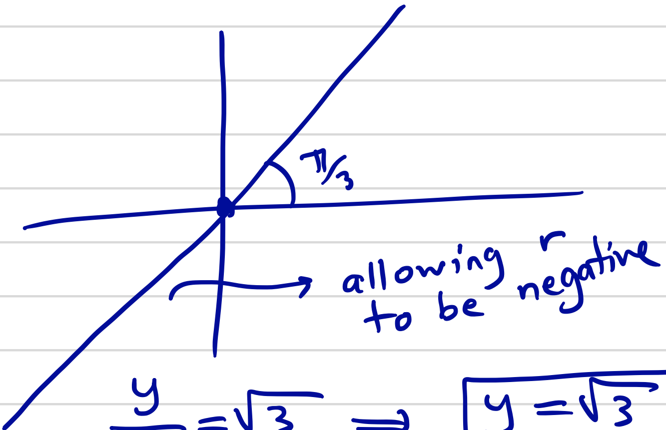
<u>Oct. 20 / 2017</u> will return MT1 papers on Monday!

- Some examples of curves in polar coor.

Ex. 13–16 in Sec. 9.3
Describe the curves given in polar Coor. below:
(by Converting it to xy – Coor.)

– $\theta = \pi/3$ (means r arbitrary).

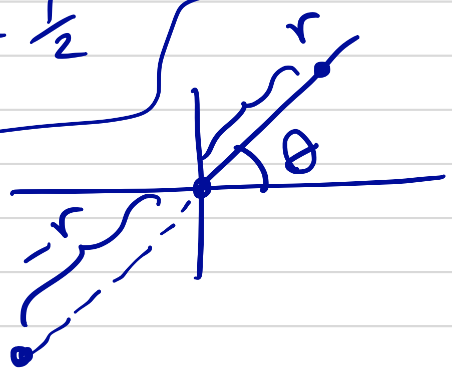


$$\tan\theta = \frac{y}{x} \rightsquigarrow \frac{y}{x} = \underbrace{\tan(\pi/3)}_{\sqrt{3}}$$

$\sin(\pi/3) = \sqrt{3}/2$

$\cos(\pi/3) = 1/2$

$$\frac{y}{x} = \sqrt{3} \Rightarrow \boxed{y = \sqrt{3}\, x}$$



$$\boxed{\begin{gathered}(r,\theta) = (-r, \theta+\pi) \\ \text{Convention}\end{gathered}}$$

- $r = 2\cos(\theta)$ circle

↳ write this equ. in x & y.

$$r^2 = 2\underbrace{r\cos(\theta)}_{x}$$

$$x^2+y^2 = 2x \quad ☺ \longrightarrow x^2+y^2-2x = 0$$

$$x^2 - 2x + 1 - 1 + y^2 = 0$$

$$(x-1)^2 + y^2 - 1 = 0 \Rightarrow (x-1)^2 + y^2 = 1$$

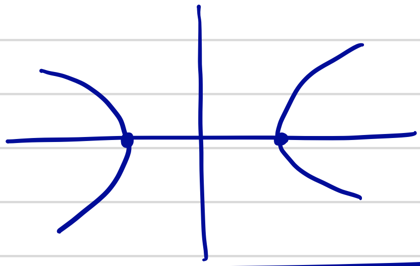
circle radius = 1
center (1, 0)

- $r^2 \underbrace{\cos(2\theta)} = 1$ ⇢ Convert to equ. in x, y.

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 1$

$x^2 - y^2 = 1$ ⇝ hyperbola



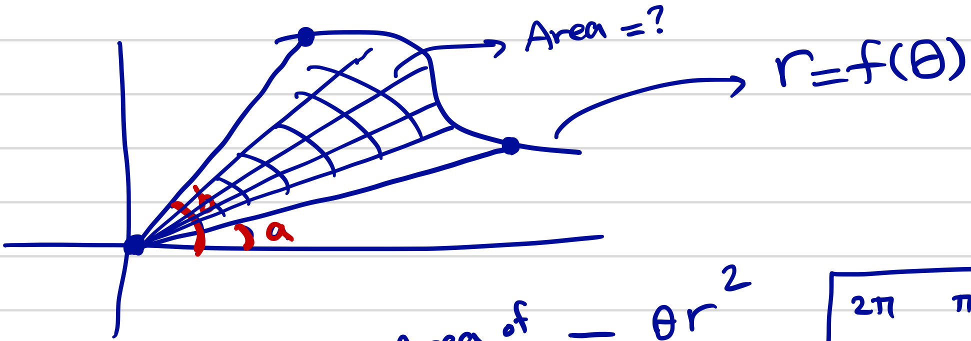
- $r = \tan\theta \sec\theta$ ⇝ $r = \dfrac{r\sin\theta}{r\cos\theta} \cdot \dfrac{1}{\cos\theta}$

$1 = \dfrac{r\sin\theta}{r\cos\theta} \cdot \dfrac{1}{r\cos\theta} = \dfrac{y}{x^2}$

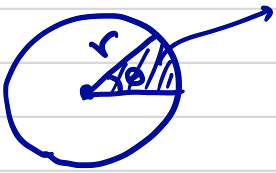
$\Rightarrow y = x^2$ parabola.

