Project Planning via PERT/CPM

Project: Any set of complex, interrelated activities aimed at some specific goal. Typically, characterized by a clear starting point and a clear end point. Examples:

- Construction (buildings, roads, bridges, etc.)
- New product introduction
- New systems installation
- One-at-a-time manufacturing (e.g., ships, aircraft)
- Space flight launches
- Large scale logistics projects
- etc.

Benefits of PERT/CPM

- Forces detailed planning
- Improves communications
- Identifies potential bottlenecks and problem areas
- Provides timely progress reports
- Assists with long range planning including budgets

Project Networks in PERT/CPM

Objective: To find the shortest time in which a project (comprising many interrelated tasks or activities) can be completed, along with a list of the "critical" activities, and the "slack" available for the noncritical ones.

Network Representation

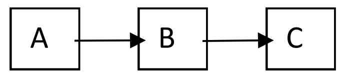
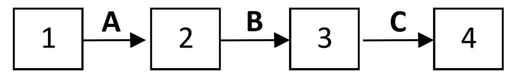
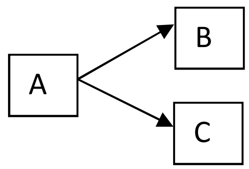
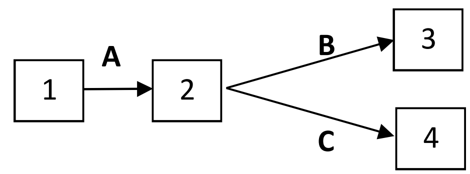
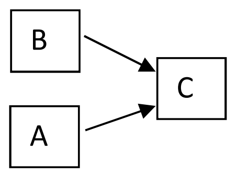
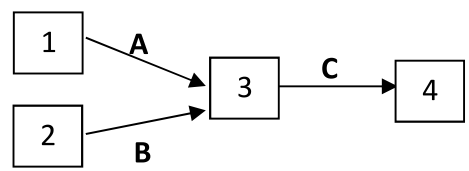
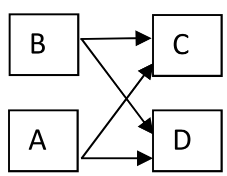
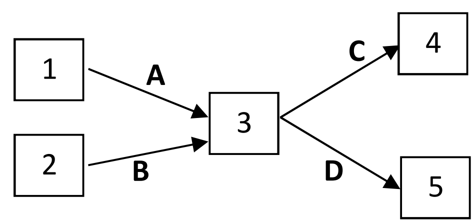
*Activity-on-node (**AON) convention:***

- Node ≡ a project task/activity with an associated duration
- Arc ≡ representation of a precedence relationship
  - one arc for each precedence relationship: if A has to precede B there is an arc from node A to node B
  - one START and one FINISH node (each of zero duration)

*Activity-on-arc (**AOA) convention:***

- Arc ≡ a project task/activity with an associated duration
- Node ≡ a project *event* signifying that
  - all activities represented by arcs entering that node have been completed, and
  - that all activities represented by arcs leaving that node can therefore commence
- Rules
  - Exactly one arc for representing each activity
  - Exactly one arc connecting any two nodes
  - Numbering Convention
    - Node 1 is the START node and signifies project commencement – has no incoming arcs
    - The highest numbered node is the FINISH node & signifies project completion – has no outgoing arcs
    - For the activity (*i-j*), *the numbering should be such that j>i, i.e., flow is always from a lower to a higher* numbered node
  - Use dummy arcs (and perhaps, extra nodes) to ensure correct precedence structures:
    - all precedence requirements must be correctly taken into account, yet no unnecessary or extraneous precedence requirement may be introduced into the problem

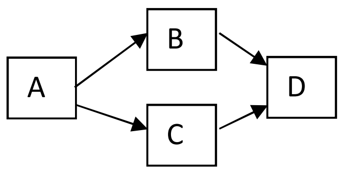
Network Representation

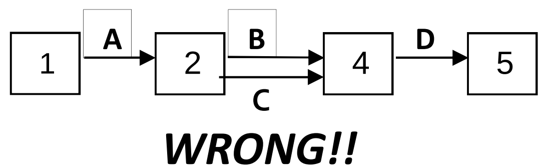
Precedence	AON Representation	AOA Representation
A<B B<C		
A<B,C		
A,B<C		
A,B<C,D		

