Oct. 18 / 2017

Length of parametric curves

Ex.

(Ellipse) $\quad x(\theta) = a\cos(\theta)$ $\qquad 0 \leq \theta < 2\pi$

$\qquad\qquad y(\theta) = b\sin(\theta)$

Circumference of ellipse $= \boxed{\displaystyle\int_0^{2\pi} \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}\, d\theta}$

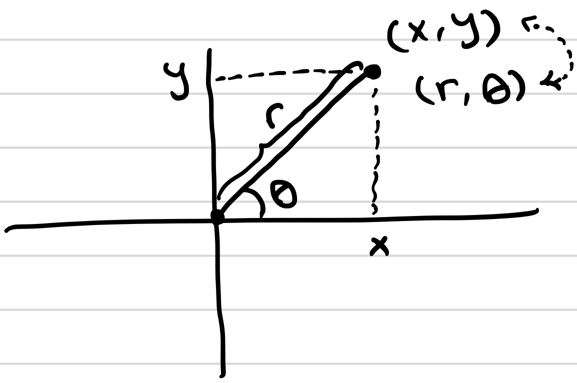
no good formula for this integral

$a = b = r$ circle

$$\int_0^{2\pi} \sqrt{r^2(\underbrace{\cancel{\sin^2 + \cos^2}}_{1})}\, d\theta = \int_0^{2\pi} \underbrace{r}_{\text{Const.}}\, d\theta = \Big[r\theta\Big]_0^{2\pi}$$

$= 2\pi r$

9.3 Polar Coordinates



$r \geq 0$

$0 \leq \theta < 2\pi$

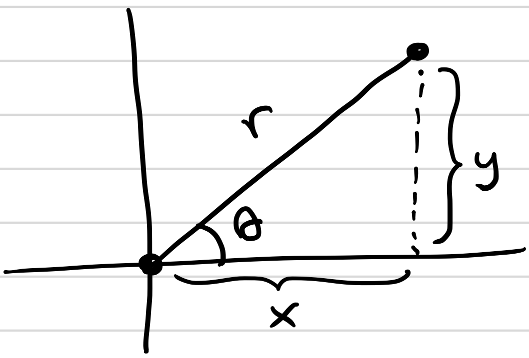
angle with x-axis

- Sometimes using $\underbrace{(r,\theta)}_{\text{polar Coor.}}$ is more Convenient than $\underbrace{(x,y)}_{\text{Cartesian coor.}}$

Note: If $r=0 \rightsquigarrow$ the Origin (whatever θ is)

Rene Descartes

How to go from (x,y) to (r,θ) & vice versa.

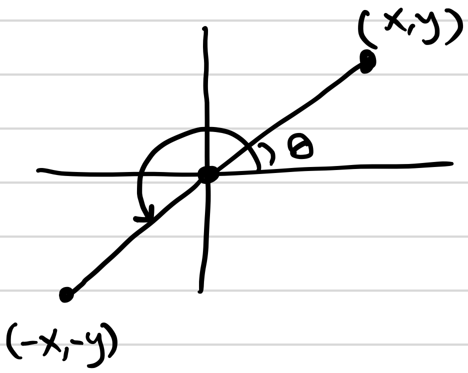


$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r = \sqrt{x^2+y^2}$$

$$\tan(\theta) = \frac{y}{x} \rightsquigarrow \boxed{\theta = \tan^{-1}\left(\frac{y}{x}\right),\quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}}$$

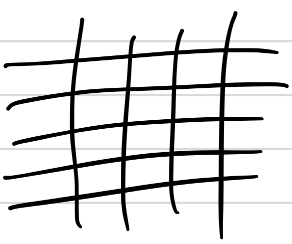


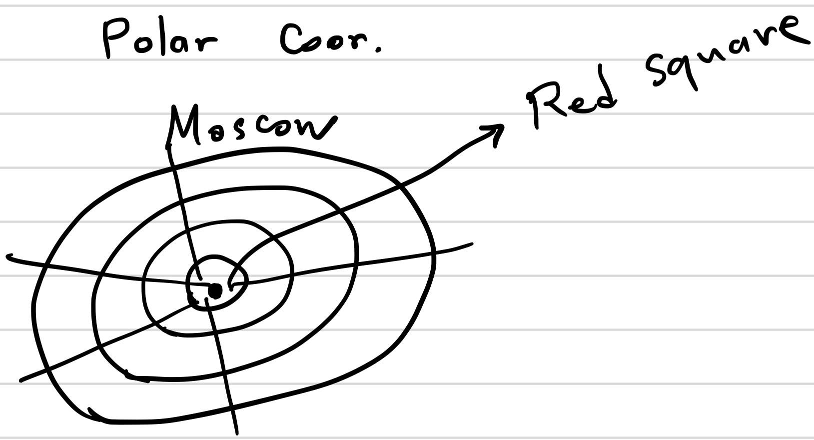
- Book by George Gamow (physicist & math.)

"one, two, three, infinity"

Cartesian Coor. vs. Polar Coor.

New York





Curves in polar Coor.

