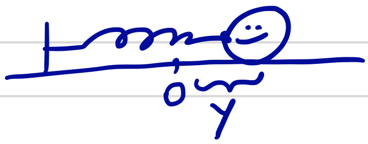
Dec. 1 /2017

Examples of Case (3) — **Char. equ. has no real roots**

Ex. $y'' + 4y = 0$

$\rightsquigarrow$ 

$4 =$ Spring or Hook's Const.

$ma = F = -ky$

$r^2 + 4 = 0$

$\Downarrow$

$r^2 = -4 \qquad r = \pm\sqrt{-4} = \underbrace{+2i}_{r_1}, \ \underbrace{-2i}_{r_2} \qquad i = \sqrt{-1}.$

$r_1 = \alpha + i\beta \rightsquigarrow \alpha = 0 \quad \beta = 2$

$r_2 = \alpha - i\beta$

General solution: $y(x) = e^{\alpha x}\left(c_1 \cos(\beta x) + c_2 \sin(\beta x)\right)$

$$= c_1 \underbrace{\cos(2x)}_{y_1(x)} + c_2 \underbrace{\sin(2x)}_{y_2(x)}.$$

Ex. $y'' - 6y' + 13y = 0$

this term corr. to a spring with "damping".

$r^2 - 6r + 13 = 0$

$$\frac{6 \pm \sqrt{36 - 4 \times 13}}{2} = \frac{6 \pm \sqrt{-16}}{2}$$

$$= \frac{6}{2} \pm \frac{\sqrt{16}}{2} i = \overset{\alpha}{3} \pm \overset{\beta}{2} i$$

$$y(x) = e^{3x}\left(c_1 \cos(2x) + c_2 \sin(2x)\right)$$

Note! No imaginary i in $y(x)$.

(For interested people)

Rem $\quad \alpha + i\beta \rightsquigarrow e^{(\alpha+i\beta)} = e^{\alpha} \cdot e^{i\beta}$

$$e^{i\beta} = \cos(\beta) + i\sin(\beta) \quad (\star)$$

(Lots of trig. formulas can be recovered from $(\star)$).

Homog. 2nd ord. lin. diff. equ. $\quad ay'' + by' + cy = 0$

with initial conditions.

Ex. $\quad y'' + y' - 6y = 0$

Find sol. $y(x)$ of the diff. equ. with

$\underbrace{y(0) = 1 \ \& \ y'(0) = 0}_{\text{two initial Condition}}$. (initial value problem)

$r^2 + r - 6 = 0 \rightsquigarrow r = 2, -3$

$y(x) = c_1 e^{2x} + c_2 e^{-3x} \rightsquigarrow \frac{3}{5}e^{2x} + \frac{2}{5}e^{-3x} = y(x)$.

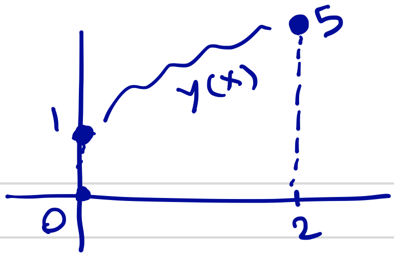
Need to find c_1, c_2 such that :

. $c_1 e^{0} + c_2 e^{0} = c_1 + c_2 = 1$

. $2c_1 e^{0} + (-3)c_2 e^{0} = 2c_1 - 3c_2 = 0$. $\quad (y'(x) = 2c_1 e^{2x} + (-3)c_2 e^{-3x})$

2 equ. & 2 unknowns c_1, c_2 $\rightsquigarrow$ $c_2 = \frac{2}{3}c_1$, $c_1 + \frac{2}{3}c_1 = 1$ $\rightarrow$ $\boxed{c_1 = \frac{3}{5},\ c_2 = \frac{2}{5}}$

Another variation of this:



$$y'' + y' - 6y = 0$$

Boundary value problem

Find sol. y such that $\boxed{y(0) = 1 \text{ and } y(2) = 5}$

$$y(x) = c_1 e^{2x} + c_2 e^{-3x}$$

$y(0) = 1 \rightsquigarrow c_1 + c_2 = 1$

$y(2) = 5 \rightsquigarrow c_1 e^{4} + c_2 e^{-\frac{3}{2}} = 5$

Solve to find c_1, c_2

Next topic is <u>non-homog. 2nd ord. lin. diff. equ.</u> (with Const. Coeff.)

$$(\star) \quad a y'' + b y' + c y = G(x) \rightsquigarrow \text{some function of } x.$$

Methods to find sol.:

- Undetermined Coeff. $\rightsquigarrow$ We will discuss.
- (Variation of para. $\rightsquigarrow$ Don't need to know for this course)

Fact: To find general sol. of equ. $(\star)$ we need to find only <u>one</u> sol. of it (called "particular sol." y_p) & <u>all</u> sol. of homog. $ay'' + by' + cy = 0$

y_c "Complementary" Sol.

That is, general sol. $y(x)$ of :

$$ay'' + by' + cy = G(x)$$

is of the form $y(x) = \underbrace{y_p(x)}_{ay''+by'+cy=G(x)} + \underbrace{y_c(x)}_{ay''+by'+cy=0}$.