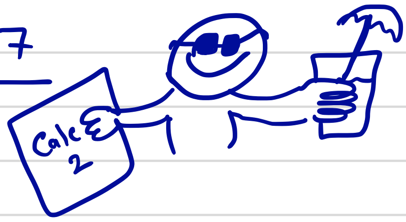
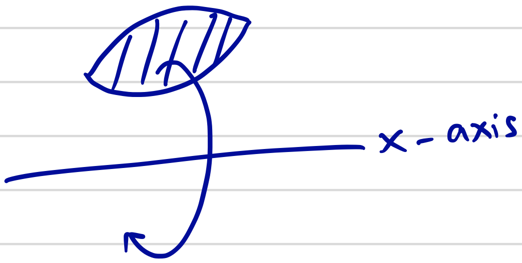
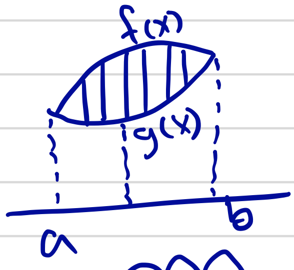
Sep. 22 / 2017

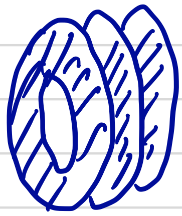


Last time: volumes of solids obtained by rotation



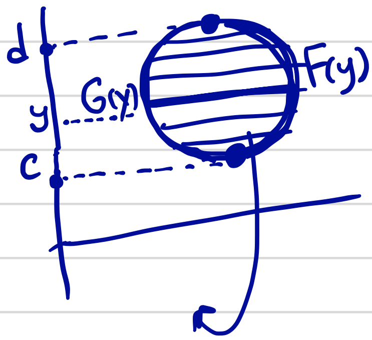


perpendicular slices

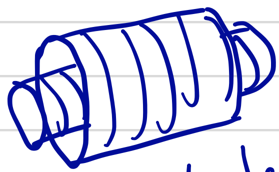


Vol. $= \displaystyle\int_{x=a}^{b} \pi\left(f(x)^2 - g(x)^2\right)dx$

• Disc/washer method



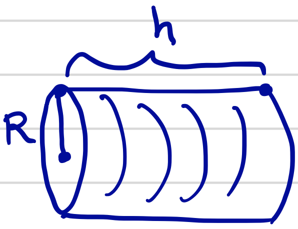
parallel slices



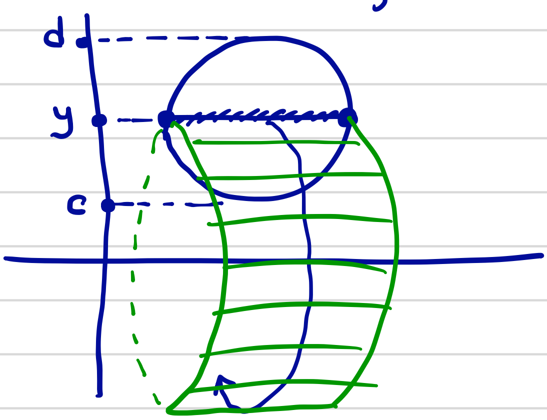
cylinders

Vol. $= \displaystyle\int_{y=c}^{d} \left(F(y)-G(y)\right) 2\pi\, y \, dy$

• Cylindrical shells



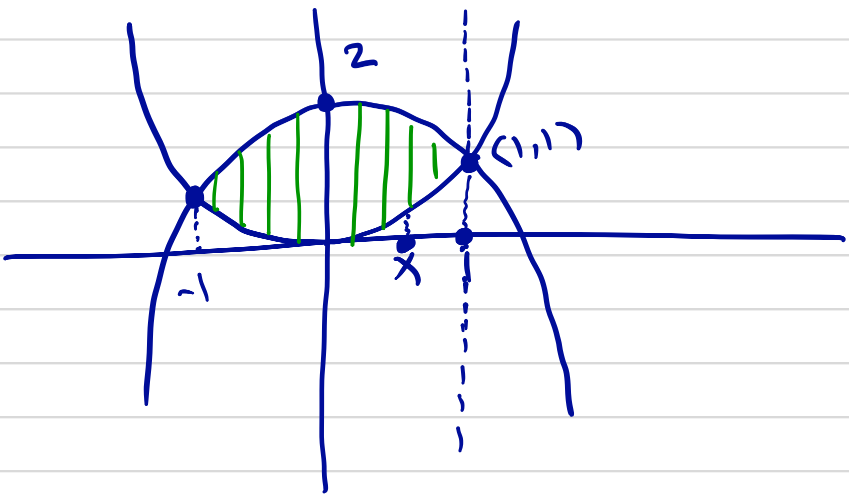
Area $= h\, 2\pi R$



• <u>Remark</u> Formula in the text book is for ratation around y-axis.

