frac{2}{3}y + 2$$

3. (a) [10 points] Find the gradient of $f(x, y) = y \ln x$.

$$\nabla f(x, y) = \langle f_x, f_y \rangle = \left\langle \frac{y}{x}, \ln x \right\rangle$$

(b) [10 points] Evaluate the gradient at the point $P(1, -3)$.

$$\nabla f(1, -3) = \left\langle \frac{-3}{1}, \ln 1 \right\rangle = \langle -3, 0 \rangle$$

(c) [10 points] Find the rate of change of f at P in the direction of the vector
 $\bar{u} = \langle -\frac{4}{5}, \frac{3}{5} \rangle$.

$$\begin{aligned}\bar{\nabla} f(1, -3) \cdot \bar{u} &= \langle -3, 0 \rangle \cdot \left\langle -\frac{4}{5}, \frac{3}{5} \right\rangle = \\ &= (-3)\left(-\frac{4}{5}\right) + 0 = \boxed{\frac{12}{5} = 2\frac{2}{5} = 2.4}\end{aligned}$$

4. [10 points] Find parametric equations and symmetric equations for the line of intersection of the planes $x + y + z = 1$ and $x + z = 0$.

Normal vectors to the planes are

$$\bar{n}_1 = \langle 1, 1, 1 \rangle, \quad \bar{n}_2 = \langle 1, 0, 1 \rangle$$

The direction vector \bar{v} of the line is orthogonal to both \bar{n}_1 and \bar{n}_2 :

$$\bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \bar{i} - \bar{k} = \langle 1, 0, -1 \rangle$$

The point $P_0(0, 1, 0)$ is on the line (it is on both planes, easy to see).

Vector equation:

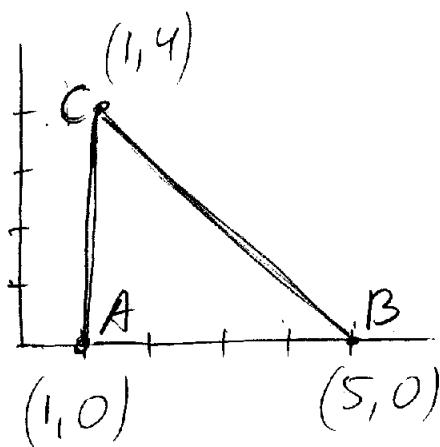
$$F(t) = \langle 0, 1, 0 \rangle + t \langle 1, 0, -1 \rangle = \langle t, 1, -t \rangle$$

param. eq's:
$$\boxed{x = t, y = 1, z = -t}$$

$$t = x, \quad t = -z$$

Symmetric eq's:
$$\boxed{x = -z, y = 1}$$

5. [20 points] Find the absolute maximum and minimum values of the function $f(x, y) = 3 + xy - x - 2y$ on the set D if D is the closed triangular region with vertices $(1, 0)$, $(5, 0)$, and $(1, 4)$.



CP's inside D :

$$f_x = y - 1 = 0 \Rightarrow y = 1$$

$$f_y = x - 2 = 0 \Rightarrow x = 2$$

$$\text{CP: } (2, 1), f(2, 1) = \boxed{1}$$

on $AB \quad y=0 \quad g_1(x) = f(x, 0) = 3-x$ is decreasing function
 $1 \leq x \leq 5$

$g_1(x)$ attains max at $x=1 \quad g(1) = \boxed{2}$
 ——— min at $x=5 \quad g(5) = \boxed{-2}$

on $CB \quad y = -x + 5, \quad g_2(x) = f(x, -x+5) = -x^2 + 6x - 7$
 $1 \leq x \leq 5$

$g_2'(x) = -2x + 6 = 0, \quad x = 3, \quad g_2(1) = \boxed{-2}, \quad g_2(3) = \boxed{2}$
 $\quad \quad \quad g_2(5) = \boxed{-2}$

on $AC \quad x=1, \quad 0 \leq y \leq 4, \quad g_3(y) = f(1, y) = 2 - y$ (decreasing)

