Nov. 13 / 2017

Midterm 2 on Wed.

NO quiz on Tuesday

Representing functions as power series

Some basic power series:

- $$\frac{1}{1-x} = 1+x+x^2+\cdots = \sum x^n \qquad 0 \leq |x| < 1$$
- $$e^x = 1+x+\frac{x^2}{2!}+\cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad \text{For all } x, \ |x|<\infty$$

Ex. Power series rep. of $\frac{1}{1+x}$

- $$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_n (-x)^n = \sum_n (-1)^n x^n = 1-x+x^2-x^3\cdots$$
  $|-x|<1$
  $|x|<1$ radius of conv.
- $$\frac{x^3}{1+x} = \sum_{n=0}^{\infty} x^3 \cdot (-1)^n x^n = \sum_{n=0}^{\infty} (-1)^n x^{n+3}$$
  $$= +x^3 - x^4 + x^5 - \cdots$$
- $$\frac{1}{(1-x)^2}$$

1 $$\frac{1}{1-x}\cdot\frac{1}{1-x} = (1+x+x^2+\cdots)(1+x+x^2+\cdots) =$$
(not the best way)
$$= \left(\sum_n x^n\right)\left(\sum_m x^m\right)$$
$$= \sum_{n,m=0}^{\infty} x^{n+m}$$
rearrange terms & write it as a power series
$$= \sum_{i=0}^{\infty} (i+1)x^i$$
$$= 1+2x+3x^2+4x^3+\cdots$$

② (better way)

$$\frac{1}{(1-x)^2} = \left(\frac{1}{1-x}\right)' = \left(\sum_n x^n\right)'$$

$$= \sum_{n=0}^{\infty} (x^n)' = \sum_{n=1}^{\infty} n x^{n-1}$$

(remember $(x^0)' = 1' = 0$.)

Ex. $$\frac{1}{(1+x)^3} = \left(\frac{1}{-2(1+x)^2}\right)' \rightsquigarrow \int \frac{1}{(1+x)^3}dx = \frac{(1+x)^{-3+1}}{-3+1} + C$$

$$= \left(\frac{1}{2(1+x)}\right)'' \qquad = \frac{(1+x)^{-2}}{-2}$$

$$\frac{1}{2(1+x)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{2} x^n \qquad (x^n)''$$

$$\frac{1}{(1+x)^3} = \left(\frac{1}{2(1+x)}\right)'' = \sum_{n=2}^{\infty} \frac{(-1)^n}{2} n(n-1)x^{n-2}$$

Ex. $$\frac{1}{1+3x^2} = \sum_{n=0}^{\infty} (-3x^2)^n$$

$$\frac{1}{1-(-3x^2)} = \sum_{n=0}^{\infty} (-3)^n x^{2n}$$

$0 \leq |3x^2| < 1$

$3|x|^2 < 1$

$0 \leq |x| < \sqrt{\frac{1}{3}}$

radius of conv.

$\Updownarrow$

$-\sqrt{\frac{1}{3}} < x < \sqrt{\frac{1}{3}}$

Some review problems

Para. Curves :

- Find (slope of) tangent line to a para. curve.

- Find angle between two para. curves.

Ex. $r_1(t) = (t, t^2)$

$r_2(t) = (\sin(t), \sin(2t))$

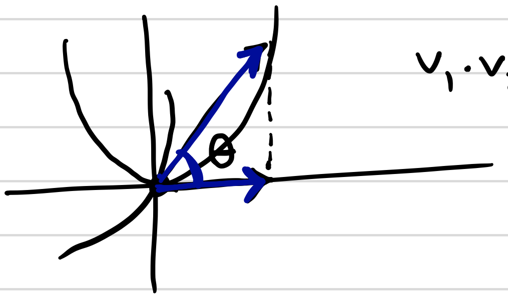
At $t=0$ the two curves intersect.

Find angle between the curves at $t=0$.

$r_1'(t) = (1, 2t) \overset{t=0}{\rightsquigarrow} (1, 0)$

$r_2'(t) = (\cos(t), 2\cos(2t)) \underset{t=0}{\rightsquigarrow} (1, 2)$.

So we want angle between $\underbrace{(1,0)}_{v_1}$ & $\underbrace{(1,2)}_{v_2}$.



$v_1 \cdot v_2 = (1,0) \cdot (1,2) = 1$

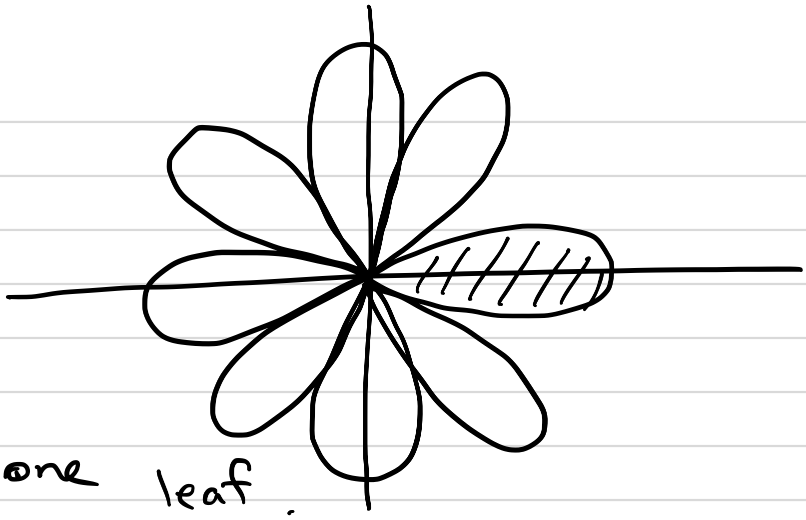
$$\cos(\theta) = \frac{v_1 \cdot v_2}{|v_1|\,|v_2|}$$

$$= \frac{1}{1 \cdot \sqrt{5}}$$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$.

Ex. Consider the polar curve $r = 3\cos 4\theta$.

$r = 3\cos 4\theta$



Find area of one leaf.

- Formula for area in polar coor.

$$\boxed{A = \int_{\theta=a}^{b} \frac{1}{2} r(\theta)^2}$$

- Range of θ in one leaf.

↳ Look at usual graph of $\cos(4\theta)$ & see <u>where</u> $\cos(4\theta)$ goes from $\underline{0}$ & comes back to $\underline{0}$.