<u>Ex.</u> (#18 Sec. 7.3) Use Cylindrical shells to find volume.

Region between $y=x^2$, $y=2-x^2$ & rotated around $x=1$.



$$\text{Vol.} = \int_{-1}^{1} \underbrace{\left(2-x^2 - x^2\right)}_{\text{height of cylider}} 2\pi(1-x)\, dx$$

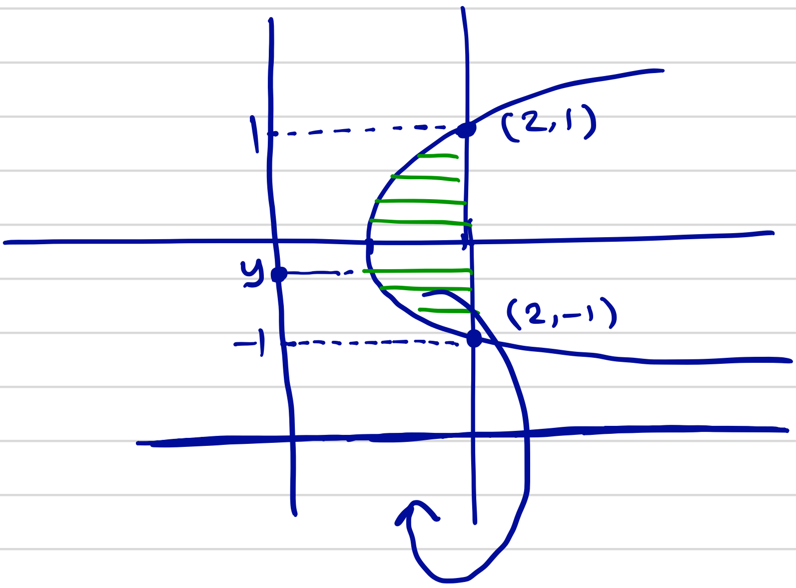
$$= 4\pi \int_{-1}^{1} (1-x^2)(1-x)\, dx .$$

Ex. (#20 Sec. 7.3)

Vol. using cylindrical shells.

region between $x = y^2+1$, $x = 2$

rotated around $y = -2$.