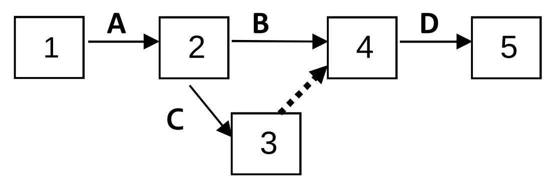
Network Representation

Precedence | **AON Representation** | **AOA Representation**

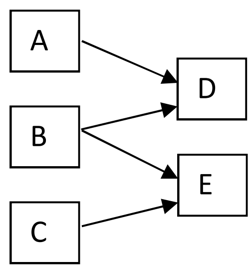
A < B, C
B,C < D

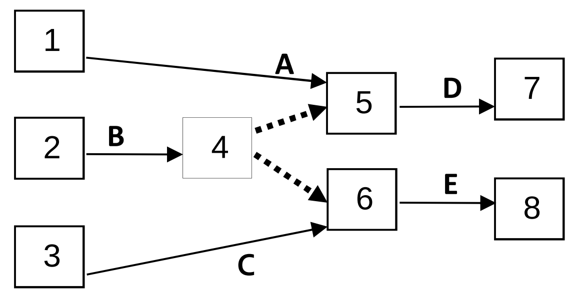


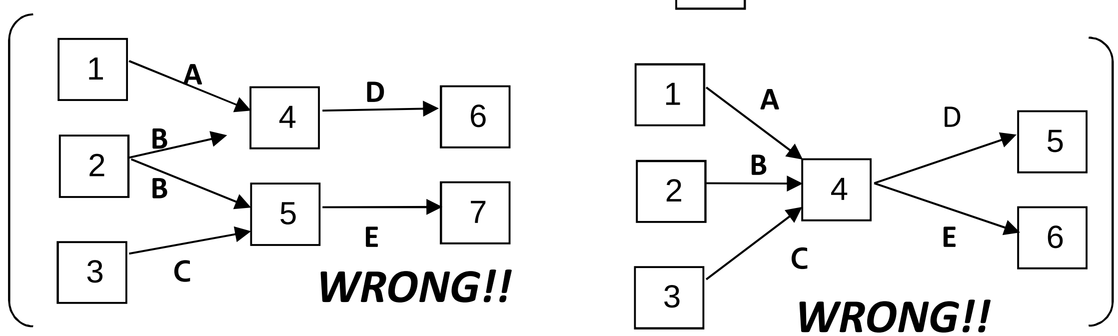




A< D
B< D,E
C< E







Project Planning via PERT/CPM

A company is considering the purchase of several units of an expensive new machine for its plants around the country. However, management would first like to set up a test facility and test the machine thoroughly before committing itself to the entire order; the machine tool manufacturer has agreed to lend them a machine for this purpose. Since it is important to complete this project quickly, it is of interest to find the shortest schedule for the entire project along with some indication of which activities in the project are critical and how much of slack is available for the various phases of the project.

The sequence of activities is as follows: first the test facility must be prepared and the paperwork for borrowing the machine must be completed; these can go on concurrently and are expected to take about 4 days. Once these formalities have been completed, the machine must be transported to the facility and repair personnel and operators need to hired. The transportation is expected to take 8 days, while the hiring of personnel for the repair team and operators are expected to take 7 and 6 days respectively. After the machine has arrived and the repair team is complete, training of the team can begin - this is expected to take 15 days, and can proceed while the actual installation of the machine is still in progress. For the machine to be installed correctly at the test facility after arrival it is expected to take 9 days. After installation, the hired operators must be trained to run the machine and this is expected to take 12 days. Finally, once the operators and the repair personnel are fully trained, the actual testing can commence and the testing process is expected to take 20 days.

Plan the project.

Constructing Project Networks - Another Example

Designer Genes (DG), a rapidly expanding genetic engineering firm, is relocating to a new office building. Jean Ettic, the project manager, has already leased a vacant office building and approved architectural plans for renovating it. To save money and to gain more control over the project, Jean herself will act as the general contractor for the renovation. She has already selected the subcontractors that she will employ. The move to the new building is complicated by the fact that DG will use this opportunity to purchase a new computer and establish an in-house Computer Services Department. The present decentralized computer capacity is simply inadequate to meet the future needs of DG.

