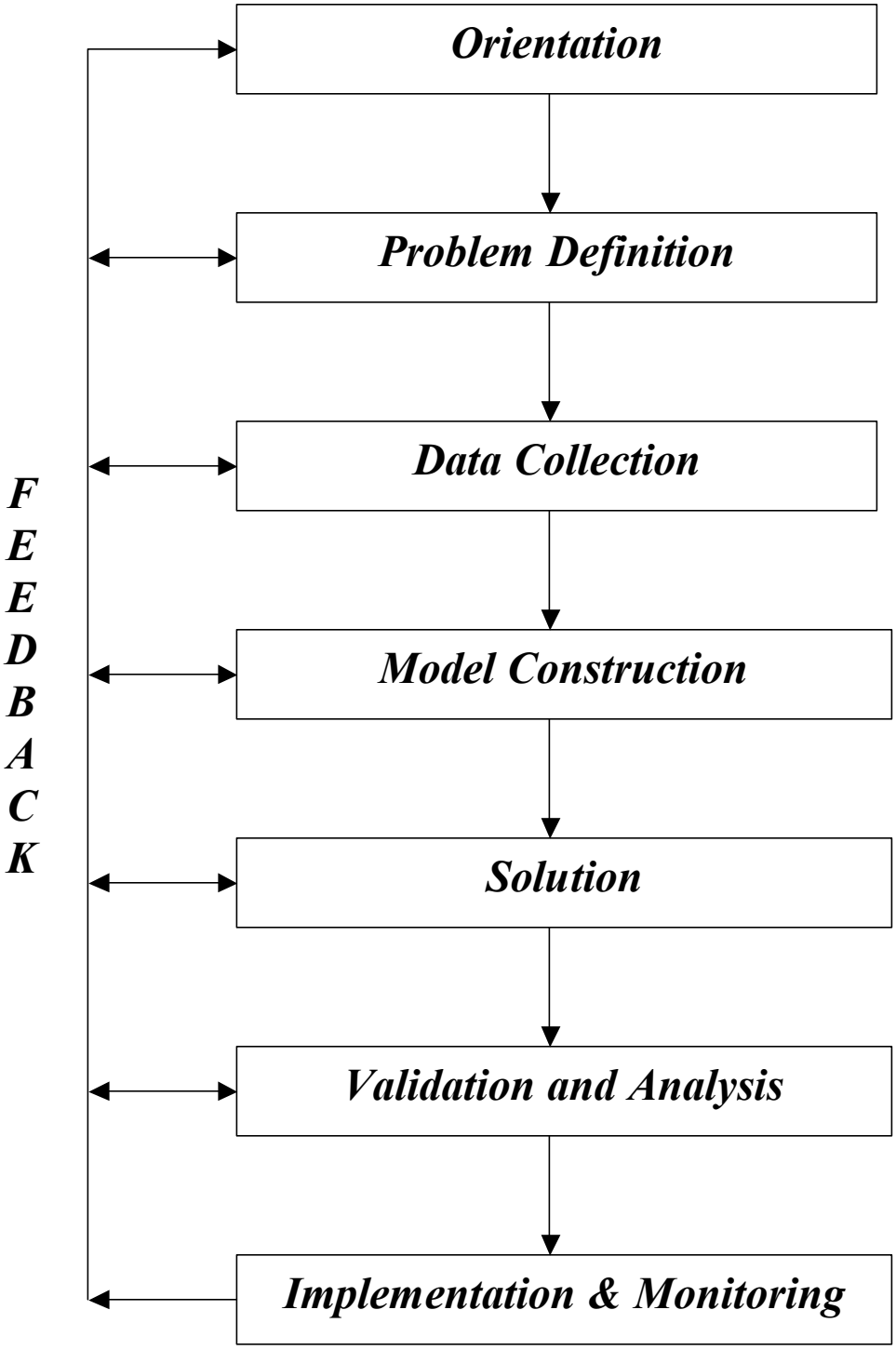
# THE "OPERATIONS RESEARCH METHOD"



# An OR Problem - A Simple Example

SCENARIO

PAR, Inc. is a small manufacturer of golf equipment and supplies. They make two models (*standard* and *deluxe*) in their line of golf bags, which are extremely popular and have a very high demand. Each bag requires three basic sets of operations, the times for which are as follows:

| | *Cutting & Dyeing* | *Sewing* | *Finishing* |
|---|---|---|---|
| *Standard* | 0.7 hrs. | 1.0 hrs. | 0.1 hrs. |
| *Deluxe* | 1.0 hrs. | 40 mins. | 15 mins. |

Each *standard* bag yields a profit of $10, while each *deluxe* bag yields a profit of $9.

