Aug 30 / 2017

First quiz next week (during recitation) Tuesday

LON CAPA PS1 due end of next week.

Ave. value of a function:

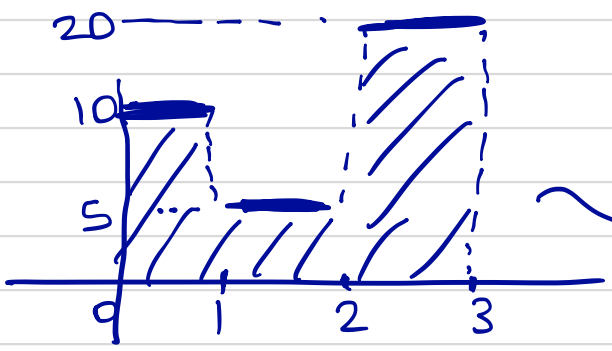
$$f : [a,b] \longrightarrow \mathbb{R}$$
$$(a,b)$$

Ave. value of f (over $[a,b]$) $:= \left(\int_a^b f(x)\,dx\right) / b-a$.

Ex. Ave. temp. during a day. (Contin. changing quantity)

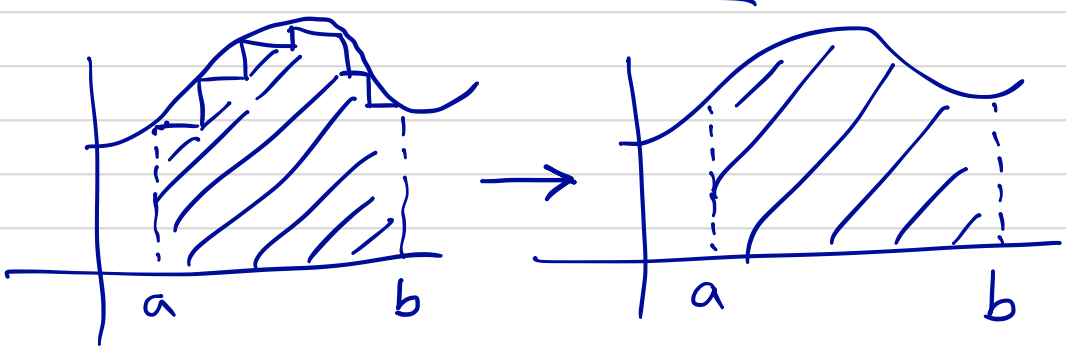
Discrete quantity $\{10, 5, 20\}$

Ave. value of $(10+5+20)/3$



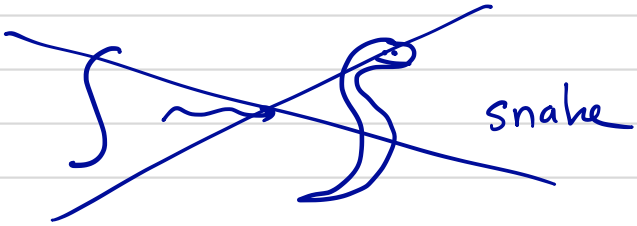
step function $\rightarrow f$

$$\left(\int_0^3 f(x)\,dx\right)/(3-0)$$



$\int$ integral ⟶ Sum ⟶ $\sum$ Greek ⟶ tooth

Phoenicians



6.1 Integration by Parts

works
- For some functions

Typical ex.
- $x \sin x$
- $x \cos x$
- $x e^x$
- $e^x \sin(x)$

- $\ln x$
- $\tan^{-1} \rightsquigarrow \arctan(x)$

$$(fg)' = fg' + f'g$$

$$fg = \int fg'\,dx + \int f'g\,dx$$

$$\int fg'\,dx = fg - \int f'g\,dx$$

$f(x) = u$ ⟶ diff. ⟶ $f'(x)dx = du$

$g(x) = v$ ⟵ int. ⟵ $g'(x)dx = dv$

$$\int u\,dv = uv - \int v\,du.$$

$$\int_a^b u\,dv = uv\Big]_a^b - \int_a^b v\,du$$

Ex. $\int x \sin(x)\,dx = x(-\cos x) - \int (-\cos x)\,dx$

u dv $\quad$ uv

$du = x'dx$ $\quad$ $v = -\cos x$

$= dx$

$= -x\cos(x) + \sin(x) + C$

Ex. $\int \ln x\,dx = \int \ln x \cdot 1\,dx$

u $\quad$ dv

$= (\ln x)\,x - \int x \frac{1}{x}\,dx$ $\quad$ $uv - \int v\,du$

$= x\ln x - x = x(\ln x - 1) + C$

Ex. $\int x e^x\,dx$ ....

u $\quad$ dv

Ex. $\int x^2 e^x\,dx = uv - \int v\,du$

u $\quad$ dv $\quad$ $v = e^x$ $\quad$ $du = 2x\,dx$

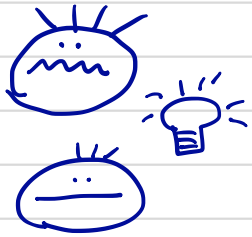
$= x^2 e^x - 2\int e^x\,x\,dx$ → we can find with one more I. by P.

Ex. $\int e^x (\cos x)\, dx$

u $\quad dv$

$du = e^x dx \qquad v = \sin(x)$

$= e^x \cdot \sin(x) - \int \sin(x)\, e^x\, dv$



$\int e^x (\sin x)\, dx = (-\cos x) e^x + \int \cos x\, e^x\, dx$

u $\quad dv$

$du = e^x dx \qquad v = -\cos x$

A

$\int e^x \cos x\, dx = e^x \sin(x) + e^x \cos x - \int e^x \cos x\, dx$ $\quad A$

$A = \frac{e^x(\sin + \cos)}{2} + C$

$\int (\cos x) e^x\, dx = \frac{e^x(\sin x + \cos x)}{2} + C$

recall

$$\tan^{-1}(x) \xrightarrow{\text{der.}} \frac{1}{x^2+1}$$

$$\tan^{-1}(x) \xleftarrow{\int} \frac{1}{x^2+1}$$

Ex. $\int \tan^{-1}(x)\,dx = \int \underbrace{\tan^{-1}(x)}_{u} \cdot \underbrace{1\,dx}_{dv}$

$$\tan^{-1}(x)\cdot x - \int x \cdot \frac{1}{1+x^2}\,dx$$

$w = 1+x^2 \qquad dw = 2x \rightsquigarrow x\,dx = \frac{dw}{2}$

$$\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{dw}{w} = \frac{1}{2}\ln(1+x^2)$$