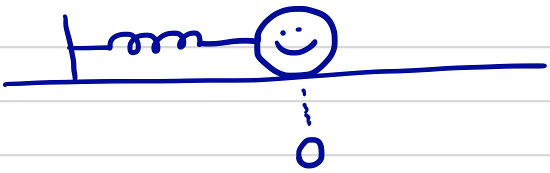
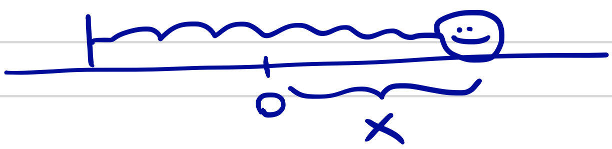
Dec. 6 / 2017

- There are 3 sample finals posted. (New) best for practice
- Please fill out online Teaching Evaluations
- Error estimates & numerical integration & oscillation NOT in the test.

Oscillation $\rightsquigarrow$ Physical interpretation of 2nd ord. lin. diff. equ.

m = mass





Simple oscillation

$x(t)$ = position of the mass at time t.

$$F = -kx$$ Hooke's Const $k > 0$.

$$F = -kx \rightsquigarrow ma = -kx \qquad a = x''(t)$$

$$mx'' + kx = 0$$

$m, k > 0$ Const.

$$mr^2 + k = 0$$
$$r^2 = \frac{-k}{m}$$

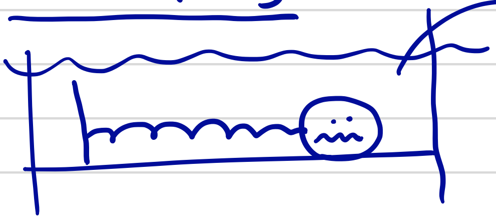
General sol. is lin. comb. of sin & cos.
(i.e. this is the case where char. equ. has complex sol.)

$$r = \sqrt{\frac{k}{m}}\, i \rightsquigarrow r = \underset{0}{\alpha} + \underset{\sqrt{\frac{k}{m}}}{\beta}\, i$$

$$x(t) = c_1 \cos\left(\sqrt{\tfrac{k}{m}}\, t\right) + c_2 \sin\left(\sqrt{\tfrac{k}{m}}\, t\right).$$

determined by initial values
e.g. $x(0)$ & $x'(0)$.

Damping



pool of some liquid (creates friction/resistance force)

damping force is proportional to velocity

$$\text{damping force} = -c\, x'(t).$$

$$ma = F = \underset{\text{spring}}{\text{force of}} + \text{damping force}.$$

mx''

$$mx'' = -kx - cx'$$

$$mx'' + cx' + kx = 0 \quad \text{(homog. equ.)}$$

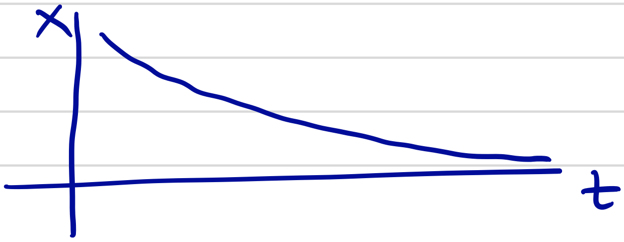
$m, c, k > 0.$

As before 3 case:

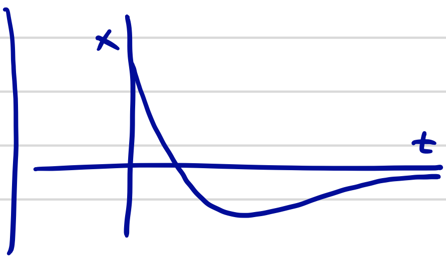
$$mr^2 + cr + k = 0.$$

① $c^2 - 4mk > 0 \rightsquigarrow r_1, r_2 < 0$

② $c^2 - 4mk = 0$



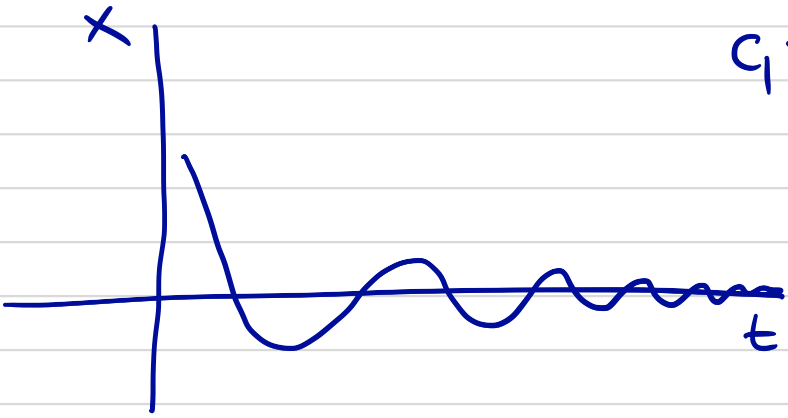
$$c_1 e^{r_1 t} + c_2 e^{r_2 t}$$



③ $c^2 - 4mk < 0$ (Complex roots)

$$c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t)$$

$(\alpha < 0)$



- More generally we can have external force $F(t)$ (Forced vibration)

$$m x'' + c x' + k x = F(t)$$

(non-homog. equ.)

Back to method of undet. coeff.

(for non-homog. diff. equ.)

#10 in Sample final 3

x var.
y function/sol.

$$y'' + 3y' + 2y = \cos x$$

$y(0) = 0$
$y'(0) = 1$

① Solve homog. equ.

② Find y_p (undet. coeff.)

$$r^2 + 3r + 2 = 0 \rightsquigarrow r = -2, -1$$

$$y_c(x) = c_1 e^{-x} + c_2 e^{-2x}$$

$$y'' + 3y' + 2y = \cos(x)$$

$$y_p(x) = A\cos(x) + B\sin(x)$$

$$y_p' = -A\sin + B\cos$$

$$y_p'' = -A\cos - B\sin$$

$$(-A\cos - B\sin) + 3(-A\sin + B\cos) + 2(A\cos + B\sin) = \cos x.$$

$$(\underbrace{-A + 3B + 2A - 1}_{0})\cos + (\underbrace{-B - 3A + 2B}_{0})\sin = 0$$

$A + 3B = 1$ $\qquad$ $B - 3A = 0$

$A + 9A = 1$ $\qquad$ $B = 3A$ $\qquad$ $y_p(x) = \frac{1}{10}\cos(x) + \frac{3}{10}\sin(x)$

$A = 1/10$ $\qquad$ $B = 3/10$

General sol. $y(x) = \underbrace{c_1 e^{-x} + c_2 e^{-2x}}_{y_c} + \underbrace{\frac{1}{10}\cos(x) + \frac{3}{10}\sin(x)}_{y_p}.$

$$y'(x) = -c_1 e^{-x} - 2c_2 e^{-2x} + \left(\frac{-1}{10}\right)\sin(x) + \frac{3}{10}\cos(x).$$

$$y(0) = c_1 + c_2 + \frac{1}{10} + 0 = 0 \Rightarrow c_1 = -c_2 - \frac{1}{10}.$$

$$y'(0) = -c_1 - 2c_2 + \frac{3}{10} = 1 \Rightarrow c_2 + \frac{1}{10} - 2c_2 + \frac{3}{10} = \frac{10}{10}$$

$$-c_2 = \frac{6}{10} \Rightarrow \quad c_2 = -\frac{6}{10}$$

$$c_1 = \frac{5}{10} = \frac{1}{2}.$$

Some 1st order diff. equ. examples

#8 (3rd sample final) (t variable)

$$\frac{dy}{dt} = \underbrace{2t(y-1)^2}_{\text{separable}} \qquad y(0)=2.$$

$$\frac{1}{(y-1)^2}\,dy = 2t\,dt$$

$$\int (y-1)^{-2}\,dy = \int 2t\,dt$$

$$\frac{(y-1)^{-2+1}}{-2+1} = -(y-1)^{-1} = \frac{-1}{y-1} = t^2 + C$$

Solve for y : $y-1 = -\dfrac{1}{t^2+C}$

$$\boxed{y = -\frac{1}{t^2+C}+1}$$

$y(0)=2 \rightsquigarrow y(0) = -\frac{1}{C}+1 = 2 \Rightarrow \frac{-1}{C} = 1 \Rightarrow \boxed{C=-1}$

$$\boxed{y(x) = -\frac{1}{t^2-1}+1}$$

#9 $y' = y + x$ variable $\qquad y(0)=2.$

Next time ...