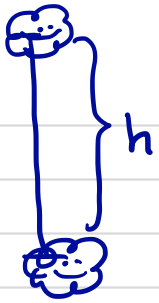
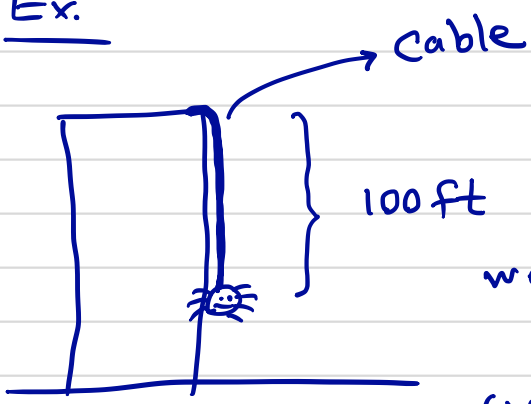
Sep 27 / 2017



weight = mg

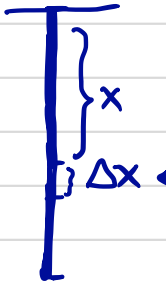
Work = (mg) h
weight

Ex.



weight = 200 lb

$\text{density} = \frac{(\text{weight})}{100} = \frac{200}{100} = 2 \text{ lb/ft}$

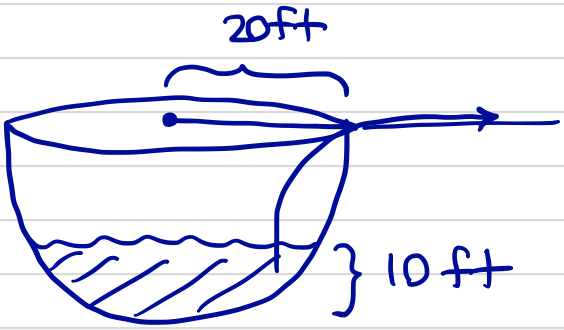


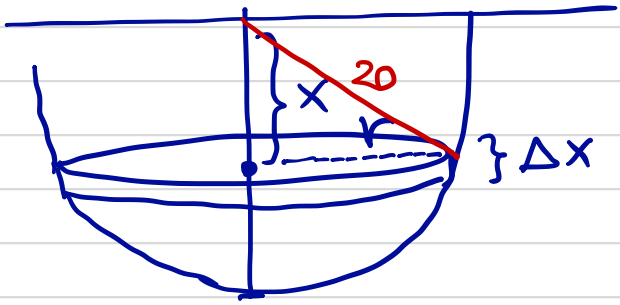
weight = $2 \cdot \Delta x$ lb.

work = $(2 \cdot \Delta x) \cdot x$

$$\text{Total work} = \int_0^{100} 2x \, dx = x^2 \Big|_0^{100} = 10000.$$

ρ = weight density of water

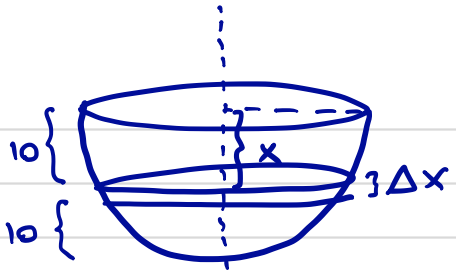




$r_{(x)} = \sqrt{20^2 - x^2}$

$A(x) = \pi r^2 = \pi (20^2 - x^2)$

weight of the layer = $\rho A(x) \Delta x = \rho \pi (400 - x^2) \Delta x$

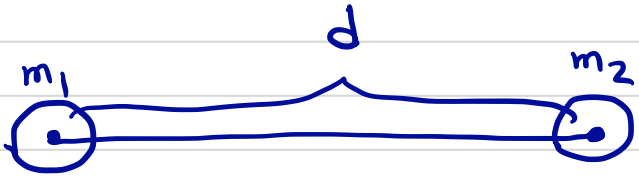


weight of layer $= \rho \pi (400 - x^2) \Delta x$

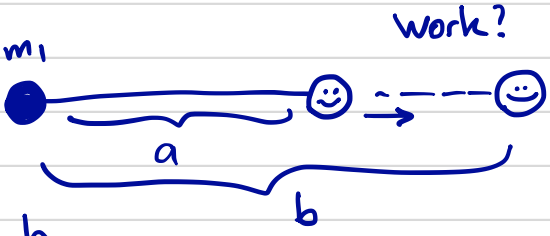
Total work $= \int_{10}^{20} \underbrace{\rho \pi (400 - x^2)}_{\text{weight of layer}} \underbrace{x}_{\text{distance}} \, dx$

Ex. (#21 in 7.6)

Law of gravity



$$F = G \frac{m_1 m_2}{d^2}$$

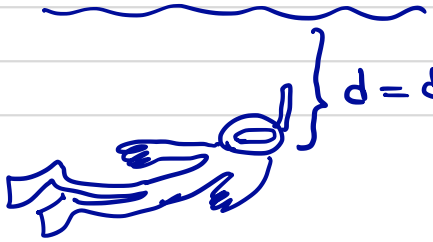


$$W = \int \text{Force} \cdot dx = \int_a^b G \frac{m_1 m_2}{x^2} \, dx$$

$$= G m_1 m_2 \int_a^b \frac{1}{x^2} dx = G m_1 m_2 \left(\frac{1}{a} - \frac{1}{b} \right)$$ ☺