Based upon departmental workload projections, it is estimated that the available production hours in each department during the next quarter are as follows

| *Department* | *Hours* |
|---|---|
| Cutting & Dyeing | 630 |
| Sewing | 708 |
| Finishing | 135 |

QUESTION

What should PAR do???

*Simple Examples of Operations Research Model Formulations*

## 1. LINEAR PROGRAMMING

A farmer owns 200 pigs that consume 90 lbs of special feed daily. The feed is prepared as a mixture of corn and soybean meal with the following compositions:

| | *Pounds per pound of feedstuff* | | | |
|---|---|---|---|---|
| *Feedstuff* | *Calcium* | *Protein* | *Fiber* | *Cost ($/lb)* |
| Corn | 0.001 | 0.09 | 0.02 | 0.20 |
| Soybean Meal | 0.002 | 0.60 | 0.06 | 0.60 |

The dietary requirements of the pigs are (a) at most 1% calcium, (b) at least 30% protein, and (c) at most 5% fiber. Determine the farmer's daily minimum cost feed mix.

## 2. NONLINEAR PROGRAMMING

A cylindrical storage tank (closed at both ends) is to be designed to hold at least 3000 $ft^3$ of oil. The tank is made of sheet metal which costs $1.00 per sq. ft. It must have a radius that is no more than 10 feet and a height that is no more than 20 feet. Further, due to safety considerations its height cannot be any more than 1.5 times its diameter. Determine the optimum design.

## 3. INTEGER PROGRAMMING

Suppose you have $150 with which you are considering buying several units of four different products. Given your needs, you do not plan to buy more than 5, 3, 3 and 2 units of items 1, 2, 3, and 4 respectively. The price per unit and the utility per unit for each of the four items is as follows:

| | *Stock* | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| *Utlity* ($) | 5 | 15 | 12 | 18 |
| *Price per unit* ($) | 20 | 40 | 30 | 50 |

What is the portfolio that maximizes utility? (This problem is also referred to as the ***knapsack problem*** in the OR literature)

## 4. 0-1 INTEGER PROGRAMMING

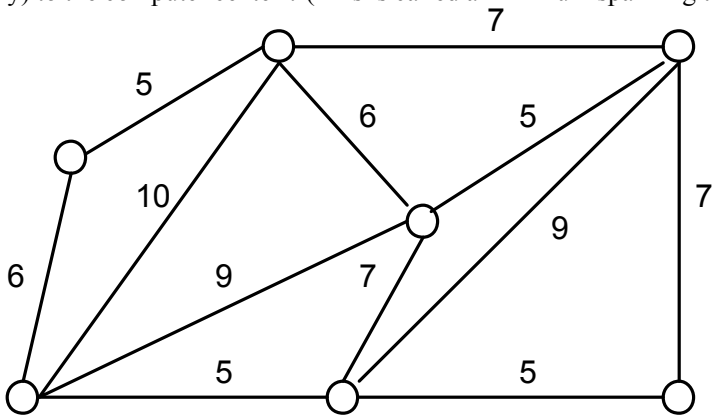
Five projects are being considered for execution over the next 3 years. The expected returns for each project and the yearly expenses (in thousands of dollars) are tabulated below:

| | *Expenses* | | | |
|---|---|---|---|---|
| *Project* | *Year 1* | *Year 2* | *Year 3* | *Returns* |
| 1 | 5 | 1 | 8 | 20 |
| 2 | 4 | 7 | 10 | 40 |
| 3 | 3 | 9 | 2 | 20 |
| 4 | 7 | 4 | 1 | 15 |
| 5 | 8 | 6 | 10 | 30 |
| *Available funds* | *25* | *25* | *25* | |

Which projects should be chosen for execution over the 3 year planning period? (This is a *capital budgeting* problem)

## 5. A NETWORK MODEL

A fiber-optic network is to be laid to connect the computer center with various locations on campus. Node A represents the computer center and each of the other nodes represents a campus location. The numbers along an arc represent the cost of a link between the two locations represented by the nodes at its ends - a missing arc between two nodes implies that it is physically impossible or prohibitively expensive to link the two locations. What is the best way of laying out the network so that every location is linked (directly or indirectly) to the computer center ? (This is called a minimum spanning tree problem)




