Sep. 15 / 2017

- Comparison tech. for improper integrals.

Ex. (#41 6.6)

$$\int_0^\infty \frac{x}{x^3+1}\,dx$$ Convergent ✓ or divergent ?

Recall general fact / theorem :

If $0 \le f(x) \le g(x)$ (for x in our range of integral (a,∞)) & $\int_a^\infty g(x)\,dx$ Conv. $\Rightarrow \int_a^\infty f(x)\,dx$ Conv.

(Also if $\int_a^\infty f(x)\,dx$ divergent $\Rightarrow \int_a^\infty g(x)\,dx$ divergent.)

$$\frac{x}{x^3+1} \le \frac{x}{x^3} \le \frac{1}{x^2}$$

Recall $\int_1^\infty \frac{1}{x^2}\,dx$ Convergent ☺

~~(but $\int_0^\infty \frac{1}{x^2}\,dx$ divergent ☹)~~ → no worry because $\int_0^1 \frac{x}{x^3+1}\,dx$ is just a const. number.

$$\left.\begin{aligned} &\int_1^\infty \frac{x}{x^3+1}\,dx \le \int_1^\infty \frac{1}{x^2}\,dx \text{ Conv.} \\ &+ \int_0^1 \frac{x}{x^3+1} \text{ is some Const. number} \end{aligned}\right\} \Rightarrow \int_0^\infty \frac{x}{x^3+1}\,dx \text{ Conv.}$$

Ex. (#43 6.6)

$$\int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx \quad \text{Conv. or div. ?}$$

$$\frac{x+1}{\sqrt{x^4-x}} \geq \frac{x}{\sqrt{x^4}} = \frac{x}{x^2} = \frac{1}{x}$$

$(x \geq 1)$

$$\int_1^\infty \frac{1}{x}\,dx \text{ is divergent} \Longrightarrow \int_1^\infty \frac{x+1}{\sqrt{x^4-x}}\,dx \text{ divergent.}$$

Chap. 7 Applications of integrations.

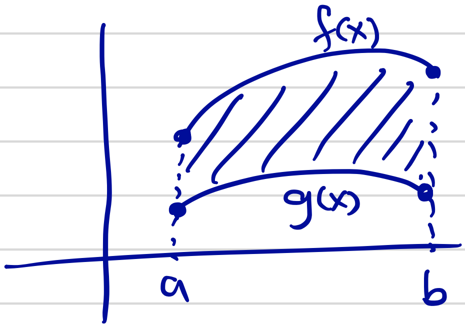
(done with integration techniques ☺)

- Reduce computation of different quantities to computation of a (definite) integral.

- Area between curves.
- Volumes (obtained by rotation of a curve).
- Quantities from physics like work .

General fact / theorem

$f(x) \geq g(x)$ for $a \leq x \leq b$.



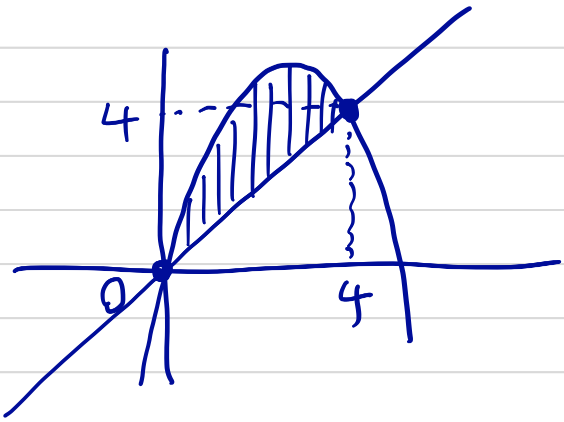
Area between the graphs of f & g & between $x=a$ & $x=b$ is:

$$\int_a^b (f(x)-g(x))\,dx$$

Note: when applying this make sure $f \geq g$.

Ex. (#17.1)

Area of region bounded by: $y = 5x - x^2$, $y = x$



Points of intersections:

$x = 5x - x^2$

$4x = x^2 \Rightarrow x = 0,\ x = 4$

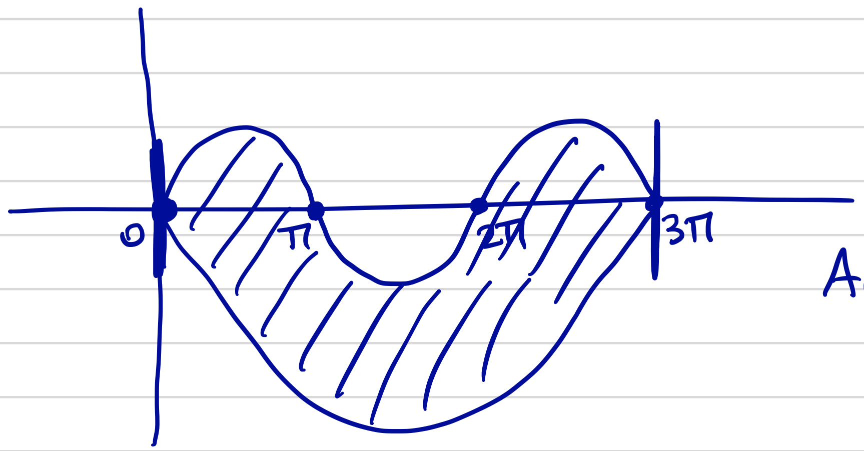
$5x - x^2 \geq x$ when $0 \leq x \leq 4$

Answer: $\displaystyle\int_0^4 (5x - x^2) - x \quad dx$

(In some questions, they may ask to just write the correct integral, without computing it)

Ex. Find area enclosed by the graphs of $y = \sin(x)$ & $y = -3\sin\left(\frac{x}{3}\right)$ & lines $x=0$ & $x = 3\pi$.

