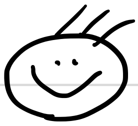
Sep. 1 / 2017



- Quizzes are from sugg. prac. problem (or very similar)

## Some remarks

linear in x

$$\int \sin(5x+3)\,dx$$

$u = 5x+3$

$du = 5\,dx$

$\frac{du}{5} = dx$

$$\frac{1}{5}\int \sin(u)\,du$$

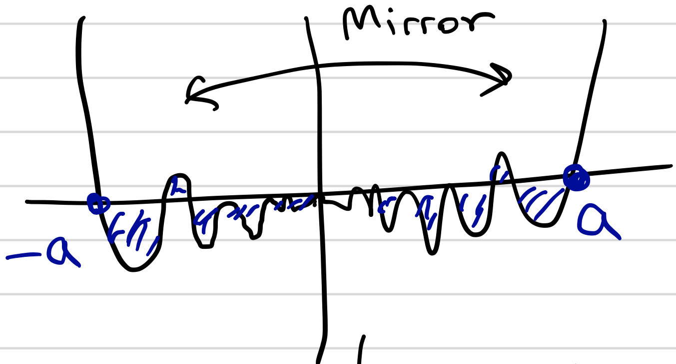
$$= \frac{1}{5}\ -\cos(5x+3) + C$$

## Odd & even functions

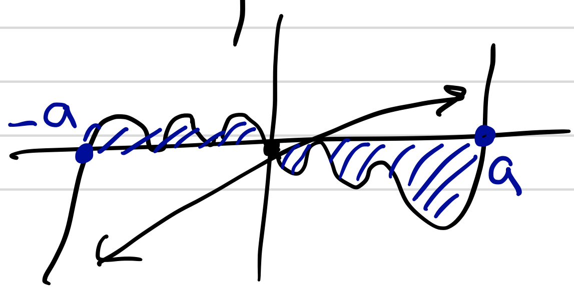
$f(-x) = -f(x)$ odd

$f(x) = f(-x)$ even

Justifing name $\rightsquigarrow$ $x^n = \begin{cases} \text{even} & n \text{ even} \\ \text{odd} & n \text{ odd} \end{cases}$




even




odd

Observation:

$f$ odd $\quad \int_{-a}^{a} f(x)\,dx = 0$

$f$ even $\quad \int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$

Ex. $\quad \int_{-1}^{1} e^{(x^2)} \sin(3x^5)\,dx = 0$

odd

$\sin(-x) = -\sin(x)$
$\cos(-x) = \cos(x)$

## 6.2 Some trig. integrals & trig. substitutions

$\boxed{\sin^3(x) = (\sin(x))^3}$

$\int \sin^3(x) \cos(x)\,dx \rightsquigarrow$ easy $\quad u = \sin(x).$

Ex. $\quad \int \sin^2(x)\,dx \longrightarrow$

$\int \frac{1}{2}(1-\cos(2x))\,dx$

$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$

$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$

$\cos(2x) = \cos^2(x) - \sin^2(x).$

Complex numbers

$$\frac{1}{2}\int (1-\cos 2x)\, dx = \frac{1}{2}\int 1\, dx - \frac{1}{2}\int \cos(\underset{u}{2x})\,\underset{\frac{du}{2}}{dx}$$

$$= \frac{x}{2} - \frac{1}{2}\cdot\frac{1}{2}\sin(2x) + C$$

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• $\int \sin^4(x)\,dx$, $\int \sin^6(x)\,dx$, ... works for $\sin^n(x)$, $n$ even

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$$\int \left(\sin^2(x)\right)^2 dx = \int \left(\frac{1}{2}(1-\cos(2x))\right)^2 dx$$

$$\frac{1}{2}(1-\cos(2x))^2 = \text{Const.} + \square\cos(2x) + \underbrace{\square\cos^2(2x)}$$

one more $\cos(2x)$ identity

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Ex. $\int \sin^n(x)\,dx$ $n$ odd

$$\int \sin^7(x)\,dx = \int \underbrace{\sin^6(x)}\sin(x)\,dx$$

write in terms of $\cos(x)$

$$\boxed{\sin^2 = 1-\cos^2}$$

$u = \cos x$

$du = -\sin(x)\,dx$

$$\int (1-\cos^2(x))^3 \underbrace{\sin(x)\,dx}_{-du} = \underbrace{\int -(1-u^2)^3\,du}_{\text{easy to do ...}}.$$

**Ex.** $\int \tan(x)\,dx$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad u = \cos(x)$$
$$du = -\sin(x)\,dx$$

$$\int \frac{-du}{u} = -\ln|u| + C$$
$$= -\ln|\cos(x)| + C \quad \rightsquigarrow \frac{1}{\cos(x)} = \sec(x)$$
$$= \ln|\sec(x)| + C$$

$$\tan'(x) = 1+\tan^2(x) = 1 + \frac{\sin^2}{\cos^2} = \frac{1}{\cos^2} = \sec^2(x)$$

**Ex.** $\int \tan^3(x)\,dx$ $\quad u = \tan(x)$

$$= \int \tan(\underbrace{\tan^2}_{+1-1})\,dx = \int \tan(1+\tan^2)\,dx - \int \tan\,dx$$

$$= \frac{\tan^2(x)}{2} - \ln|\sec(x)| + C$$

Some history $\quad f \rightsquigarrow f'$ Newton

$f \rightsquigarrow \frac{df}{dx}$ Leibnitz

Trig. substitution $\rightsquigarrow$ inverse substitution

$$\int f(x)\,dx \rightsquigarrow \int f(x(t))\,x'(t)\,dt.$$

$a > 0$ Const.

Ex. $\displaystyle\int \sqrt{a^2 - x^2}\, dx$ ⇝ This is needed to Compute area of circle & ellipse.

$$\boxed{x = a \sin(x)}$$

we will finish this next time.