Sep 11 / 2017

Quiz $\rightsquigarrow$ Sec. 6.2 & 6.3

$\dfrac{P(x)}{Q(x)}$ $\longrightarrow$ Factor into product of linear & quadratic polynomials.

Fundamental thm. of Algebra

$i = \sqrt{-1}$ $\longrightarrow$ add i to $\mathbb{R}$ real numbers.
$i^2 = -1$

 $+\ i$  Complex numbers

over complex numbers

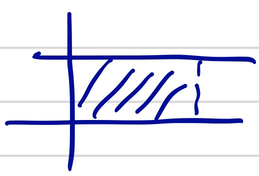
F.T.A. Any polynomial with Complex numbers as coefficients can be completely factored (i.e. into linear factors).

F.T.A. over real numbers

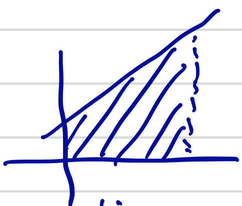
Any poly. with real numbers as coeff. can be factored into linear & quadratic polynomials.

The first proof was given by Gauss (Gauß)
ß = sz

6.5 Approx. integrals $\rightsquigarrow$ definition of integral by Riemann sums.



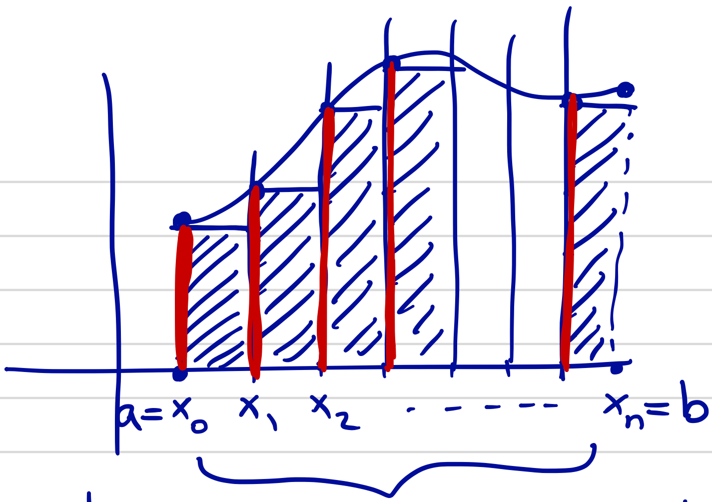
Const. function



linear function

parabola

$\longrightarrow$ Archimedes



$f(x)$

divide into n smaller intervals

- n some number (usually big)
- $\dfrac{b-a}{n} = x_{i+1} - x_i$ = size of each interval.

  Δx

. Use $f(x_0), f(x_1), \ldots, f(x_{n-1})$ as heights of rectangles.

$$\int_a^b f(x)\,dx \approx \text{sum of areas of these rects.} = f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x$$

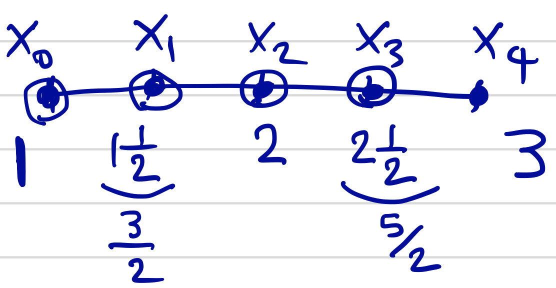
Left endpoint approx.

$$= \left(\frac{b-a}{n}\right)\left(f(x_0) + \cdots + f(x_{n-1})\right).$$

Ex. $\displaystyle\int_1^3 \frac{1}{x}\,dx$ approx. $n = 4$.