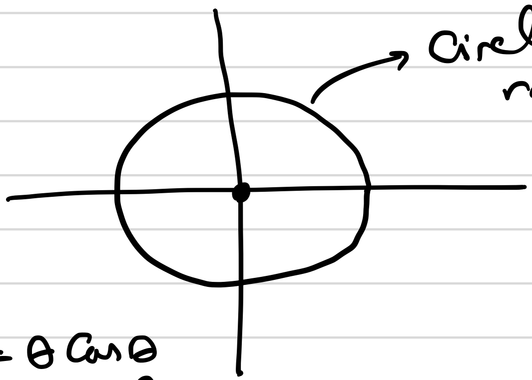
$r = f(\theta)$ (or θ as a function of r)

r → depends on θ

θ → variable

Ex. $\boxed{r=1}$

θ variable

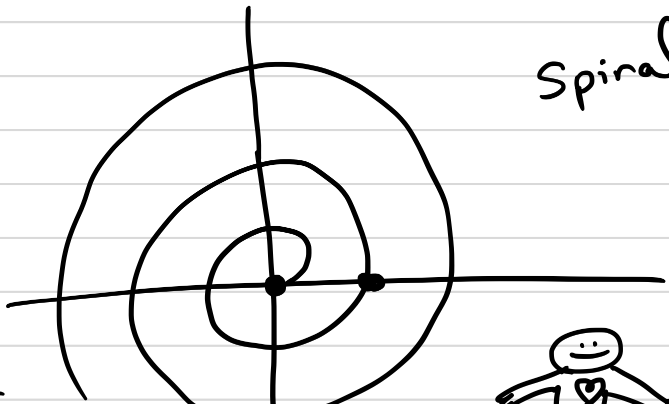


Circle of radius 1

Ex. $r=\theta \rightsquigarrow \begin{cases} x = \theta\cos\theta \\ y = \theta\sin\theta \end{cases}$

$\theta = 0 \rightsquigarrow r = 0$

As we go around $\underline{o}$ we get further away from $\underline{o}$.

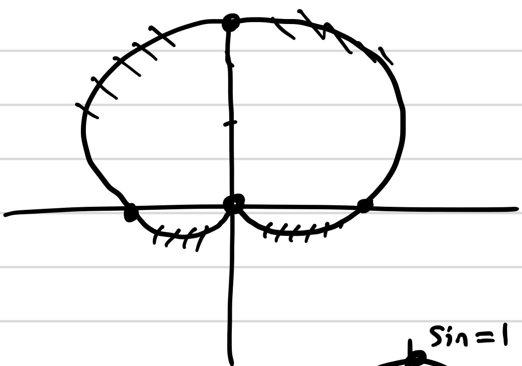


Spiral

(Cardioid) → Heart-like

Ex. $r = 1 + \sin\theta$

(Example 7. in 9.3)

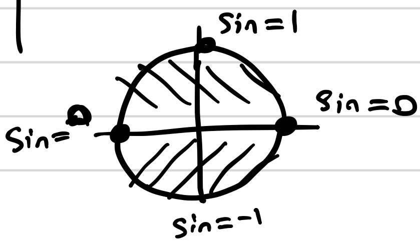


$\theta = 0 \rightsquigarrow r = 1$

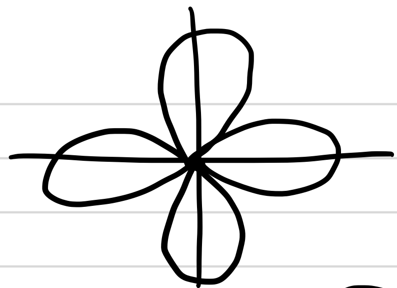
$0 \leq \theta \leq \frac{\pi}{2} \rightsquigarrow r=1 \dashrightarrow r=2$

$\frac{\pi}{2} \leq \theta \leq \pi \rightsquigarrow r=2 \dashrightarrow r=1$

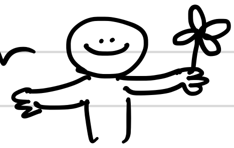
$\pi \leq \theta \leq \frac{3\pi}{2} \rightsquigarrow r=1 \longrightarrow r=0$



Ex. $r = \cos 2\theta \rightsquigarrow$



flower



- Slope of Tangent to curves given in polar Coor.

$r = f(\theta) \rightsquigarrow$ as para. curve $\quad x(\theta) = f(\theta)\cos\theta$, $\quad y(\theta) = f(\theta)\sin\theta$

$$\frac{dy}{dx} \underset{\text{Chain rule}}{=} \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \left(= \frac{y'}{x'} \right) = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

$$= \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}.$$

- Arc length of curve in polar Coor. $r = f(\theta)$

$$\int_a^b \sqrt{x'(\theta)^2 + y'(\theta)^2}\, d\theta = \int_a^b \sqrt{(r'\sin\theta + r\cos\theta)^2 + (r'\cos\theta - r\sin\theta)^2}\, d\theta$$

$a \leq \theta \leq b$ $\quad$ (simplify: $r'^2\sin^2 + r^2\cos^2 + \cancel{2rr'\sin\cos} + r'^2\cos^2 + r^2\sin^2 - \cancel{2rr'\cos\sin}$)

$$= \int_a^b \sqrt{\underbrace{r}_{f(\theta)}{}^2 + \underbrace{r'}_{f'(\theta) \text{ or } \frac{dr}{d\theta}}{}^2}\, d\theta$$