Project Formulation for Designer Genes' Relocation

Code	Description of Activity	Estimated Duration (days)	Predecessors
A	Hiring of Manager of Computer Services Dept.	10	None
B	Structural Modifications	19	None
C	Enlarging and Resurfacing of Parking Lot	13	None
D	Hiring of Staff of Computer Services Dept.	8	A
E	Purchase and Receipt of Computer	14	A
F	Electrical Modifications	4	B
G	Heating and Cooling Modifications	1	B
H	Plumbing Modifications	3	B
I	Exterior Painting & Installation of Fixtures	5	B
J	Installation of Computer	4	E,F,G
K	Sheetrocking of Walls and Ceilings	6	F,G,H
L	Training of Staff of Computer Services Dept.	8	D,J
M	Interior Painting and Interior Decorating	9	K
N	Landscaping	7	C,I

CRITICAL PATH METHOD (CPM)

Objectives: To find the minimum time required to complete the project, and to evaluate the criticality of each activity with respect to this goal.

DEFINITIONS

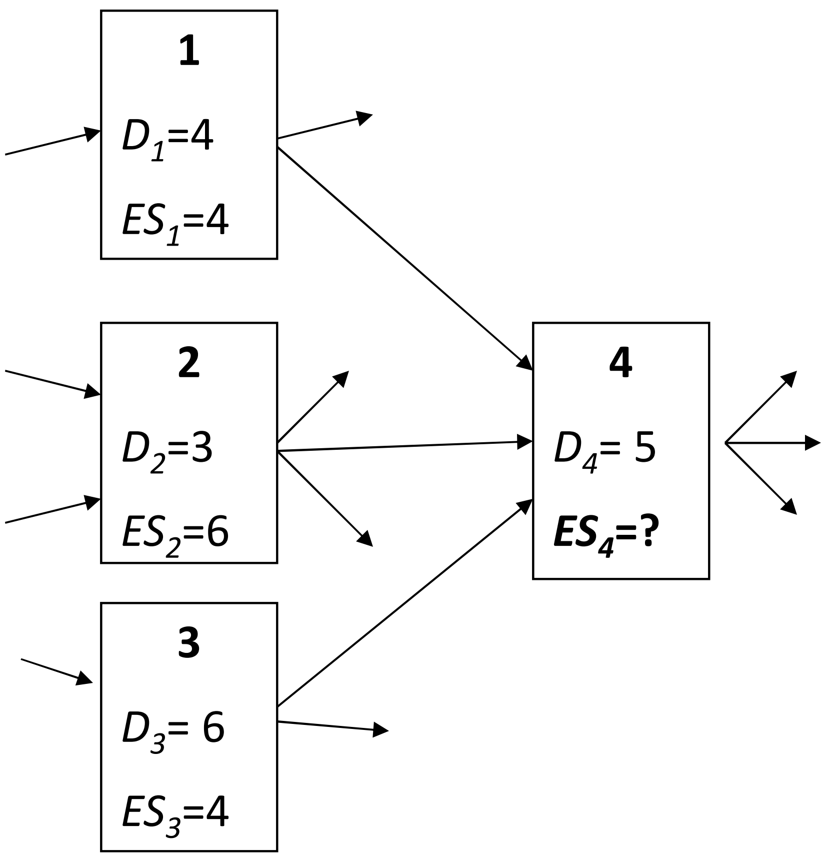
Earliest Start Time for Activity j (ES_j): Earliest time at which the activity can start:

> $ES_j = \text{Max}_{i \in P(j)} \{ES_i + D_i\}$ where $P(j)$ is the set of all prerequisite activities that immediately precede activity j

Earliest Completion Time for Activity j (EC_j): Earliest time at which the activity can end: $EC_j = ES_j + D_j$

Illustration

Consider the portion of a network shown below with activities and durations as shown in the nodes. Focus on Node *j*=4.



