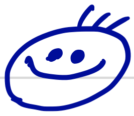
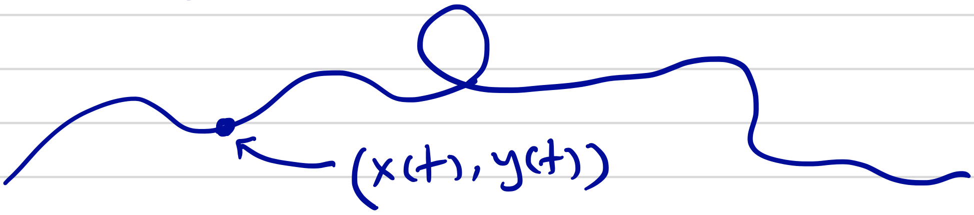
Oct. 16 / 2017



Parametric Curves

$(x(t), y(t))$ $\qquad a \leq t \leq b$

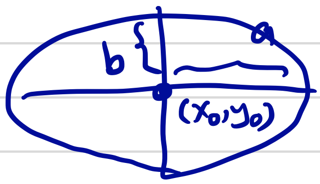


- Para. of circle & ellipse.

Ex. $\dfrac{(x-3)^2}{3} + 4(y-1)^2 = 5 \rightsquigarrow$ ellipse

Find a para. for this ellipse.

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1 \rightsquigarrow$$



$$\frac{(x-3)^2}{\underbrace{15}_{a^2}} + \frac{(y-1)^2}{\underbrace{5/4}_{b^2}} = 1$$

$\sqrt{15} = a$

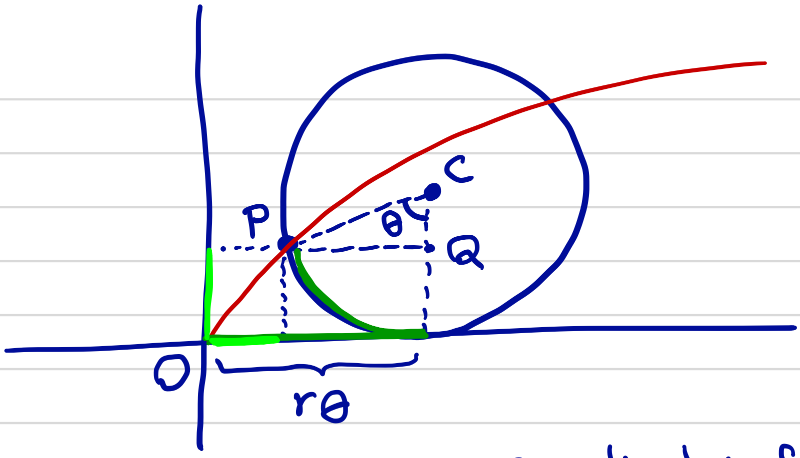
$\sqrt{5/4} = b$

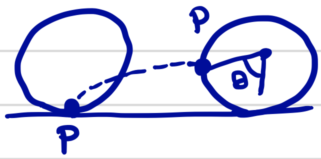
$$x(t) = \sqrt{15}\,\cos(t) + 3 \qquad 0 \leq t < 2\pi$$

$$y(t) = \sqrt{5/4}\,\sin(t) + 1$$

cycloid

r = radius





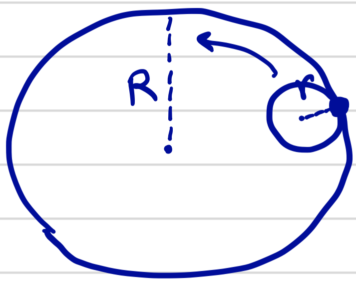
we want x & y coordinates of P in terms of θ.

parameter in radian

$$x(\theta) = r\theta - r\sin(\theta) = r(\theta - \sin(\theta))$$
$$y(\theta) = r - r\cos(\theta) = r(1 - \cos(\theta))$$

☺

Hypocycloid



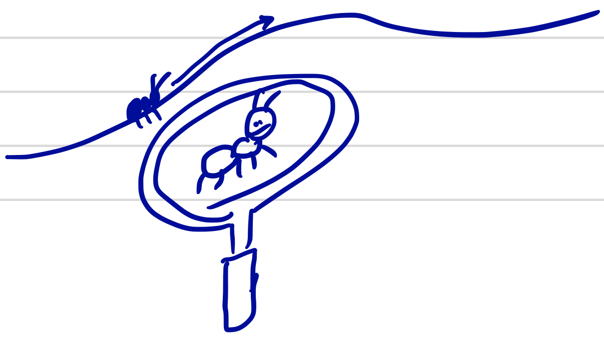
→ Curve traced by a fixed point on the smaller circle.

9.2 Calculus on parametric curves.

- Velocity / tangent vector
- length of a curve

Velocity vector

How fast the ant is moving on the curve?



Velocity at time t $= (x'(t), y'(t))$.

$x'(t) \to \frac{dx}{dt}$, $y'(t) \to \frac{dy}{dt}$

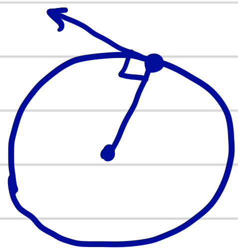
$(x'(t), y'(t))$ = ① vector tangent to the curve (at $(x(t), y(t))$) & ② the length of the curve is the speed.

