No quiz this week!

LON CAPA

PS 1 open & due next Wednesday.

10.2, 10.3, 10.4 $\quad \cdot \quad \times$

Dot product

vector $\cdot$ vector $=$ scalar.

dot!

Def. $\vec{a} = (x_1, y_1, z_1)$ $\quad \cancel{(x_1x_2, y_1y_2, z_1z_2)}$

$\vec{b} = (x_2, y_2, z_2)$

$$\vec{a} \cdot \vec{b} = x_1x_2 + y_1y_2 + z_1z_2$$

Motivation: helps in computing geo. quantities (length, angle, ...) using coordinates of vectors.

Length of vector: $\vec{a} = (x, y, z)$

$$\underbrace{\|\vec{a}\| \text{ or } |\vec{a}|}_{\text{length of } \vec{a}} = \sqrt{x^2+y^2+z^2} = \text{distance of } (x,y,z) \text{ to } O.$$

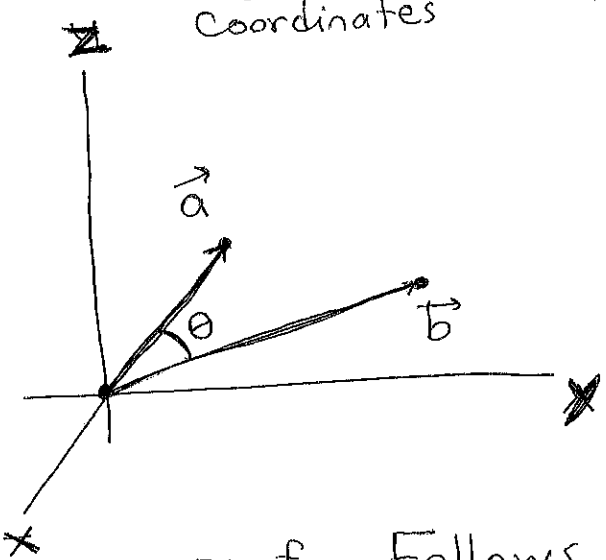
$$\vec{a}\cdot\vec{a} = x^2+y^2+z^2 \longrightarrow |\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}.$$

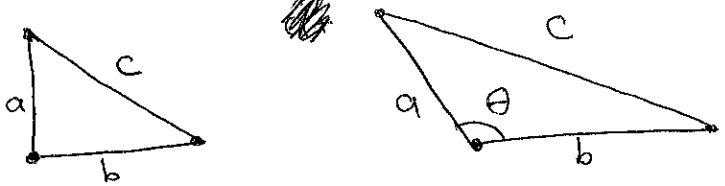
<u>Theorem</u>

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos(\theta)$$

$\vec{a}\cdot\vec{b} \rightarrow x_1x_2 + y_1y_2 + z_1z_2$ algebra and coordinates

$|\vec{a}||\vec{b}|\cos(\theta) \rightarrow$ geo. & no coordinates





$a^2+b^2=c^2$

$a^2+b^2-2ab\cos\theta = c^2$

Law of Cosinces.

<u>proof</u> Follows from "Law of Cosines"

How to find angle between vectors

$\vec{a}, \vec{b}$ $\quad \theta =$ angle between $\vec{a}$ & $\vec{b}$

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \frac{\vec{a}\cdot\vec{b}}{\sqrt{\vec{a}\cdot\vec{a}}\,\sqrt{\vec{b}\cdot\vec{b}}}$$

Rem (x, y, z) → Point

$\langle x, y, z \rangle$ → vector in the textbook.

Ex. $\vec{a} = (1,2,1)$ $\quad \vec{b} = (3,1,0)$.

$$\vec{a}\cdot\vec{b} = 1\cdot 3 + 2\cdot 1 + 1\cdot 0 = 5$$

$$|\vec{a}| = \sqrt{6} \qquad |\vec{b}| = \sqrt{10}$$

$$\cos\theta = \frac{5}{\sqrt{6}\,\sqrt{10}}. \qquad \theta = \cos^{-1}\left(\frac{5}{\sqrt{60}}\right)$$

$$\frac{5}{\sqrt{4\times 15}} = \frac{5}{2\sqrt{15}}.$$

$|\vec{a}| \neq 0 \quad |\vec{b}| \neq 0$

- $\vec{a}\cdot\vec{b} = 0$

$\Rightarrow |\vec{a}|\,|\vec{b}| \cos\theta = 0 \Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2$ or $-\pi/2$.

$\vec{a}, \vec{b}$ perpendicular perp.

