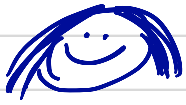
Nov. 27 / 2017  Returned MT2

Remark

About Taylor series/polynomials

- $T_n(x) = f(a) + f'(a)(x-a) + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$

  ↳ at $x=a$; $f(a)$, $f'(a)$ → at $\underline{\underline{a}}$

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

. Rem For calculus explorations ⟶ Wolfram Alpha

We continue with diff. equ.

Sec. 7.7 Last time : Separable diff. equ. (of first order)
↳ only y' appears

$y' = f(t)\, g(y).$

$$\frac{1}{g(y)}\, dy = f(t)\, dt$$

↓ integrate

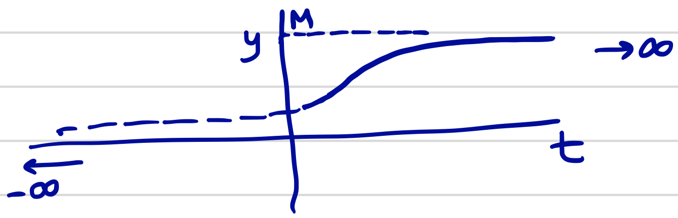
$$\int \frac{1}{g(y)}\, dy = \int f(t)\, dt$$

⟶ solve for y in terms of t.

. **Example** Logistic diff. equ. / function.

$$\frac{dy}{dt} = ky(M-y)$$

k & M Constants



- First order Linear diff. equ. ⇝ Check textbook website.

. n-th order linear diff. equ. : (I use x as variable)

$$\square\, y^{(n)} + \square\, y^{(n-1)} + \cdots + \square = 0.$$

may have x in it (functions of x)

• Linear diff. equ. with **Const.** Coeff. :
No x in the Coeff.

Ex. $3y'' + 4y' + 5y + 3 = 0$

→ In general, one uses linear alg., matrices, ... to solve Lin. diff. equ. with Const. Coeff.

Today we discuss : (non-Const. 1st order linear)

$$\frac{dy}{dx} + P(x)\,y = Q(x)$$

Ex. $$y' + \frac{1}{x} y = 2.$$

$$\underbrace{xy' + y}_{(xy)'} = 2x$$

$$(xy)' = 2x$$

Take anti-der.

$$xy = \int 2x\,dx = x^2 + C$$

$$y = \frac{x^2 + C}{x} = x + \frac{C}{x}$$

- This is example of method of "integrating factor".
  (in this example multiplying by x).

$$y' + P(x)\,y = Q(x)$$

$I(x)$ = integrating factor

$$\underline{I(x)\left(y' + P(x)y\right)} = I(x)\,Q(x)$$

we want: $I(x)\left(y' + P(x)y\right) = \left(I(x)\,y\right)'$

(Find $I(x)$ such that this happens)

$$I(x)\,y' + P(x)I(x)\,y = I(x)\,y' + I'(x)\,y$$

we want: $P(x)\,I(x) = I'(x)$.

$$I'(x) = P(x)\, I(x) .$$

(P(x) is given, we are looking for I(x))

$$\int \frac{I'}{I}\, dx = \int P\, dx$$

$$\ln |I| = \int P dx$$

$$I = A\, e^{\int P dx}$$

$\rightsquigarrow$ This I works as int. factor for $y' + Py = Q$.