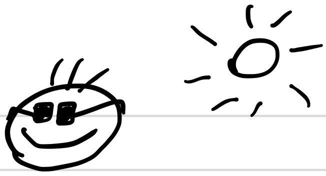
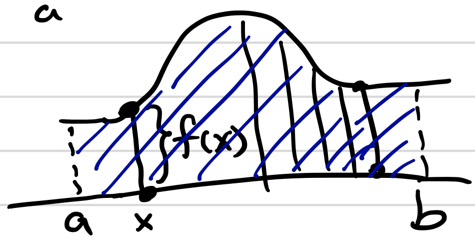
Sep. 20 / 2017



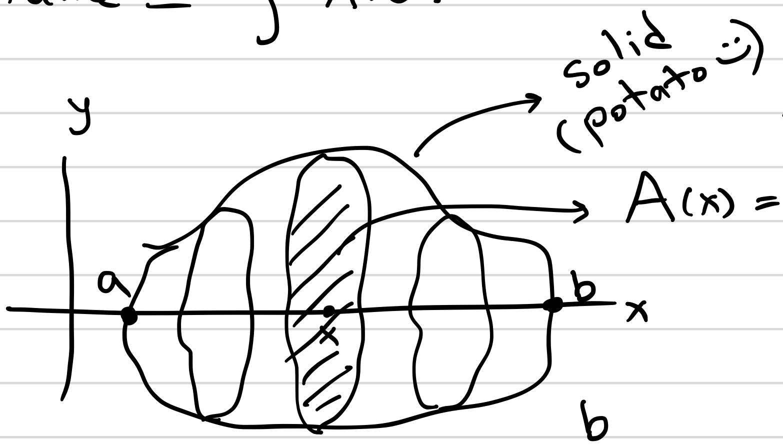
7.2 Volumes $\longrightarrow$ volumes of solids obtained by rotation (around x-axis or y-axis).

General facts

$$\text{Area} = \int \text{length} \longrightarrow \int_a^b f(x)\,dx = \text{Area}$$



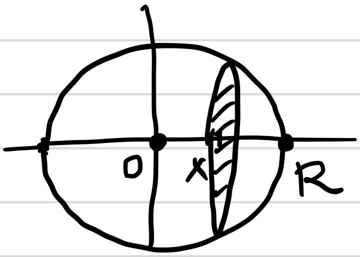
$$\text{Volume} = \int \text{Area}$$



$A(x)$ = area of slice at x

$$\text{volume (of your potato)} = \int_a^b A(x)\,dx$$

Ex. volume of sphere (radius R) $\to$ constant

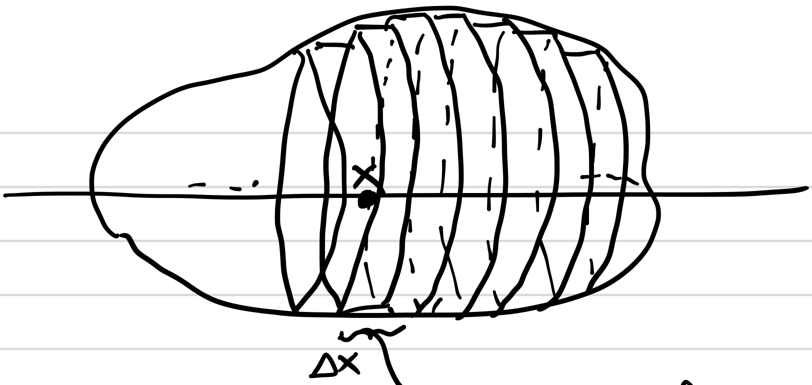


Area of slice at x $= \pi(R^2 - x^2)$

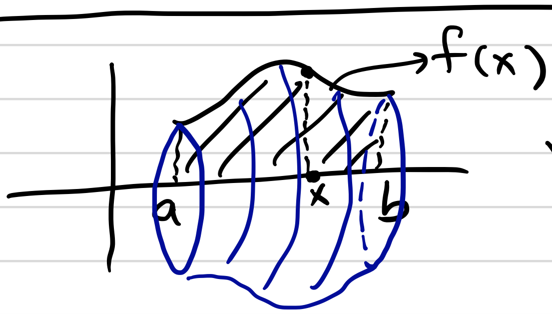
radius of the slice $= \sqrt{R^2 - x^2}$