By definition:

$ES_4 \geq EC_1$, $ES_4 \geq EC_2$, $ES_4 \geq EC_3$

i.e.,

$ES_4 \geq (ES_1+D_1) = 4 + 4 = 8$

$ES_4 \geq (ES_2+D_2) = 6 + 3 = 9$

$ES_4 \geq (ES_3+D_3) = 4 + 6 = 10$

Thus $\mathbf{ES_4 = Max\{8,9,10\} = 10}$,
(and $EC_4 = 10+5=15$)

CPM: Definitions (*cont'd*)

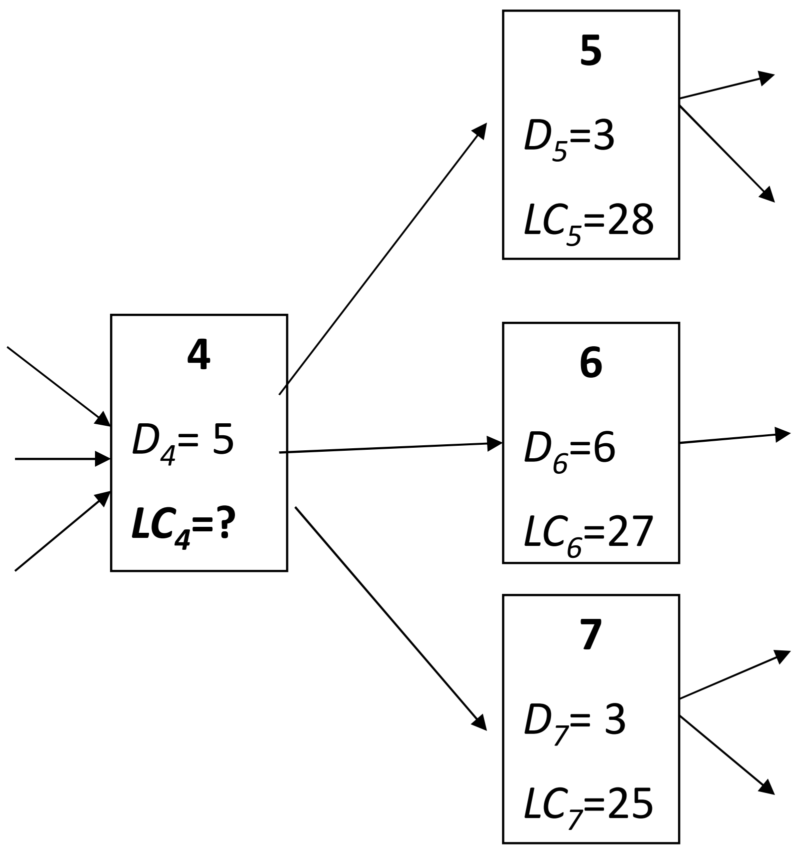
Latest Completion Time for Activity *j* (LC_j): Latest time at which activity *j* can be completed without causing the project to be delayed beyond its minimum duration:

$LC_j = \text{Min}_{i \in S(j)} \{LC_i - D_i\}$ where $S(j)$ is the set of all activities that immediately succeed activity *j*

Latest Start Time for Activity *j* (LS_j): Latest possible time at which activity *i* can start without causing the project to be delayed beyond its minimum duration: $LS_j = LC_j - D_j$

Illustration (*cont'd*)

Once again, focus on Node *j*=4.



By definition:

$LC_4 \leq LS_5$, $LC_4 \leq LS_6$, $LC_4 \leq LS_7$

i.e.,

$LC_4 \leq (LC_5 - D_5) = 28 - 3 = 25$

$LC_4 \leq (LC_6 - D_6) = 27 - 6 = 21$

$LC_4 \leq (LC_7 - D_7) = 25 - 3 = 22$

Thus **LC_4 = Min{15,21,22} = 12**,

(and LS_4 = 21-5=16)

CPM: A few more definitions

Total Float for Activity *j* (TF_j): The maximum "excess" or "slack" time available for activity *j* in the optimal project schedule:

$$TF_j = \{(LC_j - ES_j) - D_j\}$$

Note that (LC_j-ES_j) represents the "window" of time available in the optimal schedule for activity *j*.

Critical Activity: One that has a total float equal to 0, i.e., activity *j* is critical if TF_j=0. Note that $TF_j=0 \Rightarrow (LC_j-ES_j)=D_j \Rightarrow ES_j+D_j=LC_j$. Thus, a critical activity has **no excess** time available for its completion - it MUST start as soon as possible (at ES_j) so that it is completed no later than the time at which it must be completed in order for the project to not be delayed (LC_j).

Critical Path: A path of critical actvities in the project network that link the START node and the FINISH node. It is possible for a project to have more than one critical path!