Area between Curves in polar Coor.

(Curves are given as $r = f(\theta)$)

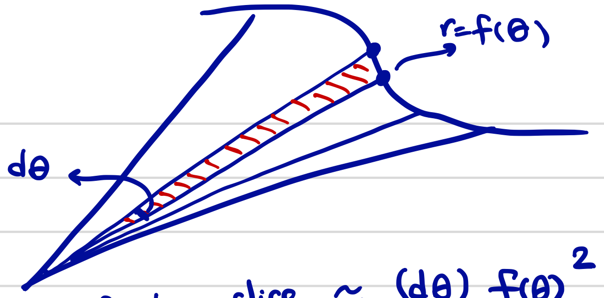


$a \leq \theta \leq b$



Area of this slice $= \dfrac{\theta r^2}{2}$

2π	πr^2
θ	$\frac{\theta r^2}{2}$



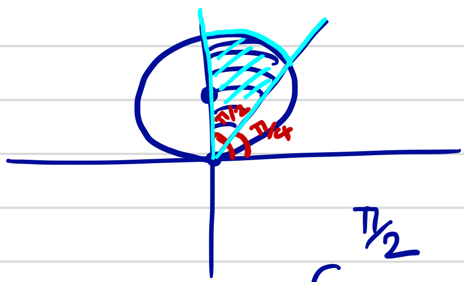
Area of tiny slice $\approx \frac{(d\theta)\, f(\theta)^2}{2}$

Total area $= \int_a^b \frac{f(\theta)^2}{2}\, d\theta$ ☺

<u>Ex.</u> $r = 2\sin\theta \qquad \pi/4 \leq \theta \leq \pi/2$.

Find area of the slice given by $r = \cos(2\theta)$ & $\pi/4 \leq \theta \leq \pi/2$.

$r^2 = 2r\sin\theta \Rightarrow x^2+y^2 = 2y \underset{\text{Complete square}}{\rightsquigarrow} x^2+(y-1)^2 = 1$

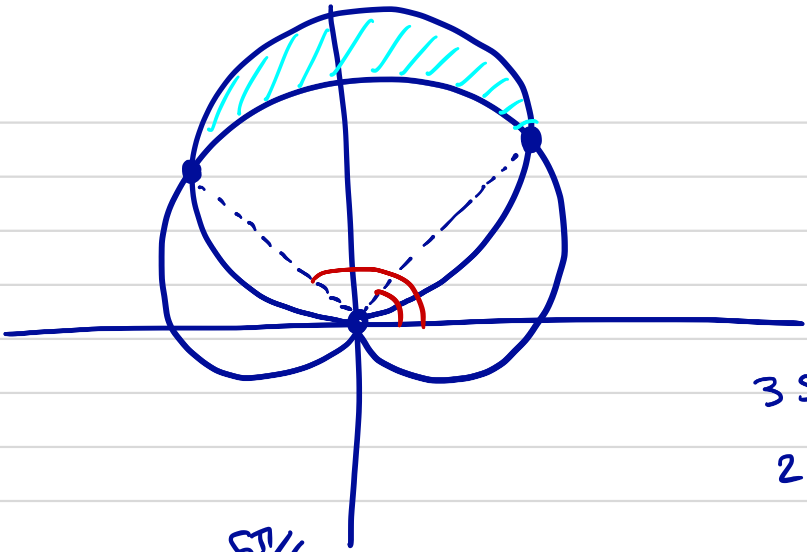


$$\text{Area} = \int_{\pi/4}^{\pi/2} \frac{(2\sin(\theta))^2}{2}\, d\theta = 2\int_{\pi/4}^{\pi/2} \sin(\theta)^2\, d\theta$$

<u>Ex.</u> (Ex. 2 in 9.4)

Find area of the region that lies inside $r = 3\sin\theta$ (circle)

& outside the cardioid $r = 1+\sin\theta$.



points of intersection of two curves:

$r = 3\sin\theta$
$r = 1 + \sin\theta$

$3\sin\theta = 1 + \sin\theta$

$2\sin\theta = 1$
$\sin\theta = \frac{1}{2} \rightsquigarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\text{Area} = \int_{\pi/6}^{5\pi/6} \frac{(3\sin(\theta))^2}{2} - \frac{(1+\sin(\theta))^2}{2} \, d\theta$$

Ex. (length of Cardiod)

$r = 1 + \sin(\theta) \qquad 0 \leq \theta \leq 2\pi$

$$\text{length} = \int_0^{2\pi} \sqrt{r^2 + r'^2} \, d\theta$$

$$= \int_0^{2\pi} \sqrt{(1+\sin(\theta))^2 + (\cos\theta)^2} \, d\theta$$
