Oct. 10 / 2017   Midterm tomorrow 

Review problems

Ex. $$\int 3x\, e^{-2x}\, dx$$

$$\underbrace{\int u\, dv}_{\int u v' dx} = uv - \underbrace{\int v\, du}_{\int v u' dx}$$

$u = 3x \rightsquigarrow u' = 3$

$v' = e^{-2x} \rightsquigarrow v = -\frac{e^{-2x}}{2}$

$$= \frac{-3x}{2} \cdot e^{-2x} - \int \frac{-3}{2} e^{-2x}\, dx$$

$$= -\frac{3}{2} x e^{-2x} + \frac{3}{2}\,\frac{e^{-2x}}{-2} + C$$

$e^{x^2} \cdot \cancel{e^{\ln x}}$

$= x e^{x^2}$.

Ex. $$\int e^{x^2 + \ln x}\, dx$$

Note!! $\boxed{e^{(x^2)} \neq (e^x)^2}$

$u = x^2$

$du = 2x\, dx$

$\frac{du}{2} = x\, dx$

$$= \int x e^{(x^2)}\, dx = \int \frac{e^u}{2}\, du = \frac{e^u}{2} + C$$

$$= \frac{e^{(x^2)}}{2} + C.$$

Ex. $\displaystyle\int_1^{\infty} \frac{\ln x}{x}\, dx$ Evaluate or show divergent.

$\displaystyle\int_1^{\infty} \frac{1}{x}\, dx$ divergent !

$\displaystyle\int \frac{1}{x}\, dx = \ln x$ $(x>0)$

$\displaystyle\int_1^{\infty} \frac{1}{x} dx = \lim_{b\to\infty} \left(\ln b - \overset{0}{\cancel{\ln(1)}}\right)$

$= +\infty$.

$\dfrac{\ln x}{x} > \dfrac{1}{x}$

If $\ln x > 1 \iff x > e$.

$\displaystyle\int_1^{\infty} \frac{\ln x}{x}\, dx = \underbrace{\int_1^{e} \frac{\ln x}{x}\, dx}_{\text{Const.}} + \int_e^{\infty} \frac{\ln x}{x}\, dx$ → divergent (by comparison with $\frac{1}{x}$)

overall divergent

Another way:

$\displaystyle\int \frac{\ln x}{x}\, dx$ → $\displaystyle\int u\, du$ ⇝ divergent

$u = \ln x \quad du = \dfrac{dx}{x}$

$= \dfrac{u^2}{2} + C = \dfrac{(\ln x)^2}{2} + C$

Ex. $\int \frac{2x+1}{(x-1)^2}\,dx$

$$\frac{2x+1}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$$ Find A & B.

$$\frac{2x+1}{(x-1)^2} = \frac{A(x-1) + B}{(x-1)^2}$$

$2x+1 = Ax + (B-A) \rightsquigarrow A=2,\ B=3$

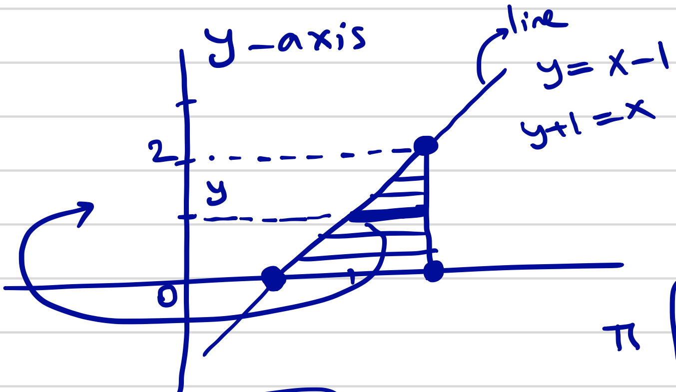
$$\int \frac{2}{x-1}\,dx + \int \frac{3}{(x-1)^2}\,dx = 2\ln(x-1) + 3\left(-\frac{1}{x-1}\right) + C$$

☺

(volumes)

Ex.

Find volume of solid obtained by rotating around y-axis triangle with vertices (1,0), (3,0), (3,2).

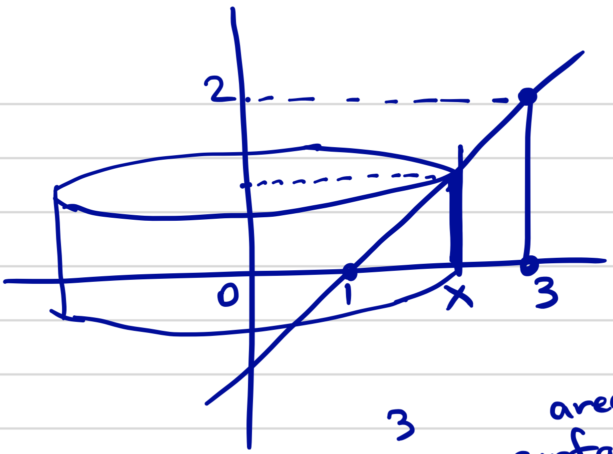


washer method:

variable $\rightsquigarrow$ y

$$\pi \int_{y=0}^{2} R_{(y)}^2 - r_{(y)}^2 \; dy$$

$$\boxed{= \pi \int_0^2 \left(3^2 - (y+1)^2\right) dy}$$



Surface area of cylinder

$= h \cdot 2\pi r$

$$2\pi \int_{x=1}^{3} \underbrace{x(x-1)}_{\text{area of cylinder surface}} \, dx$$

volume of a very thin cylindrical layer

Ex. $\displaystyle\int (9-x^2)^{3/2}\, dx$

$\sin^2 + \cos^2 = 1$

Recall • $a^2 - x^2 \xrightarrow{\text{usually}} x = a\sin\theta$ or $a\cos\theta$

a: some positive Const.

• $a^2 + x^2 \rightsquigarrow x = a\tan\theta \rightsquigarrow 1+\tan^2 = \sec^2$

• $x^2 - a^2 \rightsquigarrow x = a\sec\theta \rightsquigarrow 1-\sec^2 = \tan^2$

$x = 3\sin\theta \qquad dx = 3\cos\theta\, d\theta$

$$\int (9-9\sin^2\theta)^{3/2}\, 3\cos\theta\, d\theta = \int \sqrt{9\cos^2\theta}^{\,3} \cdot 3\cos\theta\, d\theta$$

$$= \int 3^3 \cos^3\theta \cdot 3\cos\theta\, d\theta = 3^4 \int \cos^4\theta\, d\theta \ldots$$

Use double angle formula for Cos