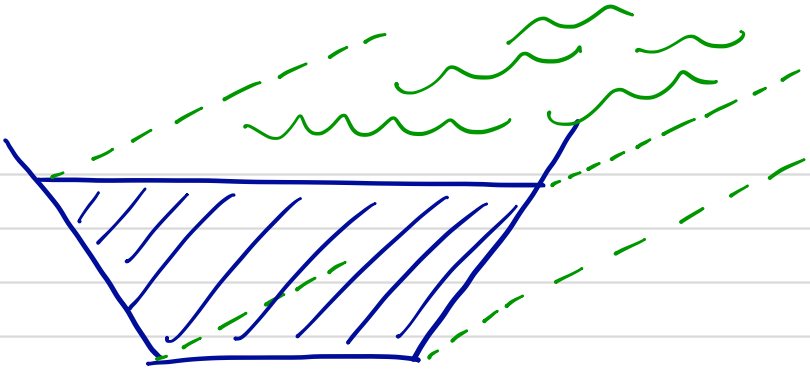
Ex. Hydro static pressure

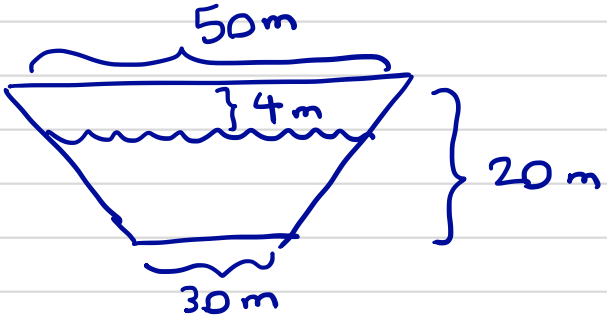
Force applied to surface of a dam by water (pressure).

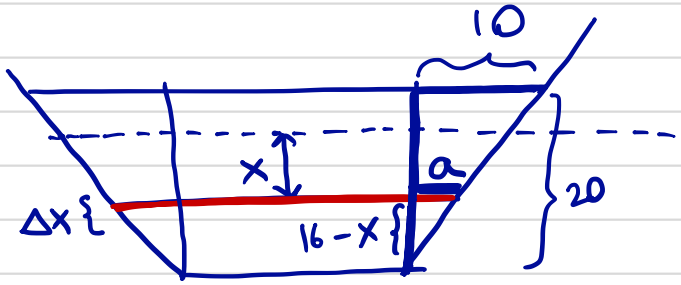


P = pressure $= \rho d$

ρ → weight density







Length of red segment $= 2a + 30 = 16 - x + 30 = 46 - x$.

$$\frac{a}{10} = \frac{16-x}{20} \rightsquigarrow a = \frac{1}{2}(16-x) = 8 - \frac{x}{2}.$$

Force of the red segment $= \underbrace{\rho x}_{\text{pressure}} \cdot \underbrace{(46-x)\,\Delta x}_{\text{area}}$

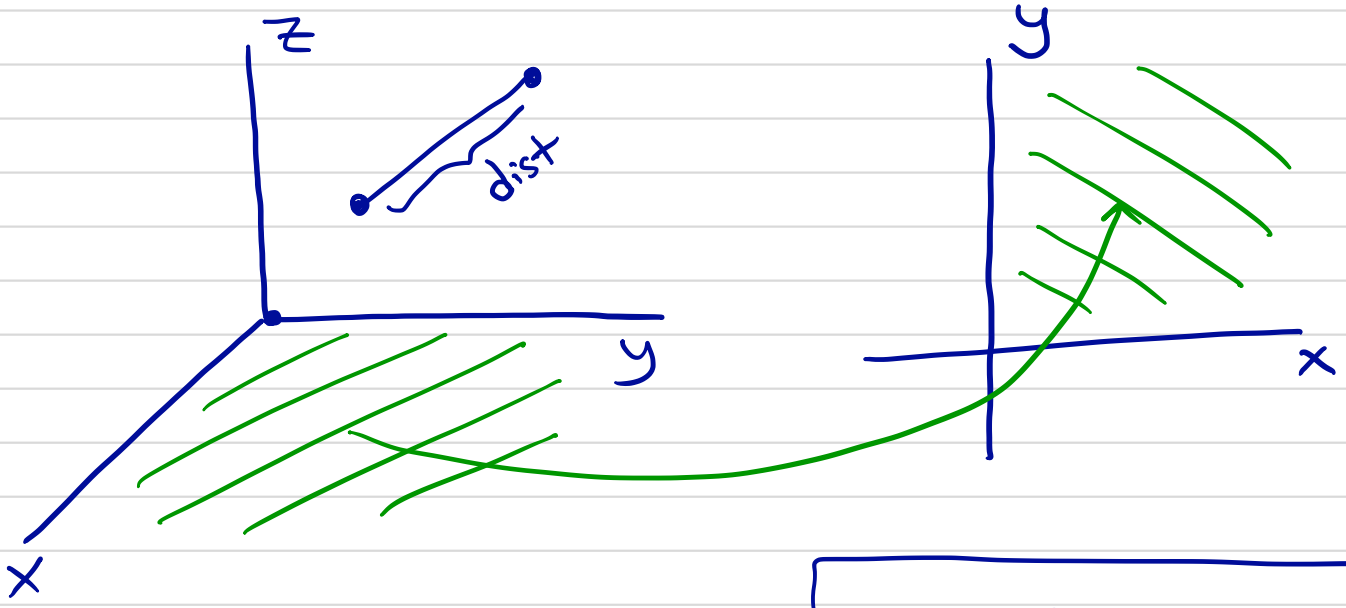
Total force

$$F = \int_0^{16} \rho\,(46x - x^2)\,dx$$

Other applications: Finding center of mass & moments.

7.7 Diff. equations $\rightsquigarrow$ we will see later.

10.1 Review of 3D geometry.



$$d\big((x_1, y_1), (x_2, y_2)\big) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

d ↳ distance

$$d\big((x_1, y_1, z_1), (x_2, y_2, z_2)\big) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}.$$