$$\text{Volume of sphere} = 2\int_0^R (\pi R^2 - \pi x^2)\,dx = 2\pi\left(R^2 x\Big|_0^R - \frac{x^3}{3}\Big|_0^R\right)$$

$$= 2\pi\left(R^3 - \frac{R^3}{3}\right) = 2\pi\,\frac{2}{3}R^3 = \frac{4\pi}{3}R^3$$ ☺

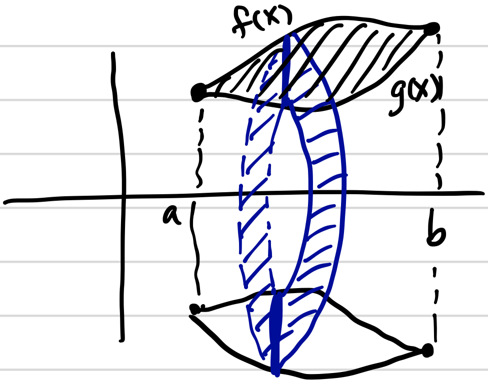


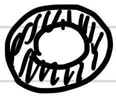
volume of a thin disc is $A(x) \cdot \Delta x$



$$\text{volume} = \int_a^b \pi (f(x))^2 \, dx$$

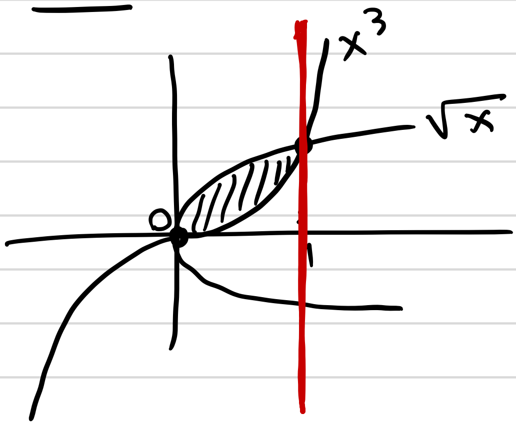
slice = circle / disc

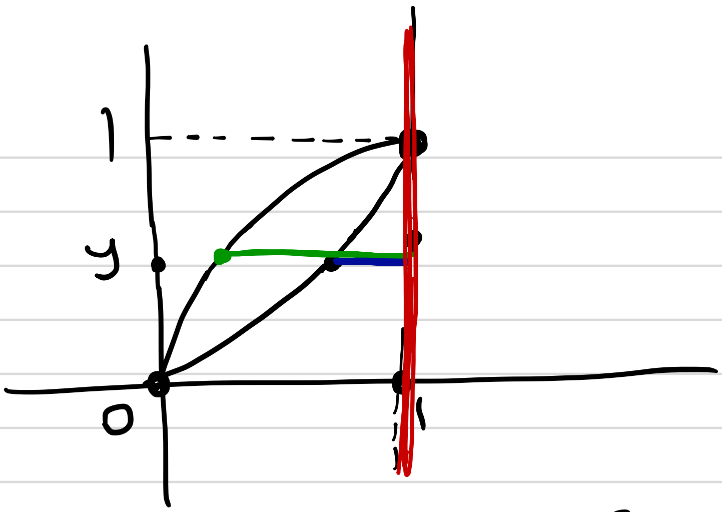


slice = washer 

$$\text{volume} = \int_a^b \pi \left(f(x)^2 - g(x)^2 \right) dx$$

Ex. (#17 in 7.2) region enclosed by $y = \sqrt{x}$, $y = x^3$. Find the vol. of rotating this region around $x = 1$.





