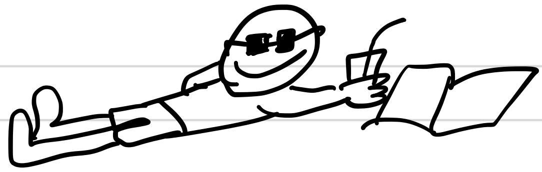
Nov. 20/2017



Diff. equ. Sec. 7.7

- Usual (algebraic) equ: Looking for an x (number).

$$x^2 - 2x + 5 = 0 \rightsquigarrow x = ?$$

- Diff. equ.: Looking for a function $y(t)$ (sometimes $y(x)$)

& we have an equ. involving t, y, y' or higher derivatives.

- If only y' appears $\rightsquigarrow$ First order diff. equ.
- If y' & y'' ------ 2nd order -----.

Ex. (1st order)

$$y'(t) = y(t) \quad (\star) \quad (\text{for all } t) \rightsquigarrow \text{models growth of population}$$

$$y' = y$$

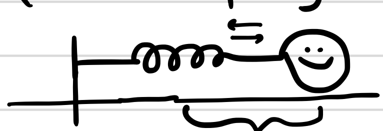
$y(t) = Ae^t$ For any Const. A.

Solution to $(\star)$

Ex. (2nd order) $\rightsquigarrow$ models oscillation in physics (or a spring)

$$y'' = -y.$$

A solution is $y(t) = \sin(t)$ or $\cos(t)$.



distance from resting position $= y(t)$

$F = ma$

$y'' = -y \rightsquigarrow$ [Newton's Law + Hook's Law

- Law's of physics or chemistry economy ... $\rightarrow$ diff. equ. $\xrightarrow{\text{calculus}}$ Find the function.

- Not all diff. equ. are solvable.
- We discuss some simpler cases.

- Separable equ. (first order).

$$y'(t) = \underbrace{F(y,t)}_{f(y)\,g(t)}$$

$f(y)\,g(t)$ separable

Ex. Diff. equ. $\boxed{y' = 3y} \rightsquigarrow 3y \times 1$ Initial value $\boxed{y(0) = 2}$.

$$\frac{dy}{dt} = 3y \rightsquigarrow \frac{1}{3y}\,dy = dt$$

chain rule or (substitution)

$$\frac{1}{3}\int \frac{1}{y}\,dy = \int \frac{1}{3y}\,dy = \int 1\,dt$$

$$\frac{1}{3}\ln(y) = t + C$$

$$\ln(y) = 3t + 3C$$

$$y = e^{3t+3C} = \boxed{A e^{3t}} \qquad A = e^{3C}.$$

$$2 = y(0) = A e^{3\cdot 0} = A \rightsquigarrow \boxed{A = 2} \rightsquigarrow \boxed{y(t) = 2e^{3t}.}$$

(Ex. # 10 is 7.7)

Ex. $\quad \dfrac{dy}{dx} = \dfrac{\ln x}{xy} \qquad y(1) = 2.$

$$\dfrac{dy}{dx} = \dfrac{1}{y} \dfrac{\ln x}{x}$$

$$y\,dy = \dfrac{\ln x}{x}\,dx$$

$$\int y\,dy = \int \dfrac{\ln x}{x}\,dx \rightsquigarrow$$

$\ln x = u$

$\frac{1}{x}dx = du$

$\int u\,du = \frac{u^2}{2} + C$

$= \frac{(\ln x)^2}{2} + C$

$$\dfrac{y^2}{2} = \dfrac{(\ln x)^2}{2} + C$$

$$y = \pm\sqrt{(\ln(x))^2 + 2C}$$

$$2 = y(1) = +\sqrt{0^2 + 2C} \rightsquigarrow 2C = 4.$$

$$\boxed{y(x) = +\sqrt{(\ln(x))^2 + 4}}$$

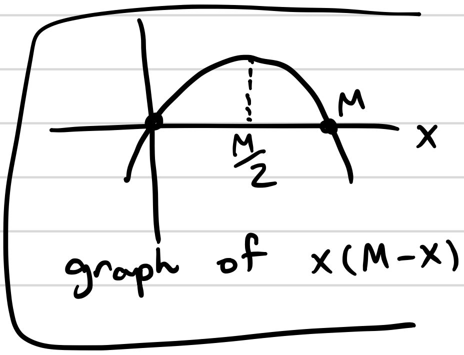
Ex. Logistic growth $\rightsquigarrow$ Verhulst 1840

(or logistic function)

M = max. population (const.)

$y(t)$ = population at time t.

$$(\star) \quad \dfrac{dy}{dt} = \underbrace{k}_{\text{Const.}}\, y(M-y)$$



$$\frac{dy}{dt} = k\,y\,(M-y)$$

$$\int \frac{1}{y(M-y)}\,dy = \int k\,dt$$

$$\frac{1}{y(M-y)} = \frac{1}{M}\left(\frac{1}{y} + \frac{1}{M-y}\right)$$ Partial fraction

$$\int \frac{1}{y(M-y)}\,dy = \frac{1}{M}\Big(\ln(y) - \ln(M-y)\Big) = kt + C$$

Solve for $y(t)$ in terms of t.

$$\ln\left(\frac{y}{M-y}\right) = M\,(kt + C).$$

Suppose $y(0) = y_0$ given.

$$\frac{y}{M-y} = e^{Mkt} \cdot e^{MC} \rightsquigarrow \frac{y}{M-y} = e^{kMt} \cdot \left(\frac{y_0}{M-y_0}\right)$$

$\rightsquigarrow$ you can solve for y.

One can see: $\lim_{t\to\infty} y(t) = M.$

max. value of $y(t)$.