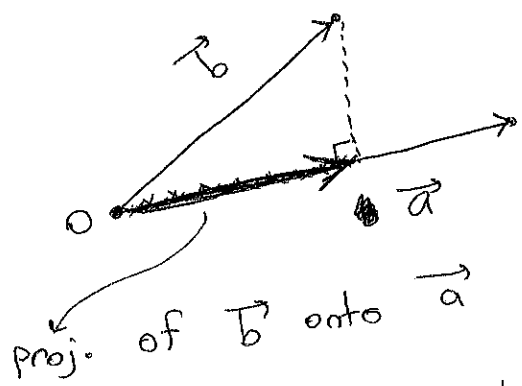
Alg. properties of dot product.

$\vec{a}, \vec{b}, \vec{c}$ $\qquad \vec{a}\cdot(\vec{b}+\vec{c}) = (\vec{a}\cdot\vec{b}) + (\vec{a}\cdot\vec{c})$

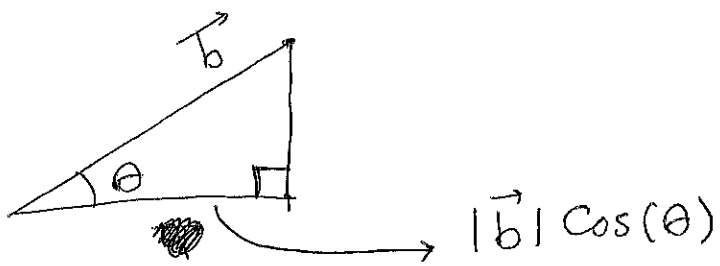
$\vec{a}, \vec{b}$ $\qquad \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

c scalar $\qquad (c\vec{a})\cdot\vec{b} = \vec{a}\cdot(c\vec{b}) = c(\vec{a}\cdot\vec{b})$

(vertical) Projection

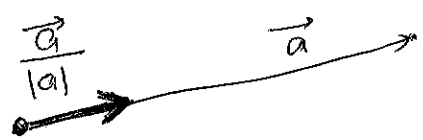


- $\vec{a}$, $\vec{b}$ known (i.e. Coor. are known)
- Find $\text{Proj}_{\vec{a}}\,\vec{b}$ → its length
  → as a vector.



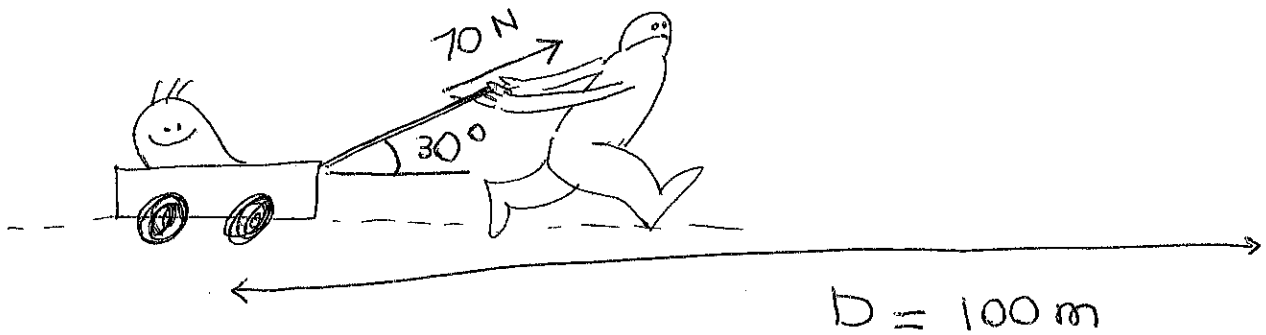
= Comp. of $\vec{b}$ along $\vec{a}$
= length of proj. = scalar proj. of $\vec{b}$ onto $\vec{a}$ = $\boxed{\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}}$

$\dfrac{\vec{a}}{|\vec{a}|}$ = unit vector in the direction of $\vec{a}$
→ length 1.



$$\boxed{\text{Proj}_{\vec{a}}\,\vec{b}} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\right)\frac{\vec{a}}{|\vec{a}|} = \boxed{\underbrace{\left(\frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\right)}_{\text{scalar}}\underbrace{\vec{a}}_{\text{vec.}}}$$

Ex. in Physics Work.



$$W \overset{\text{def.}}{=} \begin{pmatrix}\text{distance}\\ \text{traveled}\end{pmatrix}\begin{pmatrix}\text{Component of the}\\ \text{force in the direction of motion}\end{pmatrix}$$

W → work done by a constant force on some object traveling in a linear motion.

Comp. of $\vec{F}$ on $\vec{D}$

$$W = |\vec{D}|\,(|\vec{F}|\cos(\theta)) = \vec{D}\cdot\vec{F}.$$

$|\vec{D}|$ → displacement vec.

$\cos(30^\circ) \to \frac{\sqrt{3}}{2}$

$$W = 100 \cdot 70 \cdot \cos(30^\circ)$$

$$= 7000 \cdot \frac{\sqrt{3}}{2}.$$