INTEGER PROGRAMMING

- Pure Integer Programs: all decision variables restricted to integer values
  - $\Rightarrow$ 0-1 programs
  - $\Rightarrow$ general IP problems

- Mixed Integer Programs: some variables restricted to integer values; others may be continuous
  - $\Rightarrow$ 0-1 programs
  - $\Rightarrow$ general IP problems

Very versatile because many “real-world” applications involve variables that are naturally integer valued.

Also, many combinatorial optimization problems (where an optimum combination out of a possible set of combinations must be determined) can be cast as IP’s.

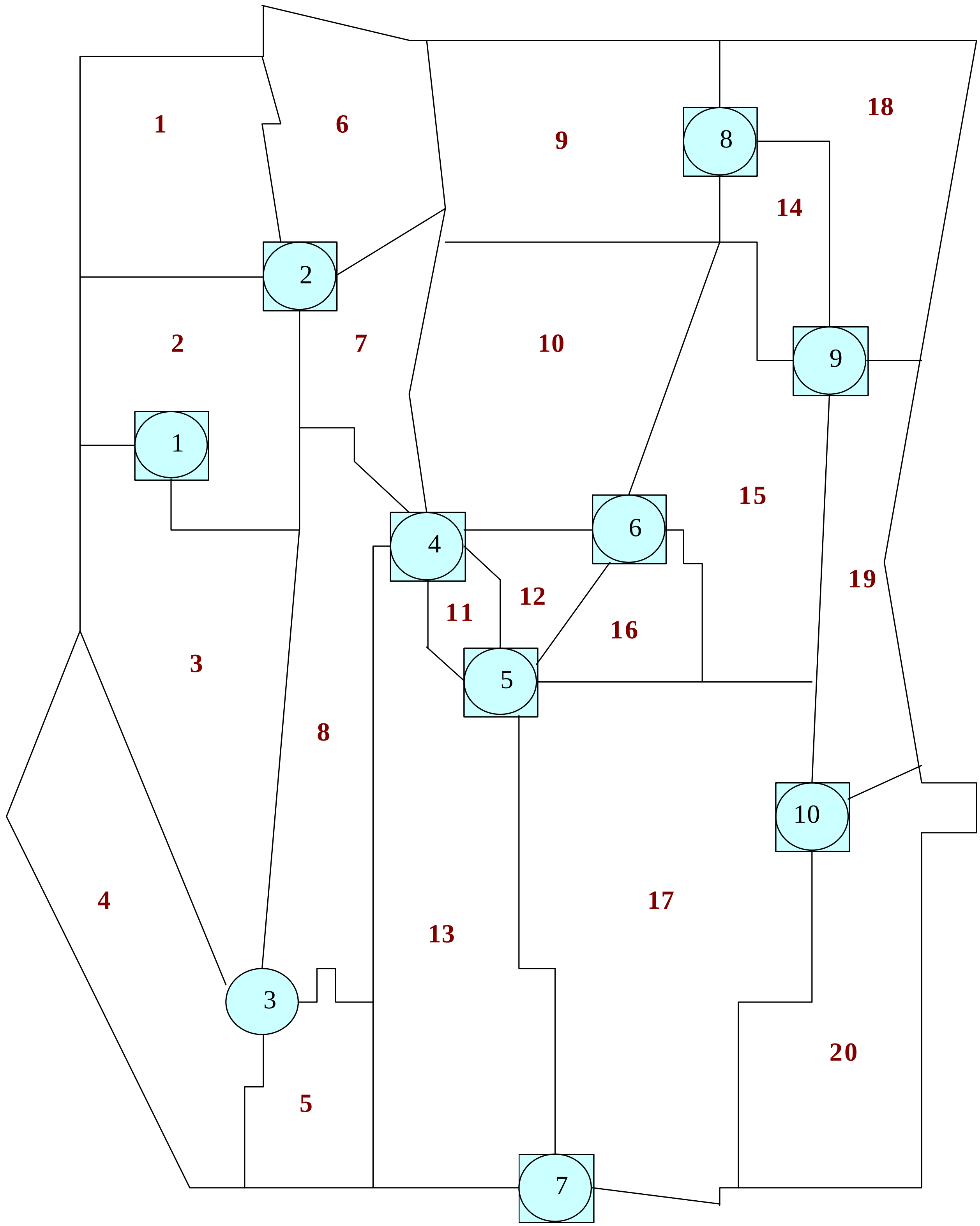
Set Covering/Partitioning/Packing

- **Γ={A,B,C,D,E,F,G,H,I,J,K}**

B1=(A,B,C,D)	**B6=(A,C,J,K)**
B2=(B,C,F,G)	**B7=(D,G)**
B3=(A,H)	**B8=(A,H,J,K)**
B4=(B,D,E,I,J)	**B9=(E,F,I)**
B5=(D,E,F,K)	**B10=(G,H,J,K)**

- Set Representation: (A,G,I,K); (A,E,G);(A,B,D,E,K)

- Set Covering: (B2,B4,B8); (B1,B2,B8,B9); (B1,B3,B5,B7,B8,B9)
- Set Partitioning: (B1,B9,B10)
- Set Packing: (B6,B7,B9); (B1,B9,B10)
- **F_A=(B1,B3,B6,B8); F_B=(B1,B2,B4); F_C=(B1,B2,B6); …; F_H=(B3,B8,B10); …; F_K=(B5,B6,B8,B10);**



Service Districts and Candidate Locations for EMS

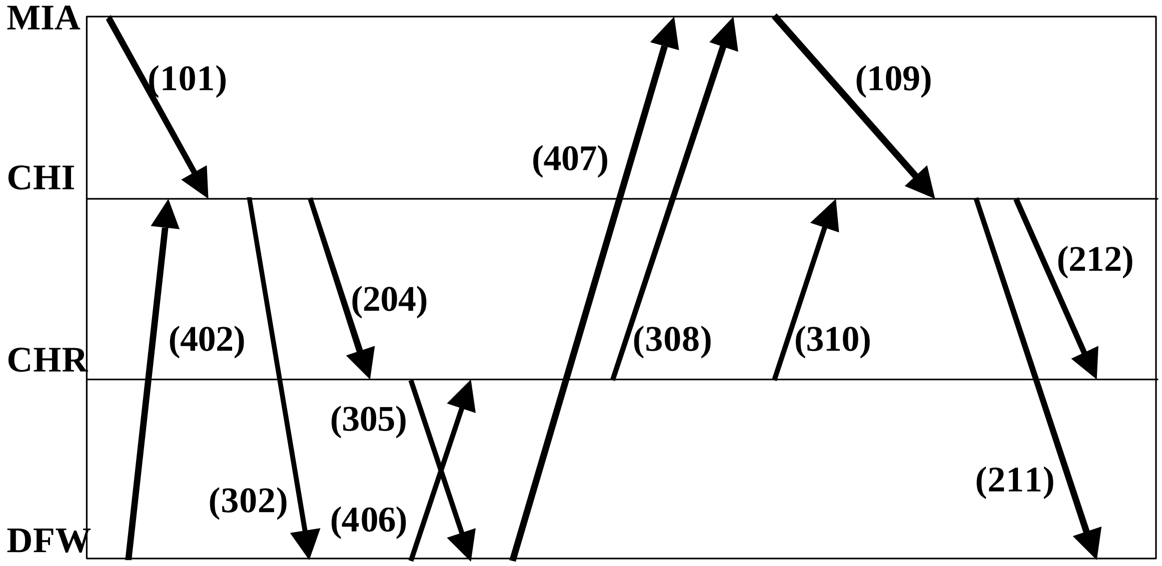


FIGURE: Flight Schedule for AA Example

TABLE: Possible Pairings for AA Example

j	Flights Sequence	Cost	j	Flights Sequence	Cost
1	101-203-406-308	2900	9	305-407-109-212	2600
2	101-203-407	2700	10	308-109-212	2050
3	101-204-305-407	2600	11	402-204-305	2400
4	101-204-308	3000	12	402-204-310-211	3600
5	203-406-310	2600	13	406-308-109-211	2550
6	203-407-109	3150	14	406-310-211	2650
7	204-305-407-109	2550	15	407-109-211	2350
8	204-308-109	2500			

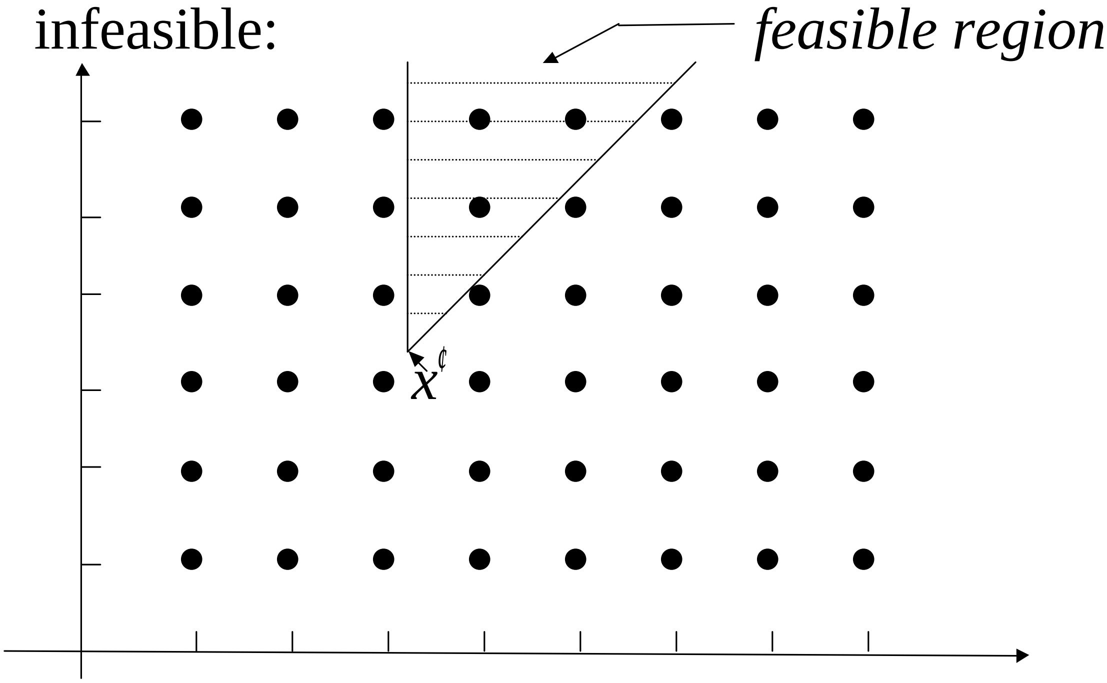
ROUNDING OFF

Why not solve the Integer LP as a regular LP and "round off intelligently?"

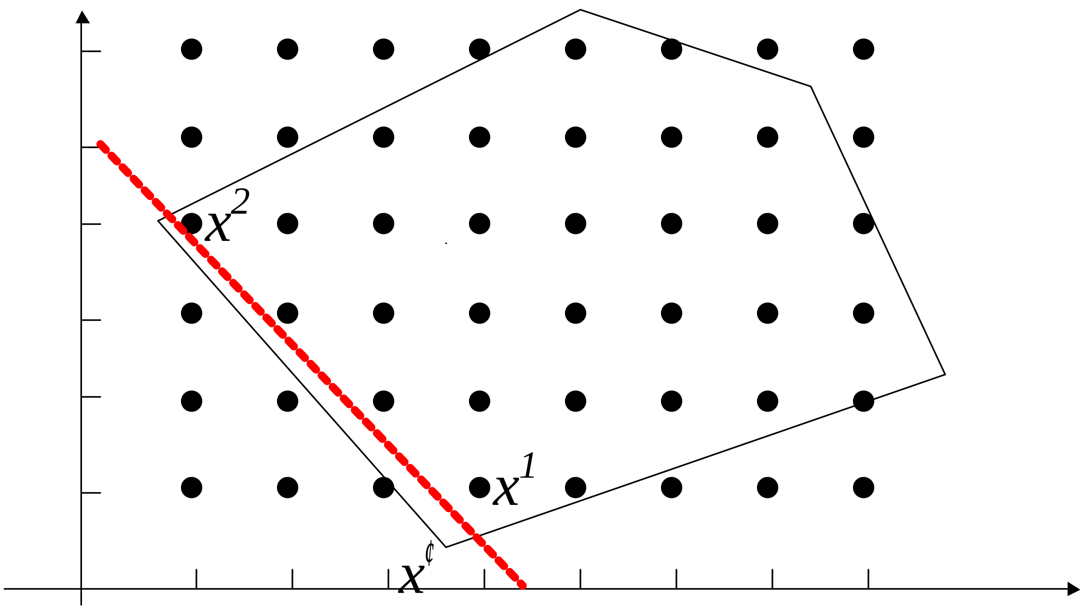
- Not an illogical approach when all variables must be general integers. May turn out to be a practical approach, especially if all the integer variables are expected to be "large" at the optimum and feasibility is easy to maintain when rounding off

However...

- Usually doesn't make sense with 0-1 programs
- Many times all "rounded-off" solutions may be infeasible:



- In other cases, the rounded solution may be quite far away from the integer optimum:

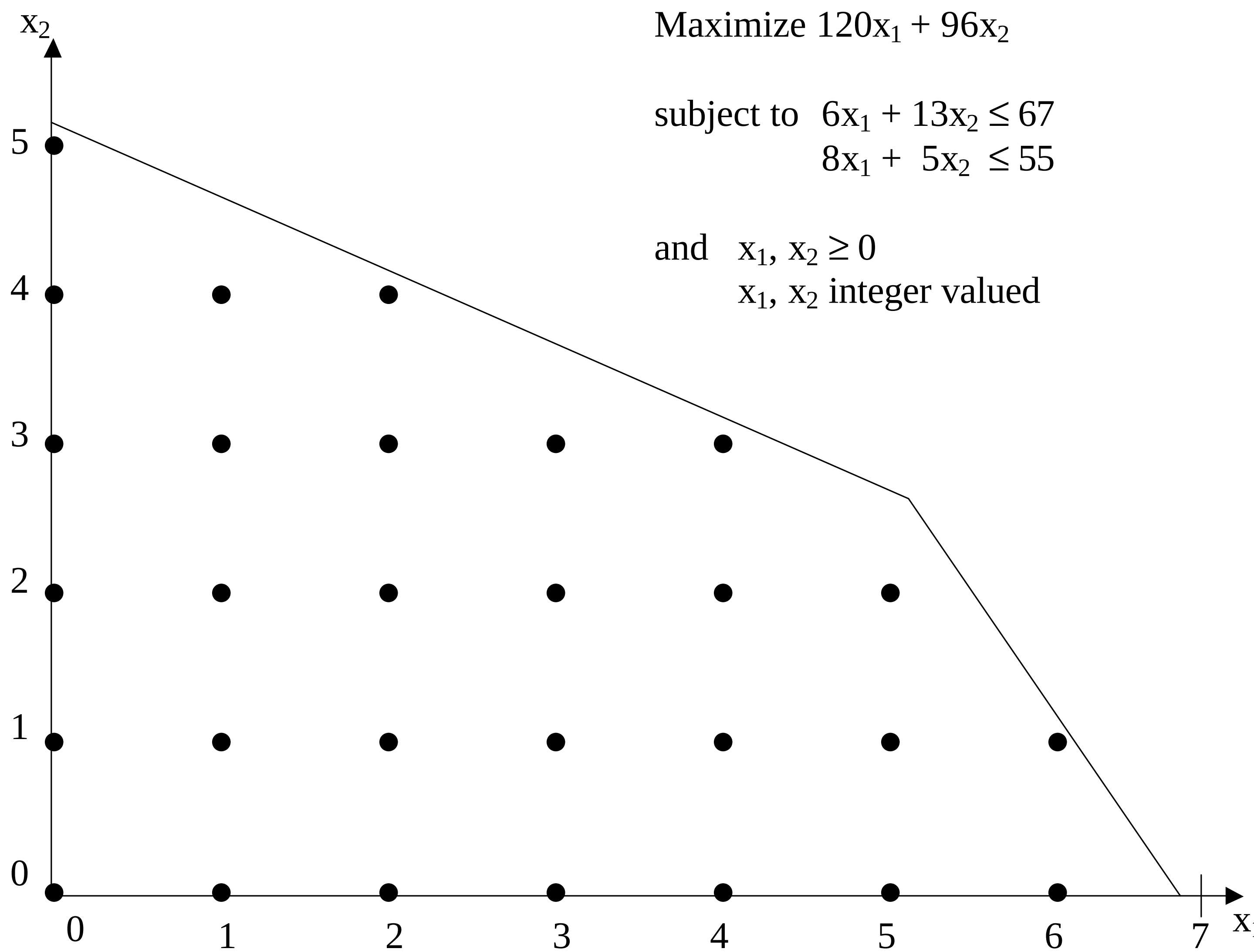


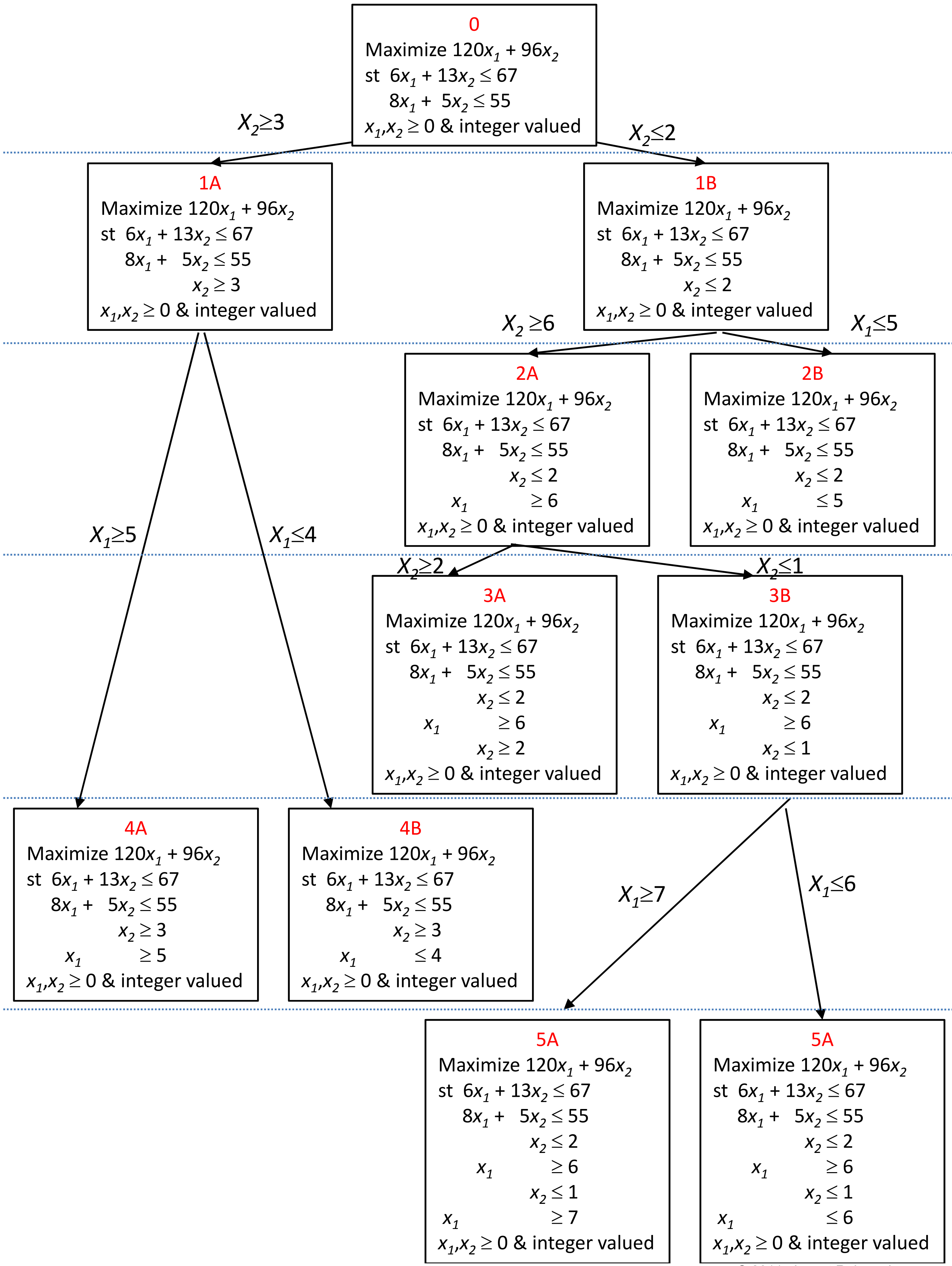
LP opt. = $x^¢$, "rounded-off" to x^1, IP opt. = x^2 !!

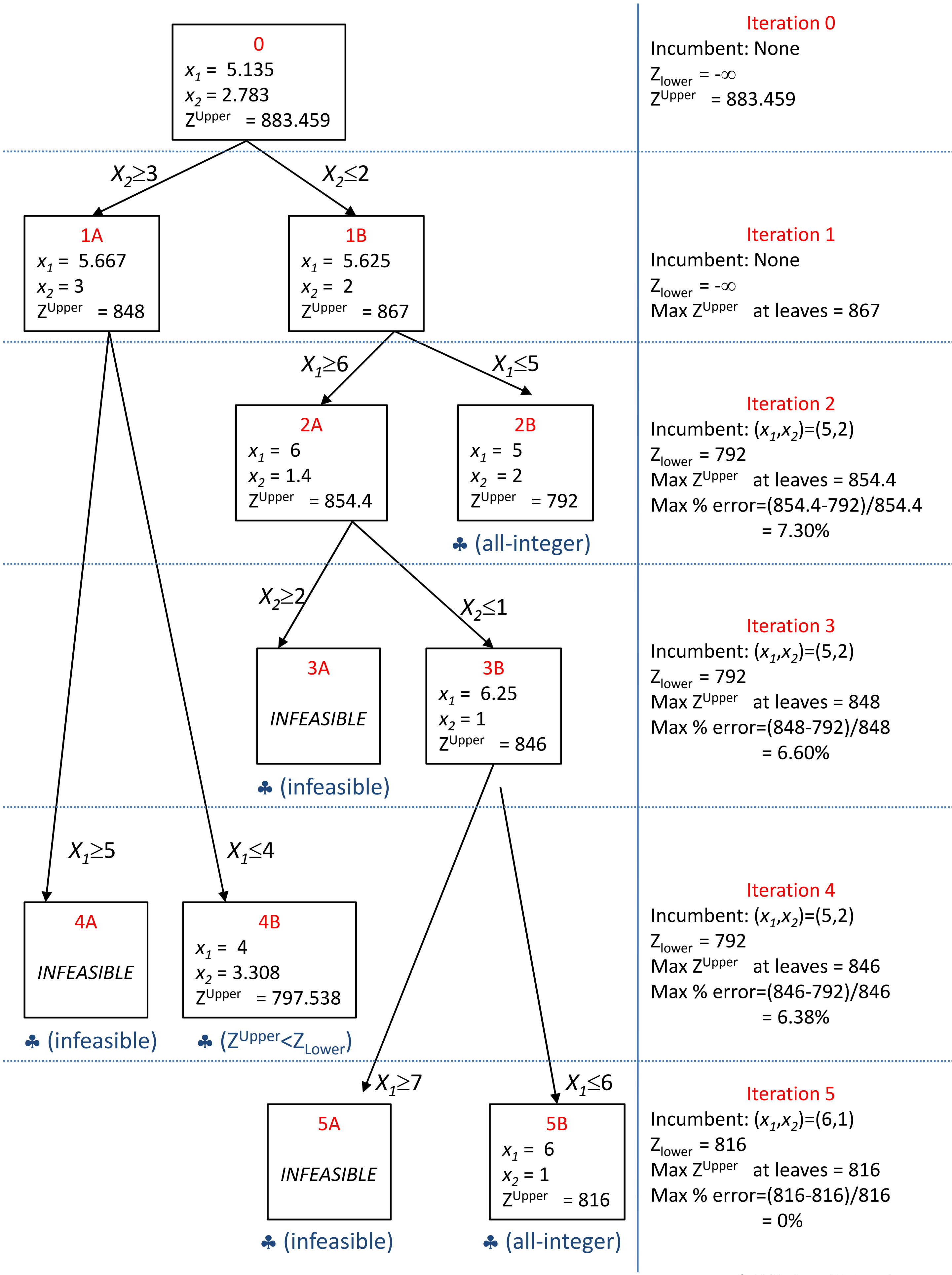
Maximize $120x_1 + 96x_2$

subject to $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$

and $x_1, x_2 \geq 0$
x_1, x_2 integer valued




0
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_1, x_2 \geq 0$ & integer valued
$X_2 \geq 3$
$X_2 \leq 2$
1A
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \geq 3$
$x_1, x_2 \geq 0$ & integer valued
1B
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \leq 2$
$x_1, x_2 \geq 0$ & integer valued
$X_2 \geq 6$
$X_1 \leq 5$
2A
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \leq 2$
$x_1 \geq 6$
$x_1, x_2 \geq 0$ & integer valued
2B
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \leq 2$
$x_1 \leq 5$
$x_1, x_2 \geq 0$ & integer valued
$X_1 \geq 5$
$X_1 \leq 4$
$X_2 \geq 2$
$X_2 \leq 1$
3A
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \leq 2$
$x_1 \geq 6$
$x_2 \geq 2$
$x_1, x_2 \geq 0$ & integer valued
3B
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \leq 2$
$x_1 \geq 6$
$x_2 \leq 1$
$x_1, x_2 \geq 0$ & integer valued
4A
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \geq 3$
$x_1 \geq 5$
$x_1, x_2 \geq 0$ & integer valued
4B
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \geq 3$
$x_1 \leq 4$
$x_1, x_2 \geq 0$ & integer valued
$X_1 \geq 7$
$X_1 \leq 6$
5A
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \leq 2$
$x_1 \geq 6$
$x_2 \leq 1$
$x_1 \geq 7$
$x_1, x_2 \geq 0$ & integer valued
5A
Maximize $120x_1 + 96x_2$
st $6x_1 + 13x_2 \leq 67$
$8x_1 + 5x_2 \leq 55$
$x_2 \leq 2$
$x_1 \geq 6$
$x_2 \leq 1$
$x_1 \leq 6$
$x_1, x_2 \geq 0$ & integer valued

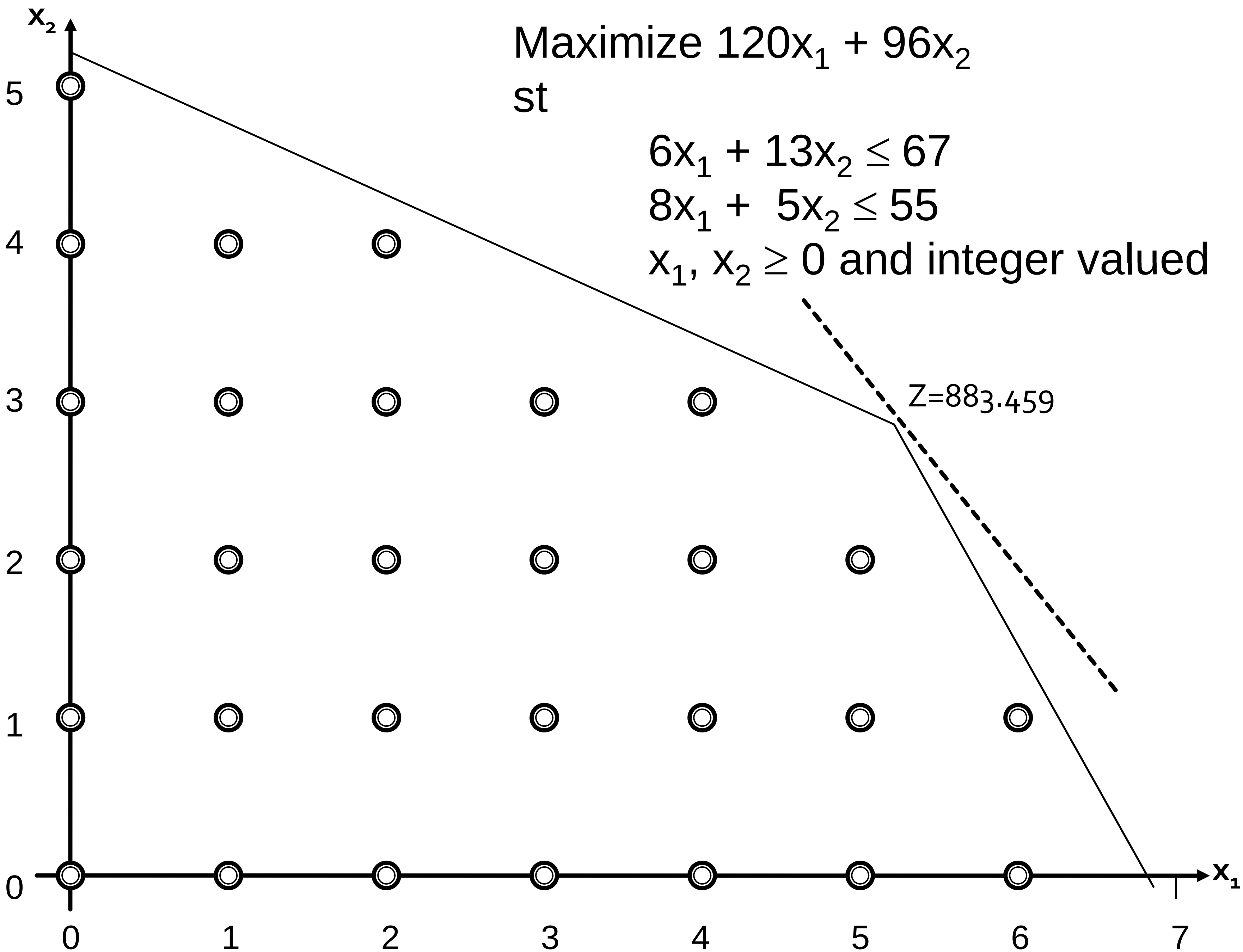


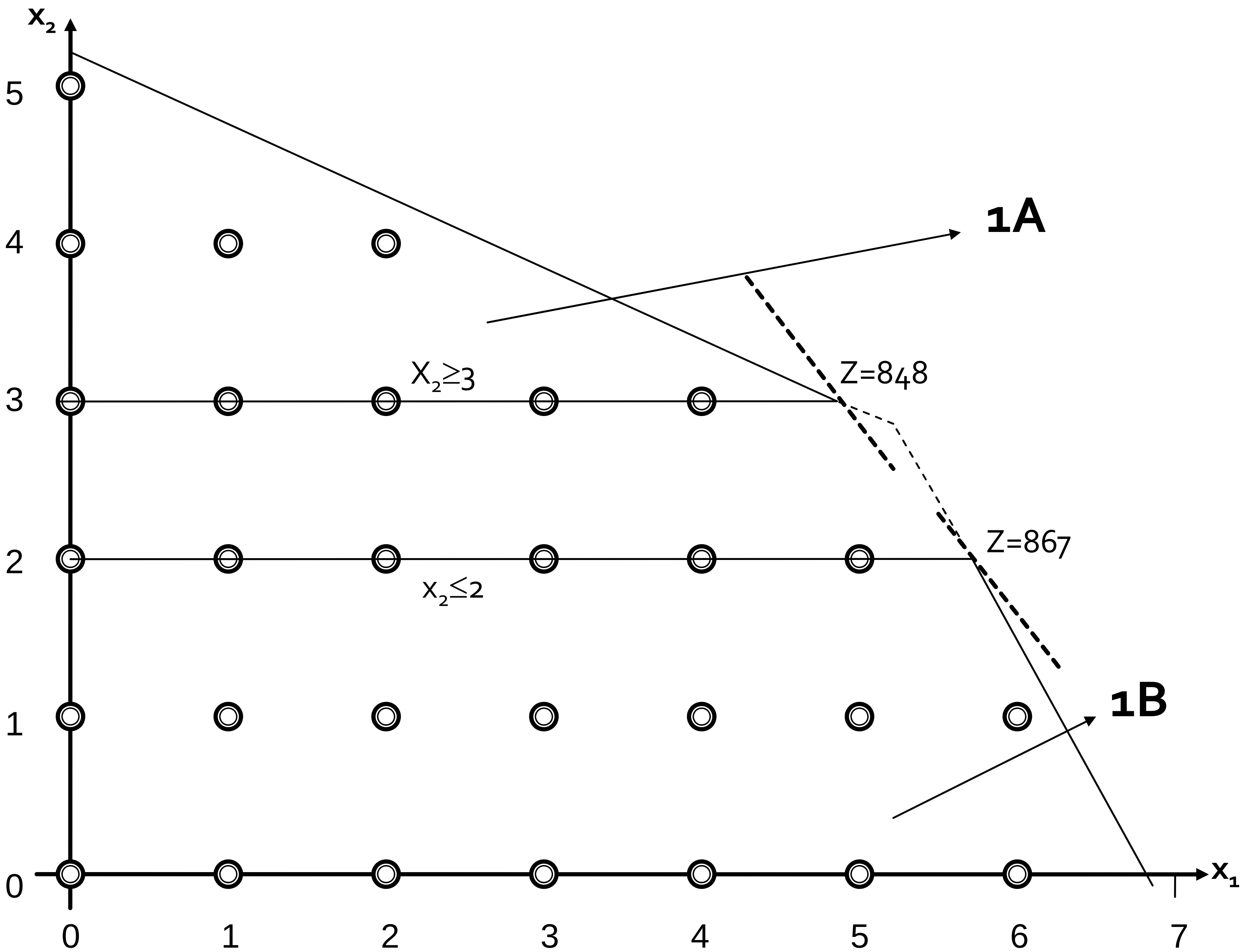
0
x1 = 5.135
x2 = 2.783
ZUpper = 883.459
X2≥3
X2≤2
1A
x1 = 5.667
x2 = 3
ZUpper = 848
1B
x1 = 5.625
x2 = 2
ZUpper = 867
X1≥6
X1≤5
2A
x1 = 6
x2 = 1.4
ZUpper = 854.4
2B
x1 = 5
x2 = 2
ZUpper = 792
♣ (all-integer)
X2≥2
X2≤1
3A
INFEASIBLE
♣ (infeasible)
3B
x1 = 6.25
x2 = 1
ZUpper = 846
X1≥5
X1≤4
4A
INFEASIBLE
♣ (infeasible)
4B
x1 = 4
x2 = 3.308
ZUpper = 797.538
♣ (ZUpper<ZLower)
X1≥7
X1≤6
5A
INFEASIBLE
♣ (infeasible)
5B
x1 = 6
x2 = 1
ZUpper = 816
♣ (all-integer)
Iteration 0
Incumbent: None
Zlower = -∞
ZUpper = 883.459
Iteration 1
Incumbent: None
Zlower = -∞
Max ZUpper at leaves = 867
Iteration 2
Incumbent: (x1,x2)=(5,2)
Zlower = 792
Max ZUpper at leaves = 854.4
Max % error=(854.4-792)/854.4
= 7.30%
Iteration 3
Incumbent: (x1,x2)=(5,2)
Zlower = 792
Max ZUpper at leaves = 848
Max % error=(848-792)/848
= 6.60%
Iteration 4
Incumbent: (x1,x2)=(5,2)
Zlower = 792
Max ZUpper at leaves = 846
Max % error=(846-792)/846
= 6.38%
Iteration 5
Incumbent: (x1,x2)=(6,1)
Zlower = 816
Max ZUpper at leaves = 816
Max % error=(816-816)/816
= 0%

Maximize $120x_1 + 96x_2$

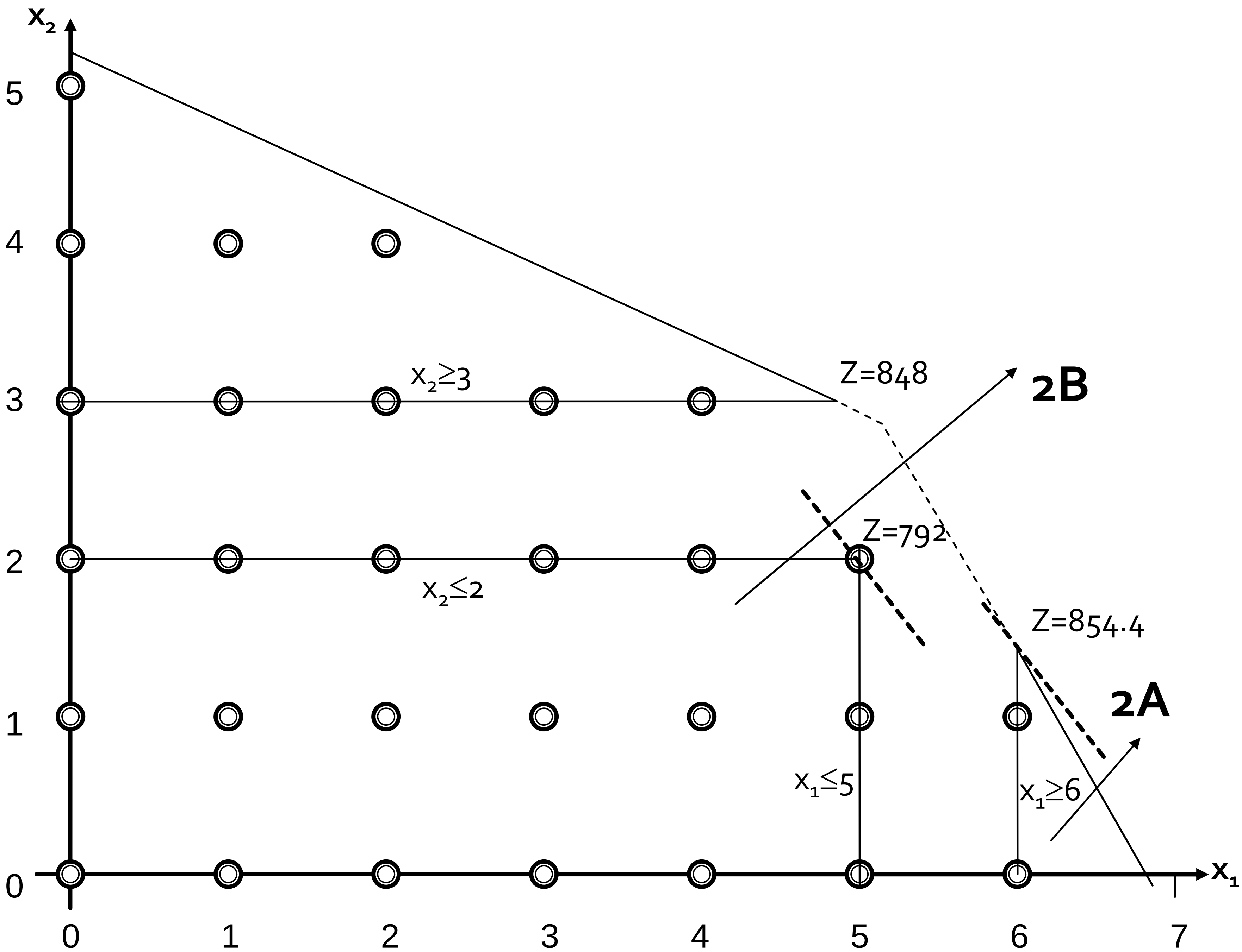
st

$$6x_1 + 13x_2 \leq 67$$
$$8x_1 + 5x_2 \leq 55$$
$$x_1, x_2 \geq 0 \text{ and integer valued}$$

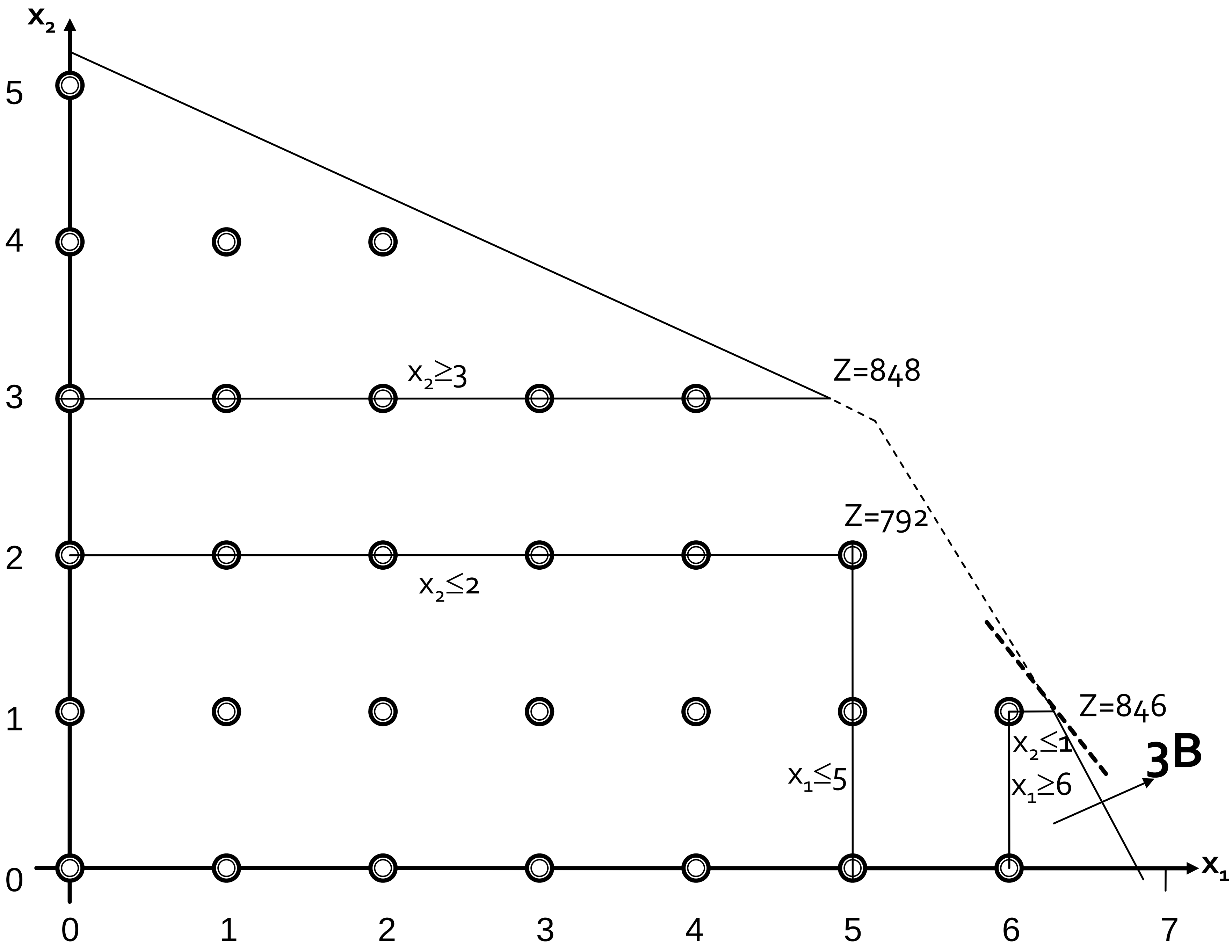




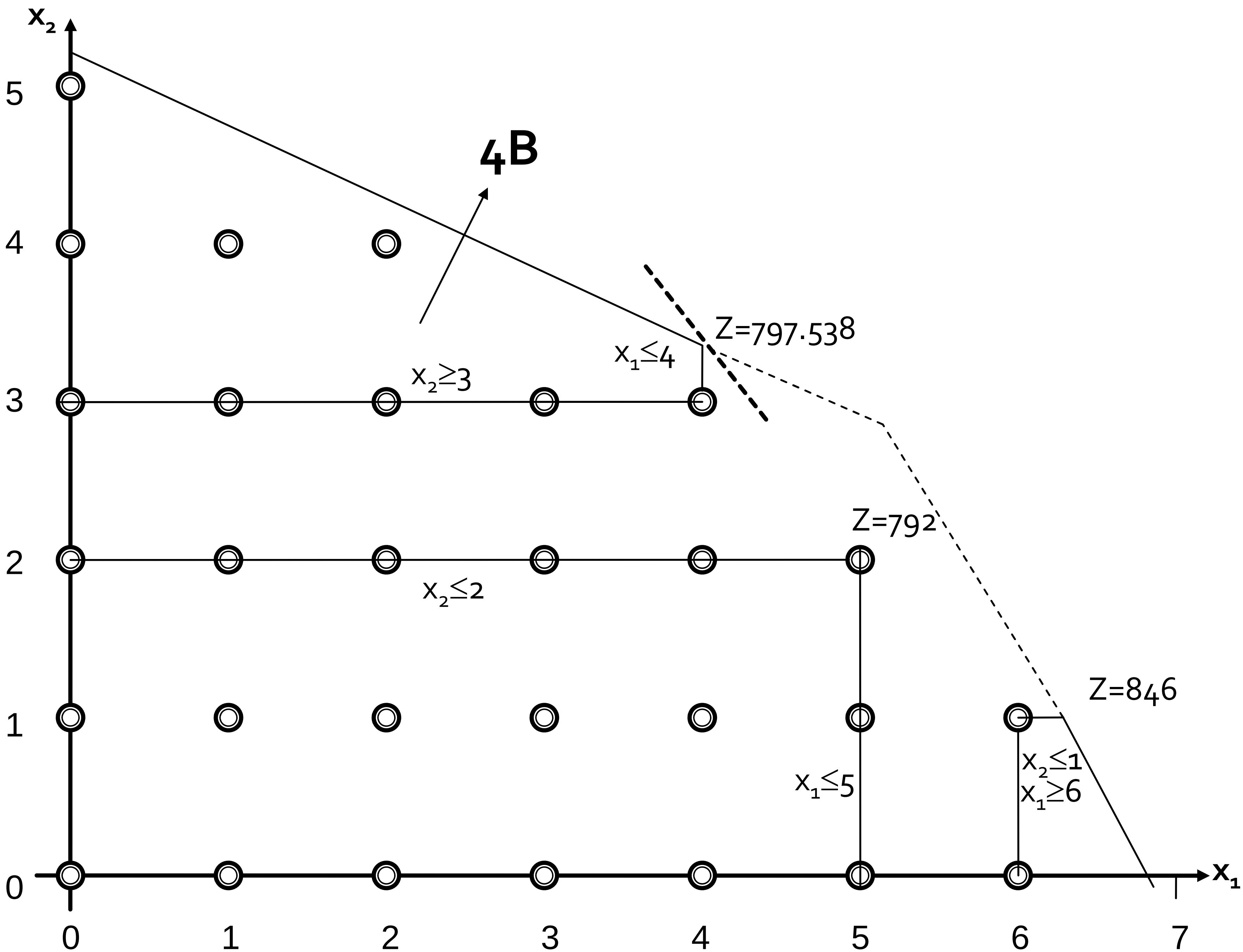
x2
x1
0
1
2
3
4
5
6
7
1A
1B
X2≥3
x2≤2
Z=848
Z=867



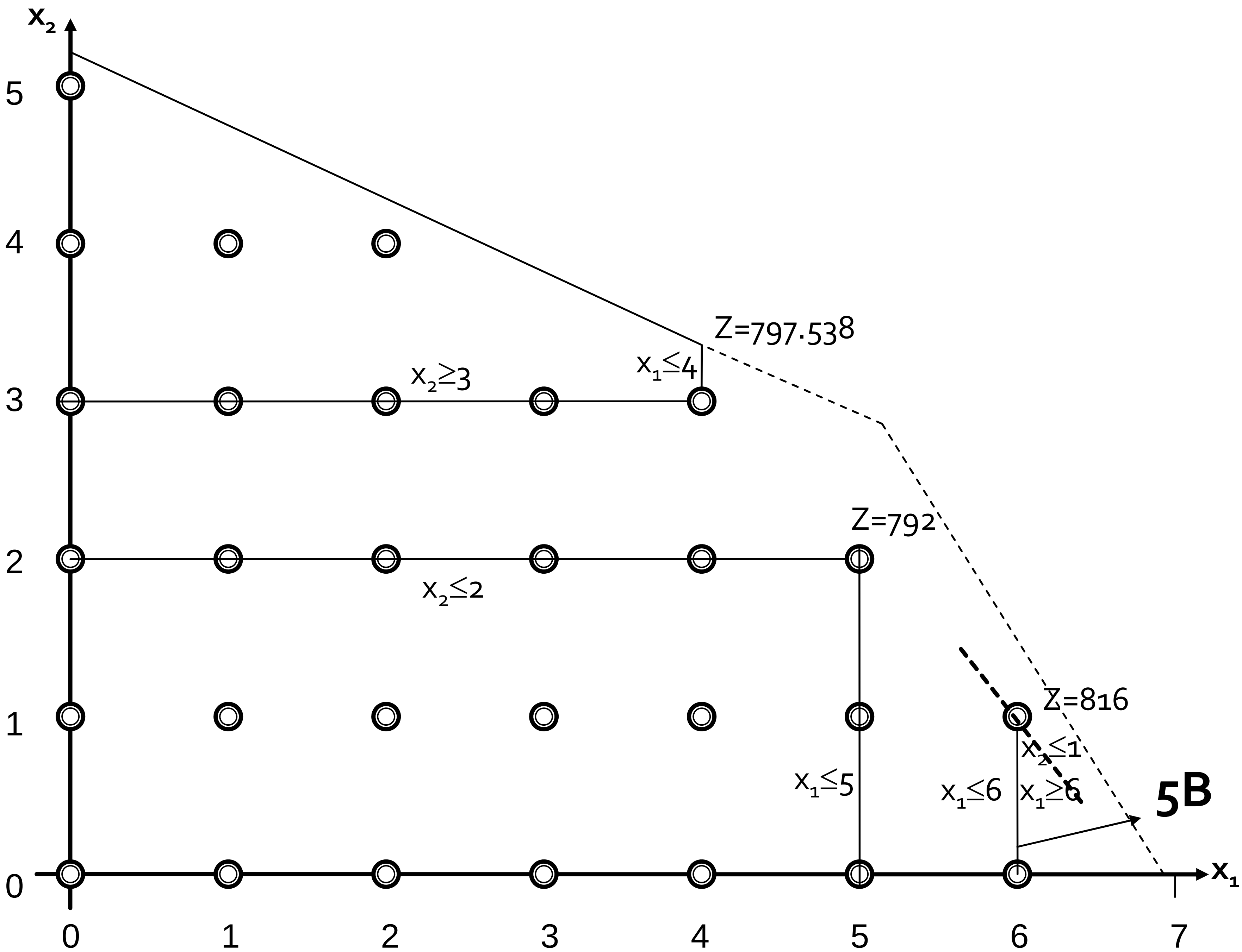
x2
x1
Z=848
2B
Z=792
Z=854.4
2A
x2≥3
x2≤2
x1≤5
x1≥6
0
1
2
3
4
5
6
7



x2
5
4
3
2
1
0
0
1
2
3
4
5
6
7
x1
x2≥3
Z=848
Z=792
x2≤2
Z=846
x2≤1
x1≥6
x1≤5
3B



4B
Z=797.538
Z=792
Z=846
$x_2 \geq 3$
$x_1 \leq 4$
$x_2 \leq 2$
$x_1 \leq 5$
$x_2 \leq 1$
$x_1 \geq 6$
x_1
x_2
0
1
2
3
4
5
6
7


x2
x1
5
4
3
2
1
0
0
1
2
3
4
5
6
7
Z=797.538
x1≤4
x2≥3
Z=792
x2≤2
Z=816
x2≤1
x1≤5
x1≤6
x1≥6
5B