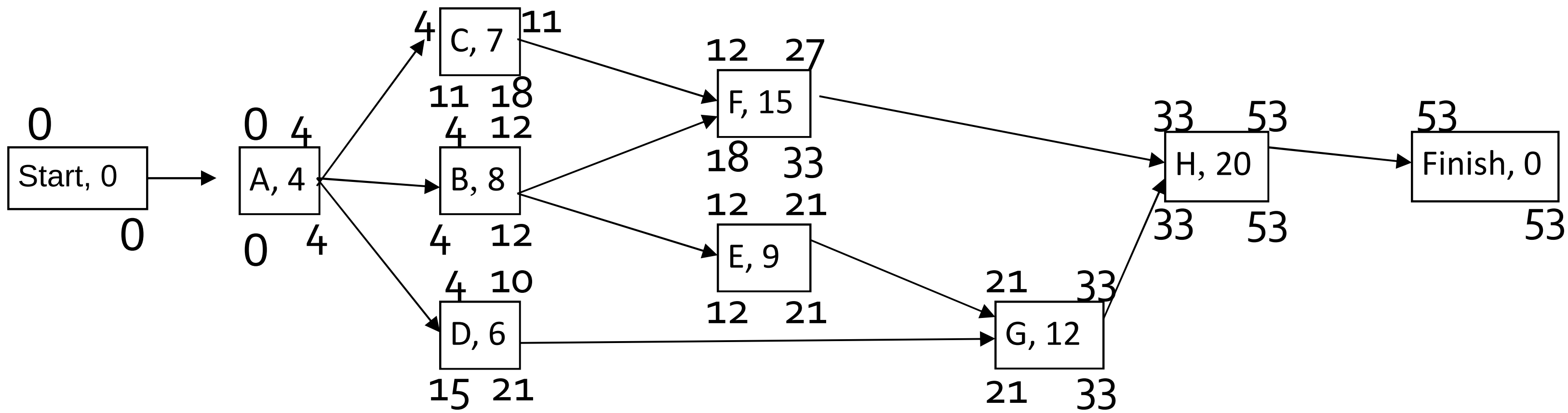
Methodology of CPM: The three phases

1. **Forward Pass:** Begin at the START node with ES_{START}=0; find ES_j=$\text{Max}_{i\in P(j)}\{ES_i+D_i\}$ for each activity *j*, going from left to right. Ensure that ES_j is computed only after finding ES_i for all $i\in P(j)$, i.e., for all activities that immediately precede activity *j*. This step ends with the computation of ES_{FINISH} (which is also equal to EC_{FINISH} …).

2. **Backward Pass:** Starting at the FINISH node with LC_{FINISH}=EC_{FINISH} find LC_j= $\text{Min}_{i\in S(j)}\{LC_i - D_i\}$ for each activity *j*, going from right to left. Ensure that LC_j is computed only after finding LC_i for all $i\in S(j)$, i.e., for all activities that immediately follow activity *j*. This step ends with the computation of LC_{START}=0 (which is also equal to LS_{START} …).

3. Compute TF_j= $\{(LC_j-ES_j) - D_j\}$ for each activity *j* in the project network to identify the critical activities and the critical path.

