

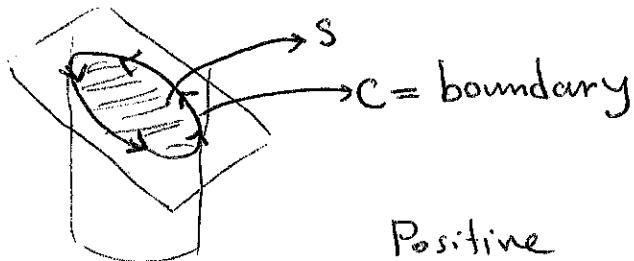


Tomorrow Quiz one question → Stokes or Divergence thm.

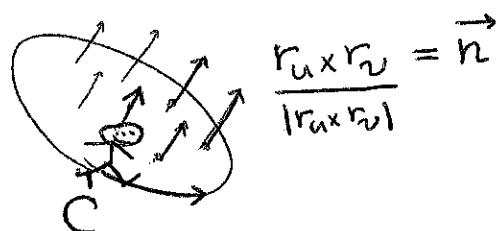
- Please do the sugg. practice problems in 13.8 & 13.9 as well as examples in 13.8 & 13.9

Recalling

Ex. (from last time)



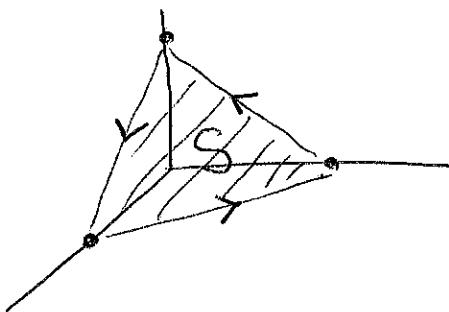
Positive orientation for C
(w.r.t. orientation of S)



: The Surface S should be to your left.

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl } \vec{F} \cdot dS \quad \dots$$

Ex. $\vec{F} = (z^2, y^2, x)$ C . triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$



Compute $\oint_C \vec{F} \cdot d\vec{r}$

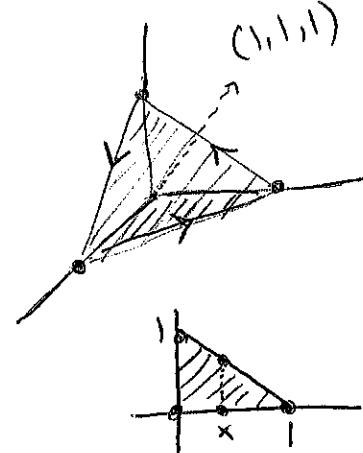
$$x + y + z = 1 \rightarrow z = 1 - x - y.$$

(2)

Let's use Stoke's thm. by computing

$$\iint_S \operatorname{curl} \vec{F} \cdot d\vec{S}$$

$$\operatorname{curl} \vec{F} = \begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z^2 & y^2 & x \end{bmatrix} = (2z-1)j$$



~~area~~ ~~area~~ ~~area~~

$$(x, y) \mapsto r = (x, y, 1-x-y).$$

$$0 \leq x \leq 1$$

$$0 \leq y \leq 1-x$$

$$r_x = (1, 0, -1)$$

$$r_y = (0, 1, -1)$$

$$r_x \times r_y = \begin{bmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$= (1, 1, 1).$$

(the positive orientation on C is counter-clockwise).

$$\begin{aligned} \iint_S \operatorname{curl} \vec{F} \cdot d\vec{S} &= \iint_{x=0}^1 \int_{y=0}^{1-x} (2z-1)j \cdot (i+j+k) dy dx \\ &= \iint_0^1 \int_0^{1-x} (2(1-x-y)-1) dy dx \\ &= \int_0^1 \left(y - 2xy - y^2 \right)_0^{x+1} dx \\ &= \left(\frac{x^3}{3} - \frac{x^2}{2} \right)_0^1 = -\frac{1}{6}. \end{aligned}$$

(2)

(3)

Recall statement of Divergence Thm. :

Thm (Divergence thm.)

E 3D domain.

\vec{F} vec. field. (is defined in E

S boundary of E

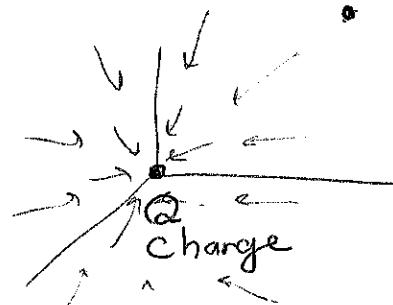
and contin. partial derivatives)

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \operatorname{div}(\vec{F}) dV$$

(usually RHS easier to compute).