## 6. GOAL PROGRAMMING

A company is upgrading three items in its product line and is interested in the optimal mix of the three to produce. The CEO has several goals: (1) maximize long run profits; a minimum goal for the NPV of profits should be $125 million, (2) stability in the workforce; ideally it should be maintained at the current level of 4,000, and (3) minimize initial capital investment, a maximum goal is $55 million. The CEO knows that he probably won't meet all three goals, so he has set priorities by penalizing deviations from the goals via "weights" as follows: 5 for each $1 million under the profit goal, 2 for every 100 employees over and 4 for every 100 employees under the employment goal, and 3 for every $1 million over the investment goal. The contributions to profit, employment and investment are shown below.
What is the optimum product mix ?

| | Unit Contribution to Product | | |
|---|---|---|---|
| Factor | 1 | 2 | 3 |
| Long-run Profit | 12 | 9 | 15 |
| Employment level | 500 | 300 | 400 |
| Capital Investment | 5 | 7 | 8 |

## 7. A PROBABILISTIC INVENTORY MODEL

A newspaper vendor at a street corner sells the local paper for 25 cents a copy. At the beginning of the day he buys papers from the local dealer at 10 cents a copy. If he runs out of papers during the day, he cannot reorder any more, and if he has any excess papers left over the dealer will refund him only 4 cents per unsold copy. The vendor has taken some statistics classes, and estimates that the daily demand for papers is approximately normally distributed with a mean of 500 and a standard deviation of 30. How many papers should he buy from the dealer each morning ?

## 8. A QUEUEING MODEL

The local bank is receiving complaints about long waiting times at its ATM machine in a shopping mall. Based upon observation over a month, it has been found that the time between arrivals of two successive customers at the ATM is exponentially distributed with a mean of 3 min. and 45 seconds. The average transaction time spent at the machine itself is 3 minutes and is also exponentially distributed. The bank would like to evaluate the impact of three different actions: (1) install a second ATM at the other end of the mall, (2) install a second ATM right next to the current one and have a common queue, (3) replace the current ATM with a state-of-the-art system which will reduce service times by a third. How do each of these compare to the current arrangement in terms of waiting times and average queue lengths?

## 9. A SEQUENCING MODEL

A batch of 5 different jobs is to be processed at a two machine workcell. Each job has its first operation on machine 1 and its second on machine 2 in that sequence. The processing times are listed below:

| **Job** | **Machine 1** | **Machine 2** |
|---|---|---|
| 1 | 5 | 2 |
| 2 | 1 | 6 |
| 3 | 9 | 7 |
| 4 | 3 | 8 |
| 5 | 10 | 4 |

What is the sequence that results in the quickest way to finish the entire batch and get it out of the system? (This is a problem of minimizing the *makespan* at a *2 machine flowshop*)

*Linear Programming Formulations*

## 1. *A MAKE-OR-BUY PROBLEM*

A company manufactures three types of chemicals which it supplies to a variety of customers. For the upcoming month the total number contracted of each are as follows:

| Chemical | Contracted Sales |
|---|---|
| 1 | 2000 |
| 2 | 3500 |
| 3 | 1800 |

Production takes place in two stages: Reactor 1 and Reactor 2. The number of hours available at each stage and the reactor processing time required per lb. for each type is given below:

| | Processing time per lb. (hrs.) | | | |
|---|---|---|---|---|
| | **Chemical 1** | **Chemical 2** | **Chemical 3** | **Available hours** |
| **Reactor 1** | 0.05 | 0.04 | 0.01 | 200 |
| **Reactor 2** | 0.02 | 0.06 | 0.03 | 150 |

Clearly, there isn't enough time available to satisfy all contracted sales. Consequently, the company must buy some chemicals from other vendors having excess capacity, for resale to its own customers. This is more expensive than in-house production; per lb. costs are as shown below:

| Chemical | In-House Production Cost | Outside Purchase Cost |
|---|---|---|
| 1 | $ 2.50/lb | $2.80/lb |
| 2 | $ 1.75/lb | $2.50/lb |
| 3 | $ 2.90/lb | $3.25/lb |

What is the cheapest combination of production and purchase for satisfying all contracted sales ?