First, get a rough idea how the graphs look like.



$0 \leq x \leq 3\pi$

$$\text{Area} = \int_0^{3\pi} \left(\sin(x) + 3\sin\left(\frac{x}{3}\right)\right) dx$$

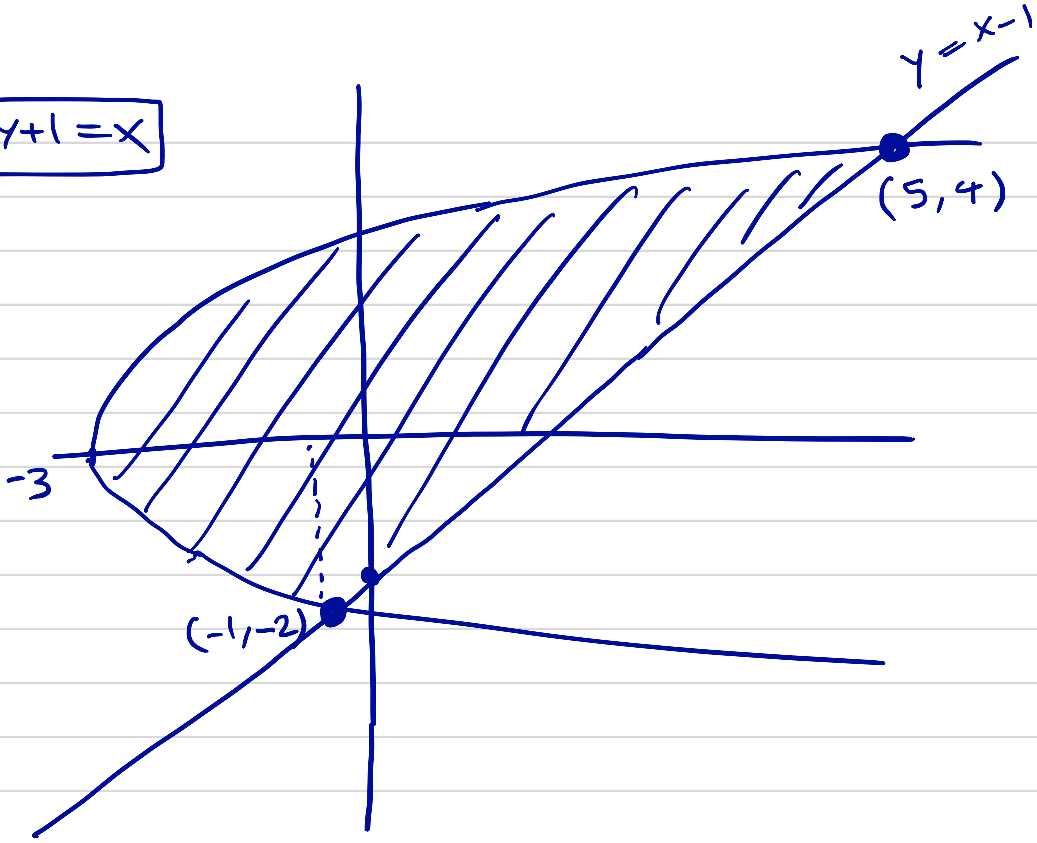
(to compute the int., compute $\int_0^{3\pi} \sin(x)\,dx$ & $\int_0^{3\pi} 3\sin\left(\frac{x}{3}\right) dx$ separately.)

Ex. (Example 4 in 7.1)

Area enclosed by $y = x-1$ & $y^2 = 2x+6$.

$y = x - 1 \longrightarrow \boxed{y+1 = x}$

$y^2 = 2x + 6 \longrightarrow \boxed{\frac{y^2}{2} - 3 = x}$



$\frac{y^2}{2} - 3 = y + 1$

$\frac{y^2}{2} - y - 4 = 0$

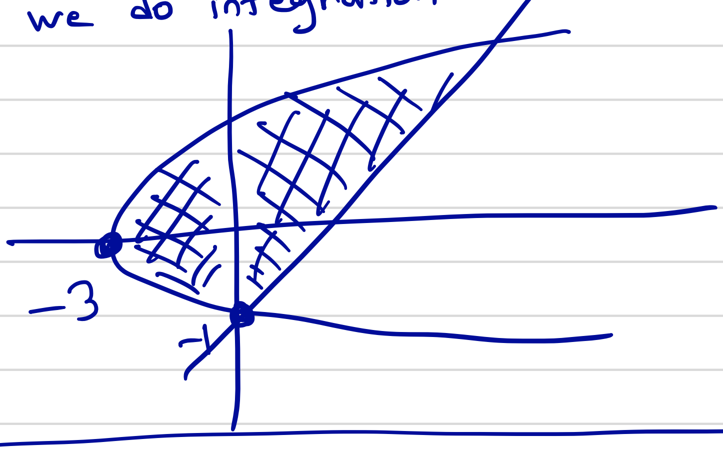
$y = 4$ and -2

($x = 5$ and -1)

Write x as a function of y (switch role of x & y)

$$\int_{y=-2}^{4} (y+1) - \left(\frac{y^2}{2} - 3\right) \, dy$$

If we do integration with resp. to x:



$$\int_{-3}^{-1} \cdots dx + \int_{-1}^{5} \cdots dx$$

More work.