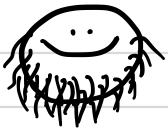
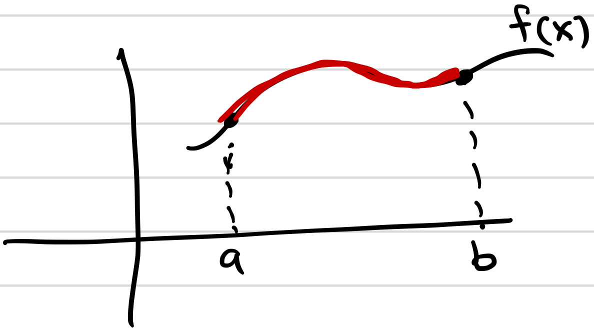
Sep. 25 / 2017



Tomorrow's quiz : Sec. 7.2 & 7.3 (volumes)

Arc length :



$$\int_a^b \sqrt{1+(f'(x))^2}\,dx$$

length of the vec.

$$(x, f(x)) \xrightarrow{\text{diff}} (1, f'(x)) \qquad (1, f'(x))$$

Ex. (#17 Sec. 7.4)

$$y = \ln(1-x^2) \qquad 0 \leq x \leq \frac{1}{2}$$

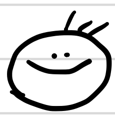
Find the arc length :

set up an integral that computes...

$$f(x) = \ln(1-x^2)$$

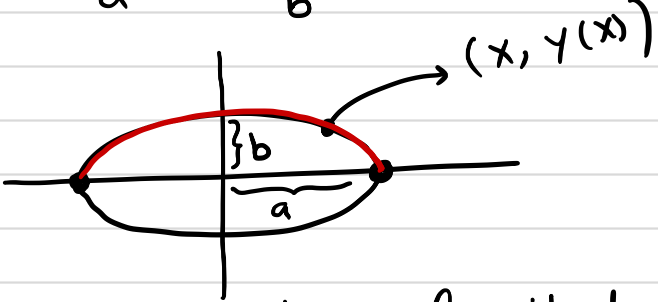
$$f'(x) = \frac{1}{1-x^2} \cdot (-2x).$$

$$\text{Arc length} = \int_0^{\frac{1}{2}} \sqrt{1 + \left(\frac{-2x}{1-x^2}\right)^2}\,dx$$



Ex (Circumference of an ellipse)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Write an integral that gives circum. of the ellipse.

$$2\int_{x=-a}^{a} \sqrt{1 + y'(x)^2}\, dx$$

$$y = \sqrt{b^2 - \frac{x^2}{a^2} b^2}$$

Alternatively, you can use implicit diff.

$$y' = \frac{1}{2\sqrt{b^2 - \frac{b^2 x^2}{a^2}}} \cdot -2x\,(b^2/a^2).$$

Circum. of ellipse $= 2\int_{-a}^{a} \sqrt{1 + \frac{4x^2\,(b^2/a^2)^2}{4\left(b^2 - \frac{b^2x^2}{a^2}\right)}}\, dx$

(you can little bit simplify this expression).

- It took centuries to finally prove that above integral can not be expressed in terms of "elementary function" e.g. sin, cos, exp, ln & polynomials. ?!

. Ramanujan found very nice approx. for this integral
(Movie : The man who knew infinity).

. We skip. 7.5 $\longrightarrow$ Surface area.

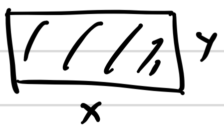
<u>7.6</u> Applications in physics.

General principle :

x, y two quantities

Third <u>quantity</u> given by xy (product).
(when x & y constant).

Simple example:



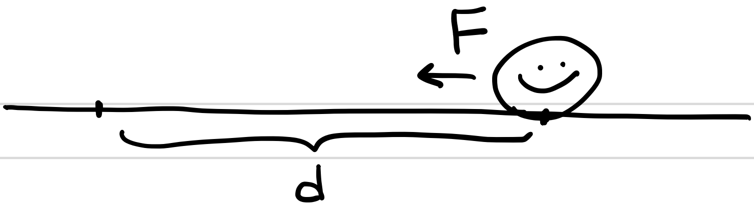
Area $= x.y$
(of rectangle)

$\rightarrow$ Now suppose x is variable & y depends on x (i.e $y = f(x)$).

Then :

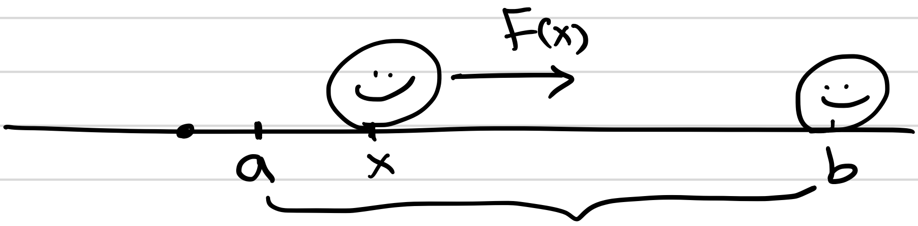
The third quantity for values of x $a \leq x \leq b$ $= \displaystyle\int_{x=a}^{b} f(x)\,dx$

<u>Ex.</u> <u>Work</u> $\underbrace{W = \text{Force} \times \text{distance}}_{\text{multiplication formula}}$



If F constant then $W = F \cdot d$

- when F is variable say $F(x)$, x distance to 0. (or position)



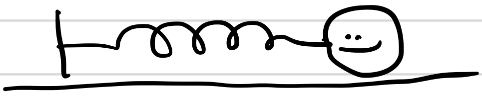
Work done by force $F(x)$ from $x=a$ to $x=b$ is:

$$W = \int_a^b F(x)\,dx \,.$$

sum

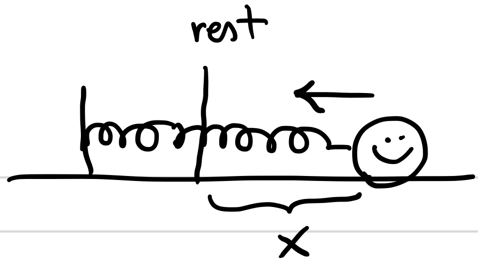
Examples we see in 7.6:

- Work $\longrightarrow$ work by force of gravity
  " by force of a spring



- Pressure (hydrostatic pressure)
- Center of mass & momentum.

Ex. Hook's law



$$F = -K \cdot x$$

Hook's const.

Work done to stretch the spring from $x=a$ to $x=b$ (Const. K known) $= \displaystyle\int_{x=a}^{b} -kx \, dx$

Next time:

work needed to empty the pool?