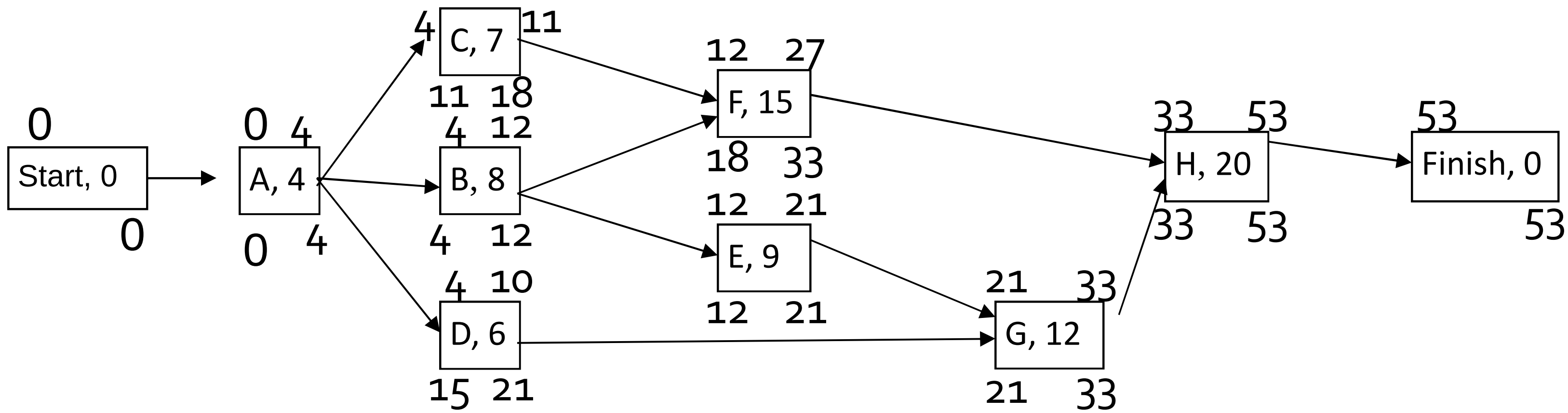
Example



The forward pass first yields the ES values, and then the backward pass yields the LC values in the table below:

Node *j*	***ES$_j$***	***LC$_j$***
Start	0	$\{LC_A-D_A\}=\{4-4\}=$ 0
A	$\{ES_{start}+D_{start}\}=0+0=$ 0	$Min\{LC_B-D_B, LC_C-D_D, LC_D-D_D\}$ = Min{12-8,18-7, 21-6}= 4
B	$\{ES_A+D_A\}=0+4=$ 4	$Min\{LC_E-D_E, LC_F-D_F\}$ = Min{21-9, 33-15}= 12
C	$\{ES_A+D_A\}=0+4=$ 4	$\{LC_F-D_F\}=33-15=$ 18
D	$\{ES_A+D_A\}=0+4=$ 4	$\{LC_G-D_G\}=33-12=$ 21
E	$\{ES_B+D_B\}=4+8=$ 12	$\{LC_G-D_G\}=33-12=$ 21
F	$Max\{ES_B+D_B, ES_C+D_C\}$ =Max(4+8,4+7) = 12	$\{LC_H-D_H\}=53-20=$ 33
G	$Max\{ES_D+D_D, ES_E+D_E\}$ =Max(4+6,12+9) = 21	$\{LC_H-D_H\}=53-20=$ 33
H	$Max\{ES_F+D_F, ES_G+D_G\}$ =Max(12+15,21+12)= 33	$\{LC_{Finish}-D_{finish}\}=53-0=$ 53
Finish	$\{ES_H+D_H\}=33+20=$ 53	$= EC_{finish}= ES_{finish}+D_{finish}=$ 53

Example



The forward pass first yields the ES values, and then the backward pass yields the LC values in the table below:

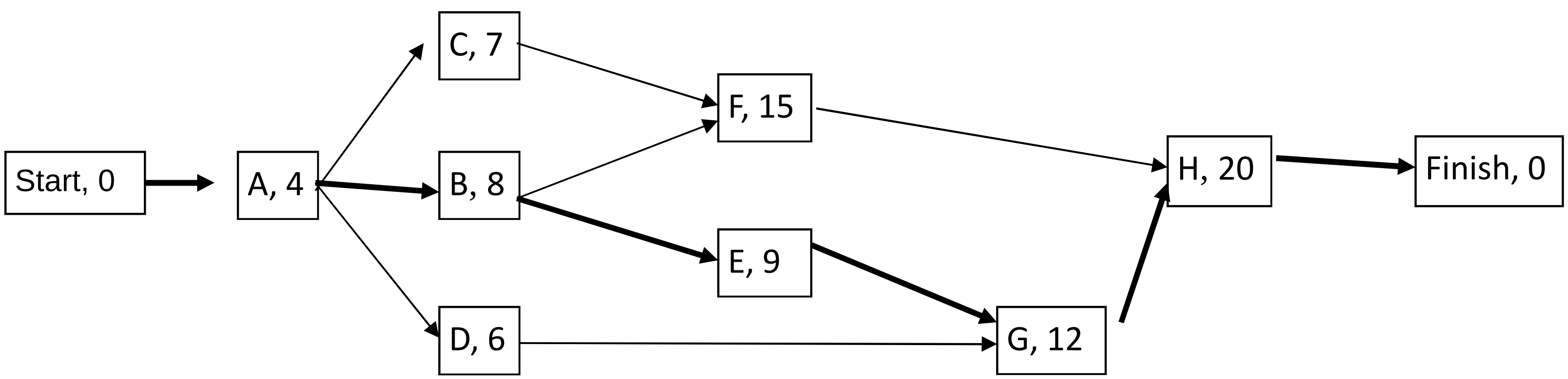
Node *j*	***ES$_j$***	***LC$_j$***
Start	0	$\{LC_A-D_A\}$= {4-4}= 0
A	$\{ES_{start}+D_{start}\}$= 0+0= 0	Min$\{LC_B-D_B, LC_C-D_D, LC_D-D_D\}$ = Min{12-8,18-7, 21-6}= 4
B	$\{ES_A+D_A\}$=0+4=4	Min$\{LC_E-D_E, LC_F-D_F\}$ = Min{21-9, 33-15}=12
C	$\{ES_A+D_A\}$=0+4=4	$\{LC_F-D_F\}$=33-15 =18
D	$\{ES_A+D_A\}$=0+4=4	$\{LC_G-D_G\}$=33-12 = 21
E	$\{ES_B+D_B\}$=4+8=12	$\{LC_G-D_G\}$=33-12 = 21
F	Max$\{ES_B+D_B, ES_C+D_C\}$ =Max(4+8,4+7) =12	$\{LC_H-D_H\}$=53-20 =33
G	Max$\{ES_D+D_D, ES_E+D_E\}$ =Max(4+6,12+9) =21	$\{LC_H-D_H\}$=53-20 =33
H	Max$\{ES_F+D_F, ES_G+D_G\}$ =Max(12+15,21+12)=33	$\{LC_{Finish}-D_{finish}\}$= 53-0 = 53
Finish	$\{ES_H+D_H\}$=33+20=53	= EC_{finish}= $ES_{finish}+D_{finish}$ = 53

Finding the Critical Path

We now go to Step 3 for TF values for each **activity**…

Activity(j)	D_j	ES_j	LC_j	$TF_j = (LC_j - ES_j) - D_j$
A	4	0	4	**0**
B	8	4	12	**0**
C	7	4	18	7
D	6	4	21	11
E	9	12	21	**0**
F	15	12	33	6
G	12	21	33	**0**
H	20	33	53	**0**

The critical activities are A, B, E, G and H and the critical path is shown below:



CPM via *STORM*

ACTIVITIES IN THE ORDER AS ENTERED

Activity Name		Symb	Activity Time	Earliest Start/Fin	Latest Start/Fin	Slack
ACT	1	A	4.0000	0.0000 4.0000	0.0000 4.0000	0.0000 c
ACT	2	B	8.0000	4.0000 12.0000	4.0000 12.0000	0.0000 c
ACT	3	C	7.0000	4.0000 11.0000	11.0000 18.0000	7.0000
ACT	4	D	6.0000	4.0000 10.0000	15.0000 21.0000	11.0000
ACT	5	E	9.0000	12.0000 21.0000	12.0000 21.0000	0.0000 c
ACT	6	F	15.0000	12.0000 27.0000	18.0000 33.0000	6.0000
ACT	7	G	12.0000	21.0000 33.0000	21.0000 33.0000	0.0000 c
ACT	8	H	20.0000	33.0000 53.0000	33.0000 53.0000	0.0000 c

The computations were based on 8 activities

Earliest project completion time = 53.0000

CPM EXAMPLE

BAR CHART: NONCRITICAL ACTIVITIES SORTED BY SLACK

```
    0                                                          53
----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
A   ccccc                                                       |
B   |    ccccccccc                                              |
E   |             cccccccccc                                    |
G   |                       ccccccccccccc                       |
H   |                                    ccccccccccccccccccccccc|
F   |             xxxxxxxxxxxxxxxxx......                       |
C   |    xxxxxxx........                                        |
D   |    xxxxxx.............                                    |
----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
         4       12       21          33
```

CPM EXAMPLE

BAR CHART: NONCRITICAL ACTIVITIES SORTED BY EARLIEST START

```
    0                                                          53
----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
A   ccccc                                                       |
B   |    ccccccccc                                              |
E   |             cccccccccc                                    |
G   |                       ccccccccccccc                       |
H   |                                    ccccccccccccccccccccccc|
C   |    xxxxxxx........                                        |
D   |    xxxxxx.............                                    |
F   |             xxxxxxxxxxxxxxxxx......                       |
----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
```