Dec. 4 / 2017 Last week of classes 

- Final exam : next Tuesday (next week)
- No quiz tomorrow!
- Check webpage for final info & Sample final.
- (The sample final problems now in the webpage is outdated & contains topics we did not discuss.)

Non-homog. 2nd ord. lin. diff. equ. (with Const. Coeff.) :

$$a y'' + by' + cy = G(x)$$

Ex. $y'' + y' - 2y = x^2 \longrightarrow G(x)$

① Solve the homog. equ. $y'' + y' - 2y = 0$
find all sol. $y_c(x)$

② Find one sol $y_p(x)$ of $y'' + y' - 2y = x^2$.

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① $r^2 + r - 2 = 0 \rightsquigarrow r = 1$ & $r = -2$.
(quad. formula)
$(r-1)(r+2)$

$y_c(x) = c_1 e^{x} + c_2 e^{-2x} \rightsquigarrow$ general sol. to the homog. equ.

② (Method of undetermined coeff.)

- If $G(x)$ is a poly. of deg. n then $y(x)$ can be taken to be a poly. of deg. n.

Determine coeff. of this poly. so it satisfies the diff. equ.

$$G(x) = x^2 \rightsquigarrow y_p(x) = Ax^2 + Bx + C \rightsquigarrow \text{Find } A, B, C$$

$$y_p'(x) = 2Ax + B$$

$$y_p''(x) = 2A$$

$$y'' + y' - 2y = x^2 \rightsquigarrow \boxed{2A + 2Ax + B - 2(Ax^2 + Bx + C) = x^2}$$

We need to find A, B, C such that for all x.

$$\underbrace{(-2A)}_{1}x^2 + \underbrace{(2A - 2B)}_{0}x + \underbrace{(2A + B - 2C)}_{0} = x^2$$

$-2A = 1 \rightsquigarrow A = -\frac{1}{2}$.

$2A - 2B = 0 \rightsquigarrow -1 - 2B = 0 \rightsquigarrow B = -\frac{1}{2}$.

$2A + B - 2C = 0 \rightsquigarrow -1 - \frac{1}{2} = 2C \rightsquigarrow C = -\frac{3}{4}$.

$y_p(x) = \left(-\frac{1}{2}\right)x^2 + \left(-\frac{1}{2}\right)x + \left(-\frac{3}{4}\right)$ is a "particular" sol.

The general sol. $y(x) = \underbrace{\left(\frac{-1}{2}\right)x^2 + \left(\frac{-1}{2}\right)x + \left(\frac{-3}{4}\right)}_{y_p} + \underbrace{c_1 e^{x} + c_2 e^{-2x}}_{y_c}$.

other examples are variations of this :

<u>Ex.</u> $y'' + 4y = e^{5x}$

$G(x) = e^{5x} \rightsquigarrow y_p$ is of the form Ae^{5x}.

(need to find A).

$$\left.\begin{aligned} y_p' &= 5A e^{5x} \\ y_p'' &= 5^2 A e^{5x} \end{aligned}\right\} \rightarrow 25Ae^{5x} + 4Ae^{5x} = e^{5x}.$$

$$25A + 4A = 1$$

$$A = \frac{1}{29} \rightsquigarrow \boxed{y_p(x) = \frac{e^{5x}}{29}}$$

. $r^2 + 4 = 0 \rightsquigarrow r = \pm\sqrt{-4} = \pm 2i$. $i = \sqrt{-1}$

$\alpha = 0,\ \beta = 2$

$y_c(x) = c_1 \cos(2x) + c_2 \sin(2x)$.

General sol. $y(x) = \frac{e^{5x}}{29} + \underline{\underline{c_1}} \cos(2x) + \underline{\underline{c_2}} \sin(2x)$.

- $G(x) = \cos(3x) \rightsquigarrow y_p(x) = A\cos(3x) + B\sin(3x)$
- $G(x) = x^2 e^x \rightsquigarrow y_p(x) = (Ax^2 + Bx + C)e^x$

  $G(x) = xe^{2x} \rightsquigarrow y_p(x) = (Ax + B)e^{2x}$
- $G(x) = x^2 + \sin(x) \rightsquigarrow y_p(x) = (Ax^2 + Bx + C) + D\cos(x) + E\sin(x)$.

Ex. $y'' + 2y' + 4y = x\cos(3x)$

$y_p(x) = (Ax + B)\cos(3x) + (Cx + D)\sin(3x).$

Find y_p' & y_p'' & plug-in the equ.

to find A, B, C, D.