$\frac{1}{x} \to f(x)$

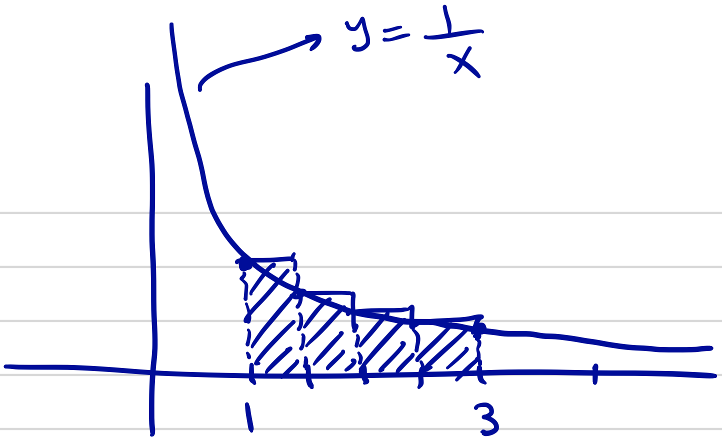
Left endpoint approx.

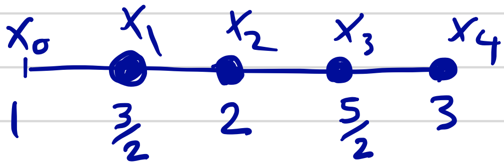


$$\Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

$f(x_0)$ $f(x_1)$ $f(x_2)$ $f(x_3)$

$$\int_1^3 \frac{1}{x}\,dx \approx \frac{1}{2}\left(1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{5}\right)$$





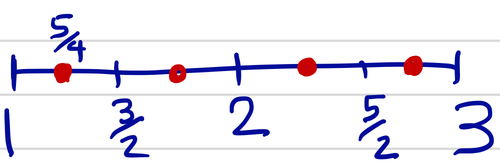
Right endpoint approx.

$$\approx \underbrace{\frac{1}{2}}_{\Delta x}\left(\underbrace{\frac{2}{3}}_{f(x_1)} + \underbrace{\frac{1}{2}}_{f(x_2)} + \underbrace{\frac{2}{5}}_{f(x_3)} + \underbrace{\frac{1}{3}}_{f(x_4)}\right)$$

Other variations of approx. integral :

- Midpoint. approx.

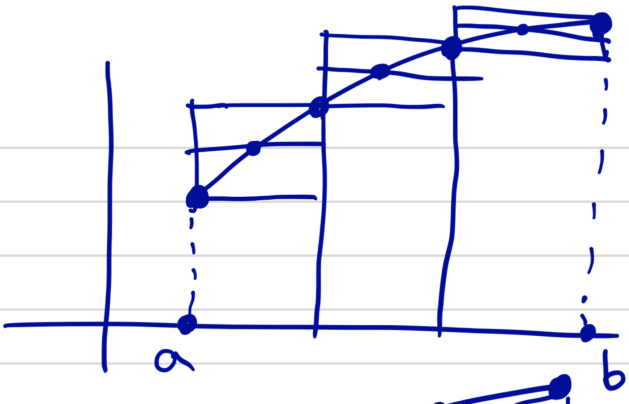
$$= f(\underbrace{\bar{x}_1}_{\substack{\text{midpoint} \\ [x_0,\, x_1]}})\Delta x + f(\underbrace{\bar{x}_2}_{\substack{\text{midpoint} \\ \text{of} \\ [x_1,\, x_2]}})\Delta x + \cdots + f(\underbrace{\bar{x}_n}_{\substack{\text{midpoint} \\ \text{of} \\ [x_{n-1},\, x_n]}})\Delta x$$