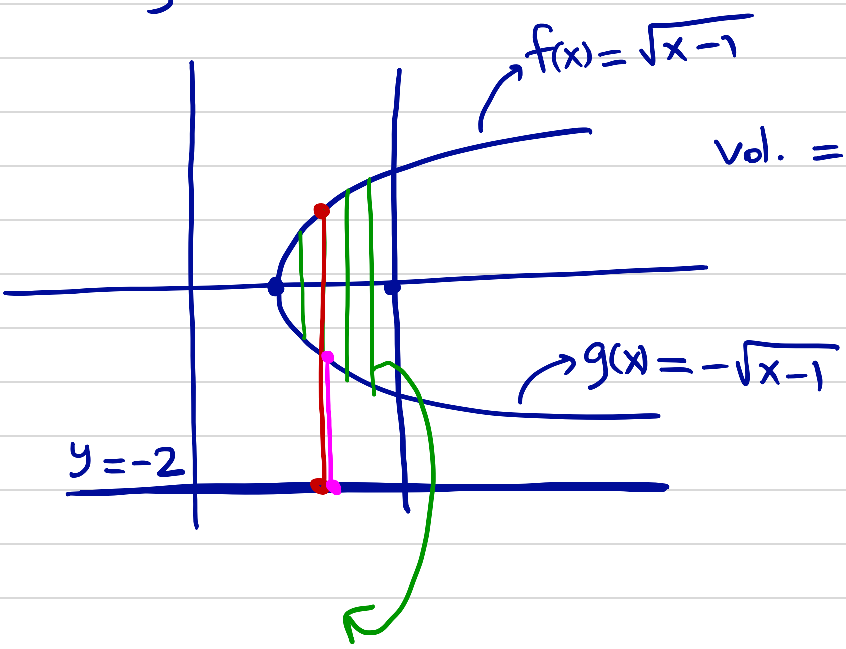


$$\text{Vol.} = \int_{y=-1}^{1} (2 - y^2 - 1)\, 2\pi (y+2)\, dy$$

☺

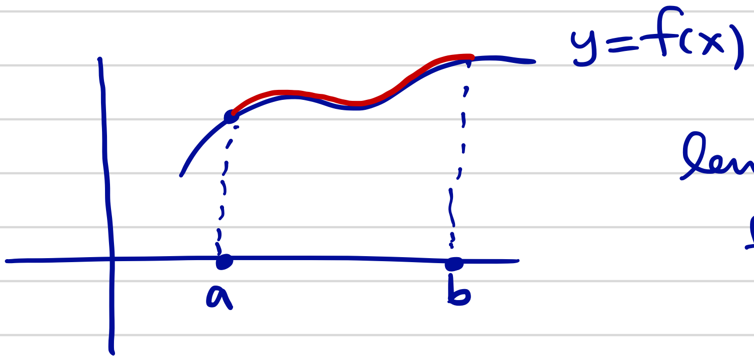
Ex. Find vol. of the same solid but using disc/washer method.

$x = 1+y^2 \rightsquigarrow y = \pm\sqrt{x-1}$



$$\text{Vol.} = \int_{1}^{2} \pi \left(\left(\sqrt{x-1}+2\right)^2 - \left(-\sqrt{x-1}+2\right)^2 \right) dx$$

7.4 Arc length.



length of the graph of f
from $(a, f(a))$ to $(b, f(b))$.

. In Calc. 3 we will see more general arc lengths.

$a \leq x \leq b$

$$x \longmapsto (x, f(x))$$

variable

(In Calc. 3 we see curves like:

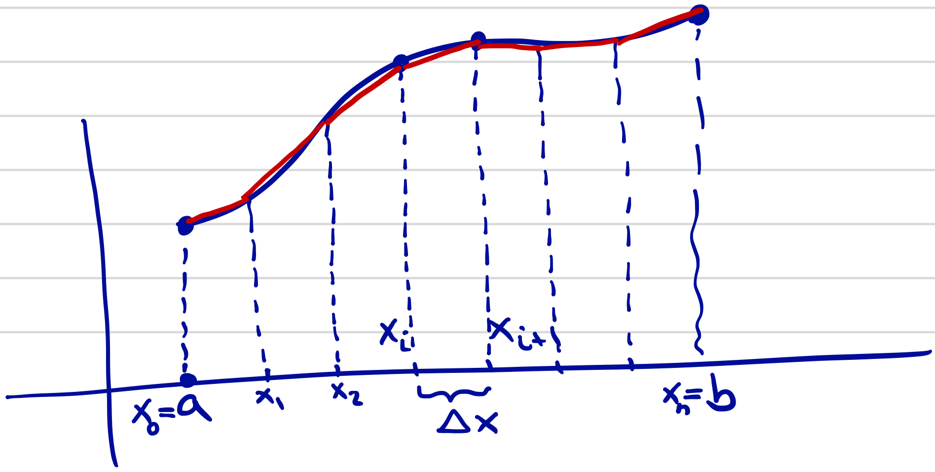
$$x \longmapsto (A(x), B(x))$$
)

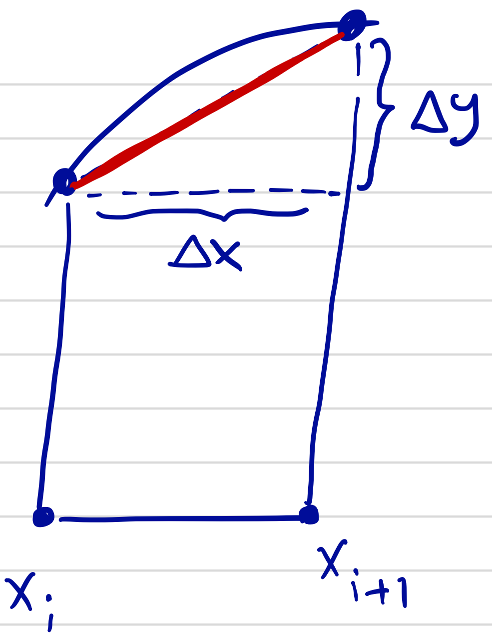
Formula for arc. length:

$$\text{Arc length} = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Proof or justification:

Arc. length $\approx$ sum of all lengths of small red. segments





$$\frac{\Delta y}{\Delta x} \approx f'(x)$$

length of red segment =

$$\sqrt{\Delta x^2 + \Delta y^2}$$

$$\sqrt{\Delta x^2 + (f'(x)\,\Delta x)^2}$$

$$\sqrt{1 + f'(x)^2}\ \Delta x$$

Sum of all red. segments $= \displaystyle\int_a^b \sqrt{1+f'(x)^2}\ dx$. ☺