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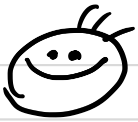

Review of topics so far

- Methods of integration → substitution, trig. sub., int. by parts, partial fraction
- Int. with infinite limits
- Volumes (using integration) & Arc length.
- Applications in physics

- Some 3D geometry

- Parametric curves & polar Coor.

New topic

Sequences & Series

A "sequence" (of real numbers):

$a_1, a_2, a_3, \ldots$ (repetition allowed)

Notation: $\{\ n\} \dashrightarrow \{a_n\}_{n=1}^{\infty}$

Ex.

$\{n\}$: $1, 2, 3, 4, \ldots$

$\{\frac{1}{n}\}$: $1, \frac{1}{2}, 1 \quad 1$

$\{\frac{1}{n^2}\}$: $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

$\{1\}$: $1, 1, 1, 1, \ldots$

$\{(-1)^{n+1}\}$: $1, -1, 1, -1, 1, -1, \ldots$

$n=1$: $(-1)^2$ $\quad$ $n=2$: $(-1)^3$

Imoprtant Concept : Limit of a sequence
(analogue of limit of a function $y=f(x)$).

$$\lim_{n\to\infty} a_n = L$$

(a real number)

means :

as $n\to\infty$
a_n's get as close as you wish to number L

L is called "limit" of the sequence $\{a_n\}$

Ex. $a_n = n$ $\qquad \lim_{n\to\infty} n = +\infty$
$\{n\}$

$a_n = \frac{1}{n}$ $\qquad \lim_{n\to\infty} \frac{1}{n} = 0$

$a_n = \frac{1}{n^2}$ $\qquad \lim_{n\to\infty} \frac{1}{n^2} = 0$

$a_n = 1$ $\qquad \lim_{n\to\infty} 1 = 1$

$a_n = (-1)^{n+1}$ $\qquad \lim_{n\to\infty} (-1)^{n+1}$ does not exist.

$1, -1, 1, -1, \ldots$

Precise def. of limit : $\lim_{n\to\infty} a_n = L$

For any small $\varepsilon > 0$ (we would to get close to L closer than ε, provided n is large enough)
there exist Large $N > 0$
such that if $n > N$ then $|a_n - L| < \varepsilon$.

dist. of a_n & L is $< \varepsilon$.

Basic
Properties of limit of sequences

- $$\lim_{n\to\infty} \{a_n \underset{\times}{\mp} b_n\} = \lim_{n\to\infty} \{a_n\} \underset{\times}{\mp} \lim_{n\to\infty} \{b_n\}$$

- $$\lim_{n\to\infty} \left\{\frac{a_n}{b_n}\right\} = \frac{\lim_{n\to\infty} \{a_n\}}{\lim_{n\to\infty} \{b_n\}}$$ provided that $\lim_{n\to\infty} \{b_n\} \neq 0$

- $$\lim_{n\to\infty} \{a_n\}^p = \left(\lim_{n\to\infty} \{a_n\}\right)^p$$

(p fixed)

- $$\lim_{n\to\infty} |a_n| = 0 \iff \lim_{n\to 0} a_n = 0$$

- $\lim_{n\to\infty} \frac{1}{n} = 0 \rightsquigarrow$ easy to show using definition.

Take any $\varepsilon > 0$

want:

$$\left|\frac{1}{n} - 0\right| < \varepsilon$$

$$\frac{1}{n} < \varepsilon$$

$$\frac{1}{\varepsilon} < n$$

Take $N > \frac{1}{\varepsilon}$

Then $n > N > \frac{1}{\varepsilon} \implies \frac{1}{n} < \varepsilon$ i.e. $\lim_{n\to\infty} \frac{1}{n} = 0$

(infinite sum)

Series: sum of elements of a sequence.

$\{a_n\} \rightsquigarrow a_1 + a_2 + a_3 + \dots$

$a_1, \; a_1 + a_2, \; a_1 + a_2 + a_3, \; a_1 + a_2 + a_3 + a_4, \; \dots \to ?$