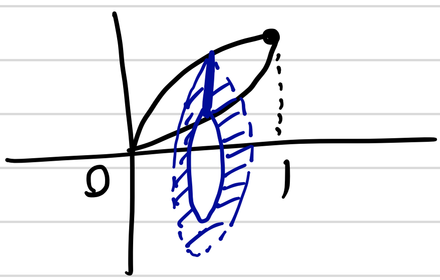
Remember: in this disc/washer method the slices are perpendicular to the axis of rotation.

$y=\sqrt{x} \longrightarrow y^2 = x$

$y=x^3 \longrightarrow \sqrt[3]{y} = x$

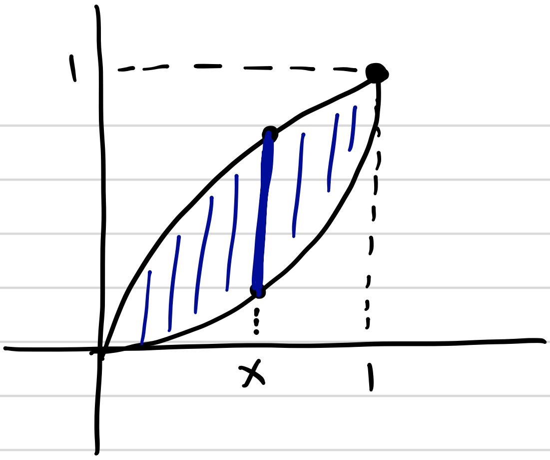
$$\text{vol.} = \int_{y=0}^{y=1} \pi(1-y^2)^2 - \pi(1-\sqrt[3]{y})^2 \, dy$$

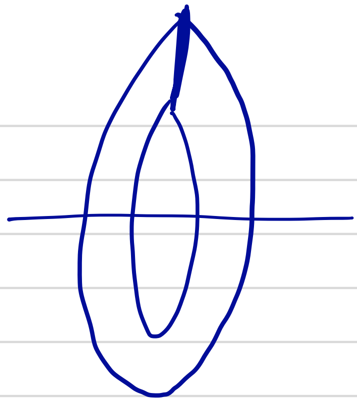
Ex. region between $y=\sqrt{x}$ & $y=x^3$ rotated around x-axis.



$$\text{Vol.} = \int_0^1 \pi x - \pi x^6 \, dx$$

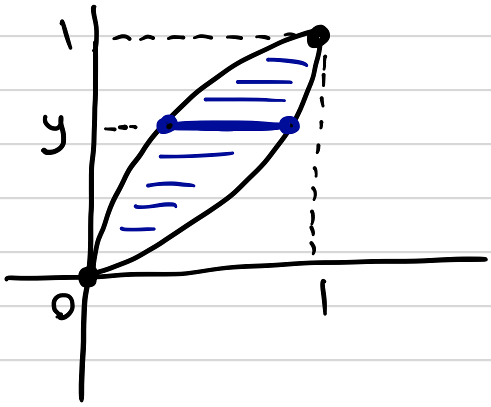
Alternative way to find the same volume
7.3 $\rightsquigarrow$ Method of cylindrical shells.

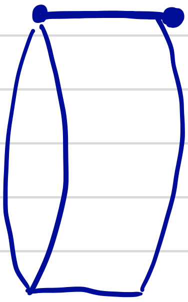




disc / washer method

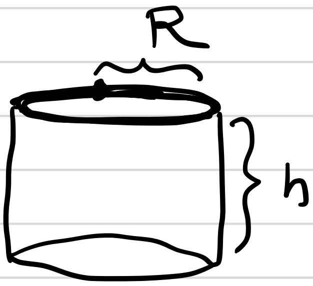
$$\text{Area} = \pi f(x)^2 - \pi g(x)^2$$





cylindrical shells method

$$\text{surface area (of cylinder)} = \underbrace{\big(F(y) - G(y)\big)}_{\text{width of the cylinder}} \, 2\pi y$$



$$\text{Surface area} = h\,(2\pi R)$$

$$\text{Vol.} = \int_{y=0}^{y=1} \left(\sqrt[3]{y} - y^2\right) 2\pi y \; dy .$$

In cylindrical shells method the slices are parallel to axis of rotation (along variable of integration).