$$\vec{r} = (x, y, z)$$

Ex. Electric field



$$\vec{E}(r) = \frac{\epsilon_0 Q}{|r|^3} \vec{r} = \frac{\epsilon_0 Q}{(\sqrt{x^2+y^2+z^2})^3} \vec{r}$$

$$P \left(\frac{x}{(\sqrt{x^2+y^2+z^2})^{3/2}}, \frac{y}{(\sqrt{x^2+y^2+z^2})^{3/2}}, \frac{z}{(\sqrt{x^2+y^2+z^2})^{3/2}} \right)$$

~~$$\vec{E} = \frac{\epsilon_0 Q}{(\sqrt{x^2+y^2+z^2})^3} \vec{r}$$~~

$$\frac{\partial}{\partial x} \left(\frac{x}{(\sqrt{x^2+y^2+z^2})^{3/2}} \right) = \frac{-x^2 - 2y^2 - 2z^2}{(\sqrt{x^2+y^2+z^2})^3}$$

$$= \frac{(x^2+y^2+z^2)^{3/2} - 2x^2 \cdot \frac{3}{2} (\sqrt{x^2+y^2+z^2})^{1/2}}{(\sqrt{x^2+y^2+z^2})^3}$$

(Note that E is undefined at $(0,0,0)$).

$$\operatorname{div} E = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = 0$$

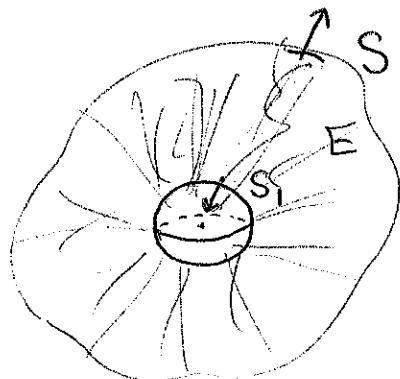
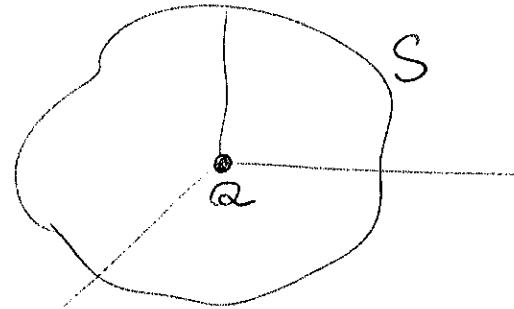
Let S be a closed surface enclosing origin. ④

The electric flux

$$\iint_S \vec{E} \cdot d\vec{S}$$

is equal to $4\pi\epsilon Q$.

(does not depend on the surface S , just on Q).



S_1 = small sphere with origin in the centre.

$$\iint_S \vec{E} \cdot d\vec{S} = \iint_{S_1} \vec{E} \cdot d\vec{S}$$

* Apply Div. Thm. to the region E between S_1 & S .

$$\iint_{S_1 \cup S} \vec{E} \cdot d\vec{S} = \iiint_E \text{div } \vec{E} dV = 0$$

because
 $\text{div } \vec{E} = 0$

$$\iint_S \vec{E} \cdot d\vec{S} - \iint_{S_1} \vec{E} \cdot d\vec{S} = 0 \Rightarrow$$

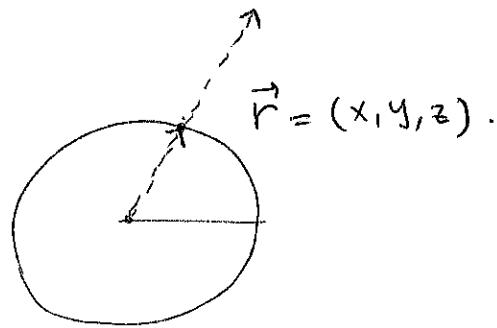
$$\iint_S \vec{E} \cdot d\vec{S} = \iint_{S_1} \vec{E} \cdot d\vec{S}$$

Let's calculate

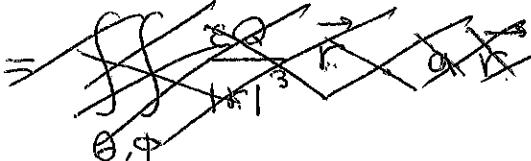
S_1 = Sphere of rad. a

$$\vec{n} = \frac{\vec{r}_\theta \times \vec{r}_\phi}{|\vec{r}_\theta \times \vec{r}_\phi|} = \frac{a \vec{r}}{|\vec{r}|}$$

$$\iint_{S_1} \vec{E} \cdot d\vec{S}$$



$$\iint_{S_1} \vec{E} \cdot d\vec{S}$$



$$\begin{aligned}
 &= \iint_{S_1} \frac{\epsilon Q}{a^2} dS = \frac{\epsilon Q}{a^2} \text{Area}(S_1) \\
 &= \epsilon Q \frac{4\pi a^2}{a^2} \\
 &= 4\pi \epsilon Q.
 \end{aligned}$$