$$\frac{1+\frac{3}{2}}{2} = \frac{5}{4}$$

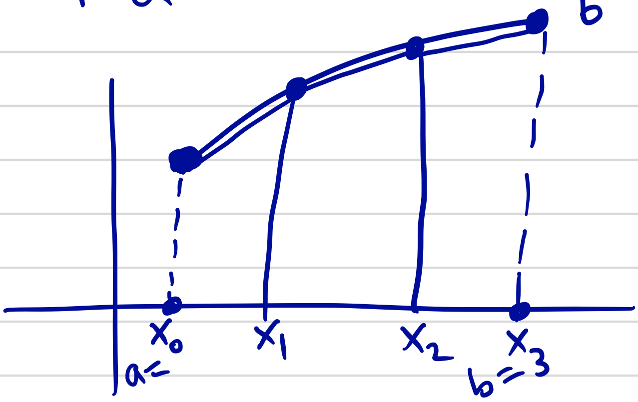
Midpoint approx. $= \underbrace{\frac{1}{2}}_{\Delta x}\left(\underbrace{\frac{4}{5}}_{f(\bar{x}_1)} + \cdots\right).$

$f(x) = \frac{1}{x}.$

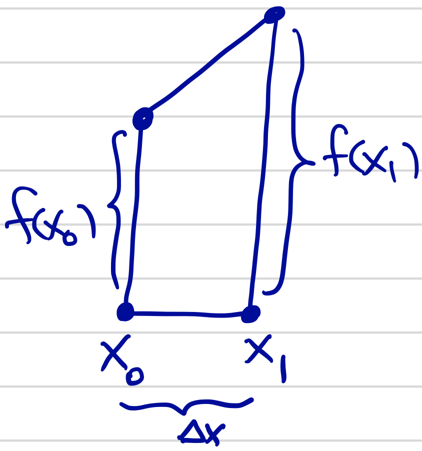


$f(x)$

$n=3$



Trapezoid approx.



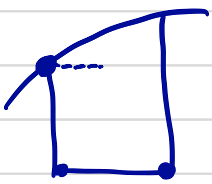
Area of trap. $= \Delta x \left(\dfrac{f(x_0)+f(x_1)}{2} \right)$.

Trap. approx. $=$ area of 1st trap. + area of 2nd trap + ... + area of n-th trap.

$$= \Delta x \left(\frac{f(x_0)+f(x_1)}{2} + \frac{f(x_1)+f(x_2)}{2} + \cdots + \frac{f(x_{n-1})+f(x_n)}{2} \right)$$

$$= \frac{\Delta x}{2} \left(f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right)$$

- Left endpt. approx
- Right - - - - - -
- Midpoint - - - - - -



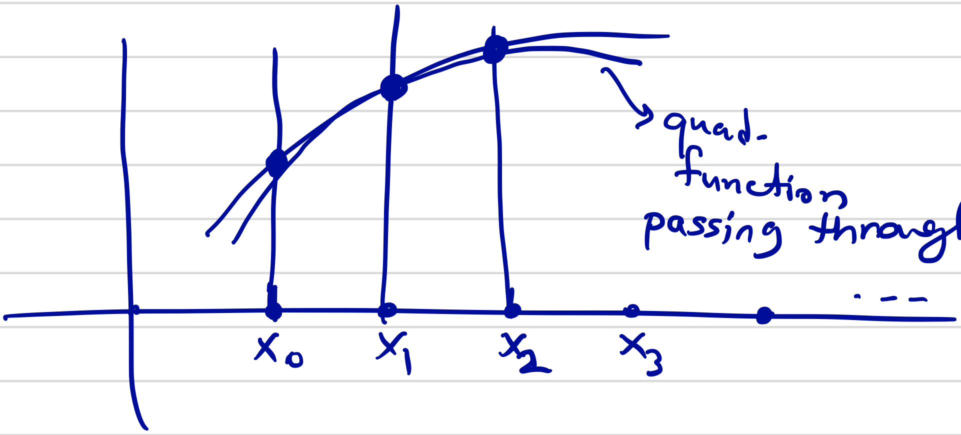
$\to$ approx. f by Const. function

- Trapezoid - - - - $\to$ approx f by linear function

$\hookrightarrow$ One expects trapezoid method to be more accurate.

One can also approx. f by say quadratic poly.

$\rightsquigarrow$ Simpson's method.



Error bounds $\rightsquigarrow$ There are formulae to estimate the errors of these approx. integrals.

Next time $\rightsquigarrow$ improper integrals