<u>Ex.</u> (circle) $x(t) = 2\cos(t)$

$y(t) = 2\sin(t)$

velocity $= (-2\sin(t), 2\cos(t))$

speed $= 2 = 2\sqrt{\cancel{\sin^2 + \cos^2}}$



<u>Rem</u> If we want $\frac{dy}{dx}$ (slope of tangent line)

$$\underset{\text{chain rule}}{=} \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)} .$$

<u>Ex.</u> $x(t) = t^3$

$y(t) = t^4$

$(x'(t), y'(t)) = (3t^2, 4t^3) \longrightarrow$ slope $= \frac{4t^3}{3t^2} = \frac{4}{3}t$

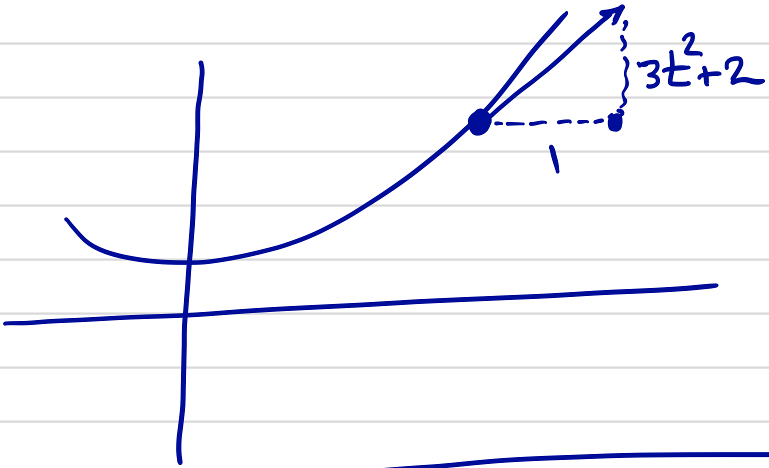
Ex. (graph of a function)

$y = f(x) \rightsquigarrow y = x^3 + 2x + 1$

$x = t$

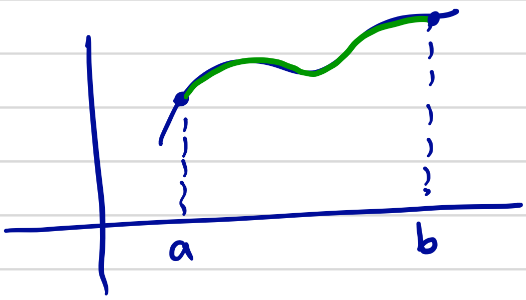
$y = t^3 + 2x + 1$ $\rightsquigarrow$ $x' = 1$

$y' = 3t^2 + 2 = f'(x)$



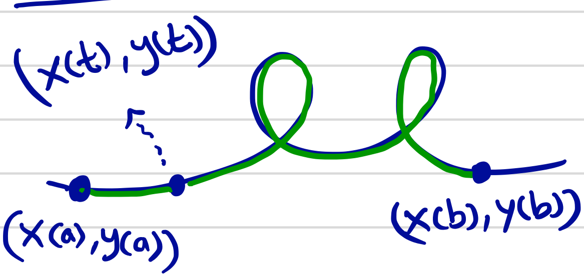
Length of a curve

Recall: length of graph of a function.



$y = f(x)$

length from $\underline{\underline{a}}$ to $\underline{\underline{b}}$ $= \displaystyle\int_a^b \sqrt{1 + f'(x)^2}\, dx$

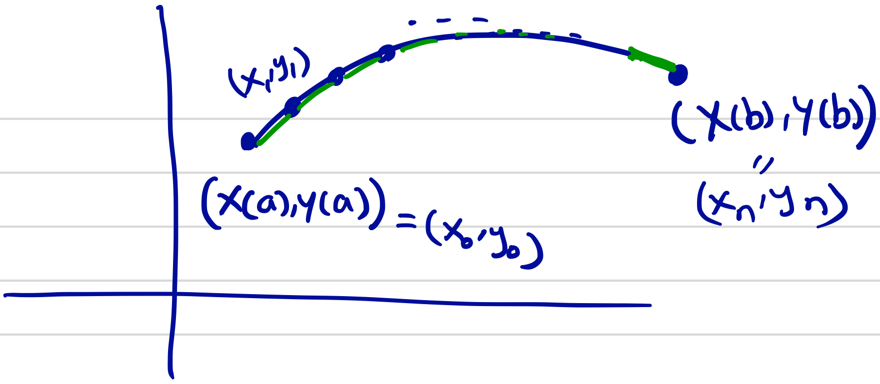


$a \leq t \leq b$

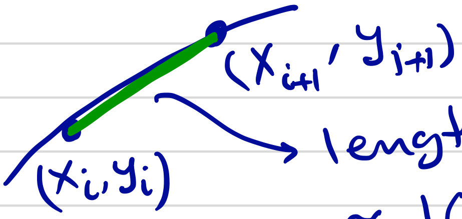
length from $t=a$ to $t=b$ $= \displaystyle\int_a^b \boxed{\sqrt{x'(t)^2 + y'(t)^2}}\, dt$

speed = length of velocity vector

Justification



n very large



length

$\approx \left|(x'(t_i), y'(t_i))\right| \Delta t$

$$\underbrace{\sum_{i=0}^{n-1}}_{\text{sum}} \sqrt{x'(t_i)^2 + y'(t_i)^2}\ \Delta t \rightsquigarrow \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\ dt$$