$g_3(0) = \boxed{2}, \quad g_3(4) = \boxed{-2}$

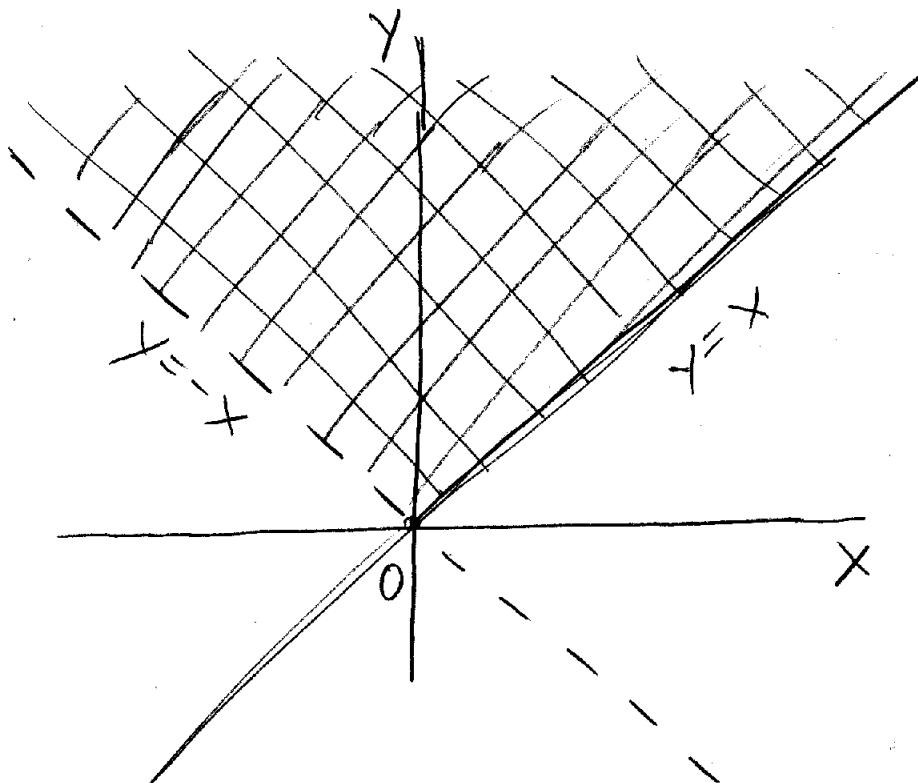
Abs max value: $\boxed{2}$ Abs. min value: $\boxed{-2}$

6. [10 points] Find and sketch the domain of the function

$$f(x, y) = \sqrt{y-x} \ln(y+x)$$

$$D = \{(x, y) \mid y-x \geq 0, y+x > 0\}$$

$$= \{(x, y) \mid y \geq x, y > -x\}$$



7. [10 points] Find the scalar and vector projections of \bar{b} onto \bar{a} and \bar{b} onto \bar{a} if $\bar{a} = \bar{i} + \bar{j} + \bar{k}$ and $\bar{b} = \bar{i} - \bar{j} + \bar{k}$.

$$\bar{a} = \langle 1, 1, 1 \rangle, \bar{b} = \langle 1, -1, 1 \rangle$$

$$|\bar{a}| = \sqrt{3}, |\bar{b}| = \sqrt{3}, \bar{a} \cdot \bar{b} = 1 - 1 + 1 = 1$$

$$\text{comp}_{\bar{a}} \bar{b} = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}|} = \boxed{\frac{1}{\sqrt{3}}}$$

$$\text{proj}_{\bar{a}} \bar{b} = \frac{\bar{a} \cdot \bar{b}}{|\bar{a}|^2} \bar{a} = \frac{1}{3} \langle 1, 1, 1 \rangle = \boxed{\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \rangle}$$

$$\text{comp}_{\bar{b}} \bar{a} = \frac{\bar{b} \cdot \bar{a}}{|\bar{b}|} = \boxed{\frac{1}{\sqrt{3}}}$$

$$\text{proj}_{\bar{b}} \bar{a} = \frac{\bar{b} \cdot \bar{a}}{|\bar{b}|^2} \bar{b} = \frac{1}{3} \langle 1, -1, 1 \rangle = \boxed{\langle \frac{1}{3}, -\frac{1}{3}, \frac{1}{3} \rangle}$$

bonus problem [7 points extra] Reduce the equation $-x^2 + 2y^2 + 3z^2 = 0$ to one of the standard forms, classify the surface and sketch it.

$$x^2 = 2y^2 + 3z^2 \Leftrightarrow x^2 = \left(\frac{y}{\sqrt{2}}\right)^2 + \left(\frac{z}{\sqrt{3}}\right)^2$$

a cone along x -axis:

