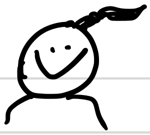
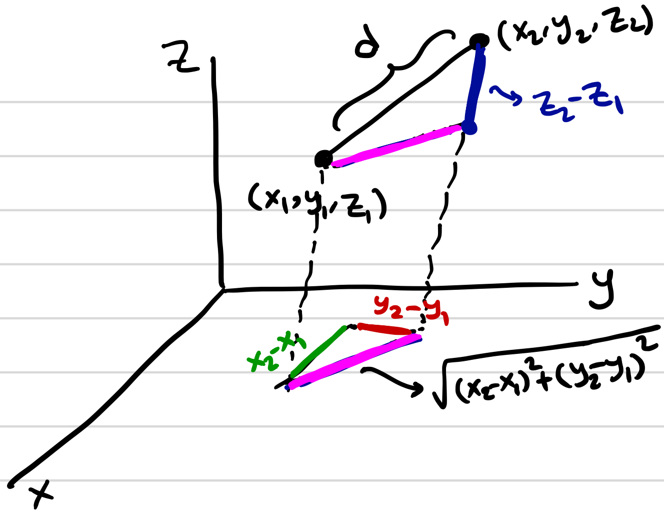
Sep 29 / 2017





$$d = \sqrt{\underline{(x_2-x_1)^2 + (y_2-y_1)^2} + \underline{(z_2-z_1)^2}}$$

(To get the formula we use Pythogras thm. twice).

Equ. of sphere

- $\boxed{x^2 + y^2 + z^2 = R^2}$ ~ distance of (x,y,z) to $(0,0,0)$ is equal to R.

  (x,y,z) such that

  $C = (h, k, \ell)$ fixed
- center of sphere

  R = radius of sphere

$$\boxed{(x-h)^2 + (y-k)^2 + (z-\ell)^2 = R^2}$$

(x,y,z) such that

(distance of (x,y,z) to (h,k,ℓ) is R).

(#13 10.1)

Ex. $x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$ (★)

Show this is equ. of a sphere, & find radius & center of this sphere.

$$(x - \bigcirc)^2 + (y - \bigcirc)^2 + (z - \bigcirc)^2 = R^2$$

$\boxed{x^2 - 2x + 1} - 1$ (Completing square)

$(x-1)^2 - 1$

$(x-a)^2 = x^2 - 2ax + a^2$

$y^2 - 4y + 4 - 4$

$(y-2)^2 - 4$

$z^2 + 8z + 16 - 16$

$(z+4)^2 - 16$

(★) $(x-1)^2 + (y-2)^2 + (z+4)^2 = 15 + 1 + 4 + 16$

$= 36 = 6^2$

Center $= (1, 2, -4)$

radius $= 6$ ☺

Aside Some quad. expression in x, y, z = Constant

What does this look like geometrically?

Some quad. expression.

Ex. $x^2 + xz - y^2 + 3xy + 10x + y + 100 = 2$

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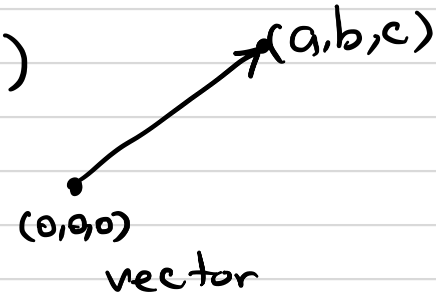
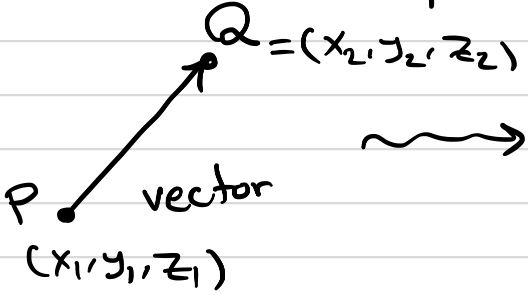
sphere, ellipsoid, hyperboloid, paraboloid...

10.2 Vectors (2D or 3D)

vectors (a,b,c) point $\rightsquigarrow$

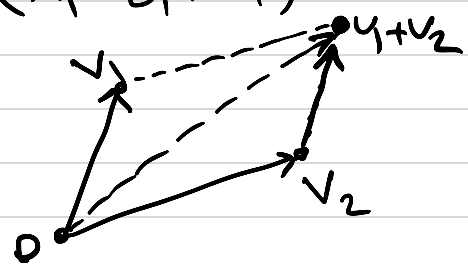




$\rightsquigarrow$ represented by $\underbrace{Q - P}_{(x_2-x_1,\ y_2-y_1,\ z_2-z_1)}$

- Addition of vectors

$v_1 = (x_1, y_1, z_1)$ $\quad v_2 = (x_2, y_2, z_2)$



$$v_1 + v_2 = (x_1+x_2,\ y_1+y_2,\ z_1+z_2)$$

$0 = (0,0,0)$ $\quad v + 0 = v$ for every v

$v \rightsquigarrow -v$ $\quad v + (-v) = 0$

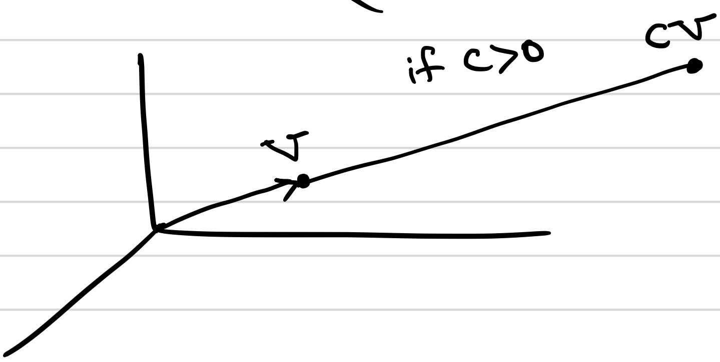
$v_1 + v_2 = v_2 + v_1$.

vectors ⟶ · velocity
· force
· Momentum
⋮

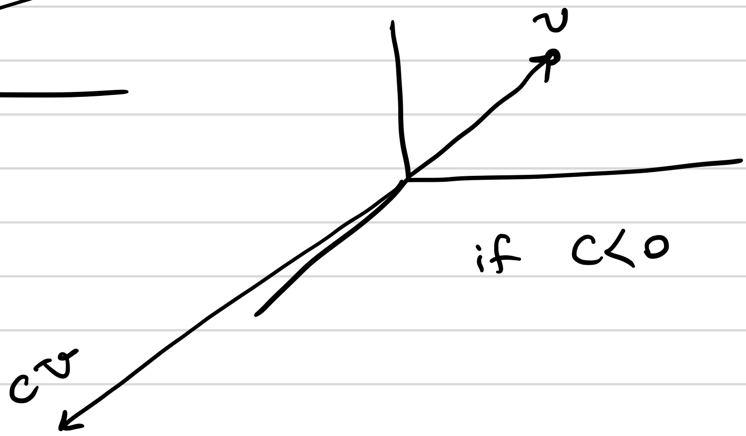
- <u>Scalar multiplication</u>

c scalar = number $\qquad v = (x, y, z)$

$$cv = (cx, cy, cz)$$



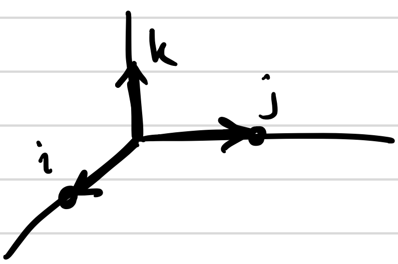
scaled c times.



- $c(v_1 + v_2) = cv_1 + cv_2$

$\vec{i} = (1, 0, 0)$
$\vec{j} = (0, 1, 0)$
$\vec{k} = (0, 0, 1)$

standard basis vectors



$$(3,5,7) = 3\vec{i} + 5\vec{j} + 7\vec{k}$$

$$3\vec{i} \to (3,0,0) \qquad 5\vec{j} \to (0,5,0) \qquad 7\vec{k} \to (0,0,7)$$

$$(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|v| = \sqrt{x^2+y^2+z^2}$$

(sometimes $\|v\|$)

$v = (x,y,z)$

v "unit" vector if $|v|=1$.

. v any vector
c any scalar $\rightsquigarrow$ $|cv| = |c|\,|v|$

abs. value of a number

length of vector

. v any vector $\rightsquigarrow$ $\dfrac{v}{|v|}$ unit vector.

$$v=(x,y,z) \rightsquigarrow \frac{v}{|v|} = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

unit vector

Ex. $v=(1,2,3)$ $\quad w=(1,-1,1)$

. $|2v| = |(2,4,6)| = \sqrt{2^2+4^2+6^2}$.

. $2v-w = (2,4,9)-(1,-1,1)$
$= (1,5,8)$.

10.3 Dot product

("product" $\rightsquigarrow$ some operation that distribute into addition)

$$v \cdot (u + w) = (v \cdot u) + (v \cdot w).$$

$V_1 = (x_1, y_1, z_1)$
$V_2 = (x_2, y_2, z_2)$ $\rightsquigarrow$ $V_1 \cdot V_2 \overset{\text{def.}}{=} \underbrace{x_1x_2 + y_1y_2 + z_1z_2}_{\text{scalar number}}$

- $v_1 \cdot v_2$ has info about $|v_1|$, $|v_2|$, the angle between them.

Theorem / general fact

θ = angle between v_1 & v_2

$$V_1 \cdot V_2 = x_1x_2 + y_1y_2 + z_1z_2 = |v_1|\,|V_2| \cos(\theta)$$