## 2. *A PLANNING PROBLEM*

A building contractor has acquired a 10-acre site and plans to develop it as a residential area. He has two basic designs for homes: a low-income home and a middle-income home, and plans to finish the project in no more than 5 years. Low-income homes cost $30,000 to build and occupy 1/20th of an acre, while middle-income homes cost $40,000 and occupy 1/12th of an acre. The contractor expects to sell a maximum of 100 low-income and 75 middle-income homes, but a local zoning regulation will allow no more than a total of 155 homes. In order to make sure that enough low-income homes are built, the city council has also stipulated that the number of low-income homes exceed half the number of middle-income homes by at least 35. The contractor has a workforce of 60 people and each low-income home will take 2 labor years to build, while each middle-income home needs 2.5 labor years. The contractor expects to sell a low-income home for $50,000 and each middle-income home for $70,000. What should the contractor do ?

## 3. *A TRANSPORTATION PROBLEM*

A company has two manufacturing plants that supply three warehouses (which in turn supply a series of retail outlets). The monthly production capacities at Plant 1 and Plant 2 are 3000 units and 3500 units respectively. Maximum sales potentials at the three warehouses are 1500, 2000 and 3500 respectively. Each unit is base priced at $130; however the warehouses serve different geographical areas and incur different operating costs, so that a premium of $10 at warehouse 2 and $20 at warehouse 3 is added on to determine the actual selling price. Plant 1 is an older plant while Plant 2 is a more modern facility; the per-unit cost of manufacturing at these are respectively $60 and $50. The shipping and handling cost depend on the distance moved and average $0.10 /unit/mile moved. The distances (in miles) from Plant i (for i=1,2) to Warehouse j (j=1,2,3) are as below:

| | Warehouse (j) | | |
|---|---|---|---|
| **Plant (i)** | **1** | **2** | **3** |
| **1** | 200 | 400 | 600 |
| **2** | 600 | 400 | 500 |

Formulate a linear program to find how many units should be shipped from each plant to each warehouse.

## 4. *A PERSONNEL SCHEDULING PROBLEM*

The personnel manager at your plant must schedule security officers so as to satisfy certain minimum requirements at all times as shown below:

| Time | Minimum no. of officers required |
|---|---|
| Midnight - 4 AM | 5 |
| 4 AM - 8 AM | 7 |
| 8 AM - Noon | 15 |
| Noon - 4 PM | 7 |
| 4 PM - 8 PM | 12 |
| 8 PM - Midnight | 9 |

Officers work 8 hour shifts paying different amounts as follows :

| Shift | Start | End | Hourly Wage ($) |
|---|---|---|---|
| **1** | Midnight | 8 AM | 16 |
| **2** | 4 AM | Noon | 18 |
| **3** | 8 AM | 4 PM | 15 |
| **4** | Noon | 8 PM | 15 |
| **5** | 4 PM | Midnight | 16 |
| **6** | 8 PM | 4 AM | 18 |

Formulate an LP to meet staffing requirements at a minimum cost.

## 5. *A MULTIPLE KNAPSACK TYPE PROBLEM*

A cargo plane has three compartments for storing cargo: front, center and rear. These compartments have limits both on space and weight as summarized below

| Compartment | Weight Capacity | Space Capacity |
|---|---|---|
| Front | 12 tons | 7,000 cu ft. |
| Center | 18 tons | 9,000 cu ft. |
| Rear | 10 tons | 5,000 cu ft. |

Furthermore, the weights in each compartment must be the same proportion of that compartment's weight capacity to maintain the balance of the airplane. The following 4 cargoes have been offered for shipment:

| Cargo | Weight (tons) | Volume (cu ft/ton) | Profit $/ton |
|---|---|---|---|
| 1 | 20 | 500 | 320 |
| 2 | 16 | 700 | 400 |
| 3 | 25 | 600 | 360 |
| 4 | 13 | 400 | 290 |

Any portion of these cargoes can be accepted. How much (if any) of each should be accepted and how should it be distributed in the plane ?

## 6. *A MULTIPERIOD PRODUCTION PLANNING PROBLEM*

An appliance company manufactures dishwashers and washing machines. Its sales target for each of the four quarters of the next year (the planning horizon) are shown below:

| Product | Symbol | Q1 | Q2 | Q3 | Q4 |
|---|---|---|---|---|---|
| Dishwashers | $D_t$ | 2000 | 1300 | 3000 | 1000 |
| Washing Machines | $M_t$ | 1200 | 1500 | 1000 | 1400 |

At the end of the current year 75 dishwashers and 50 washing machines are forecast to be in inventory, and the company is expected to have a workforce equivalent to 5000 labor-hours. Management does not like too many changes in the workforce level and has stipulated that the change from one quarter to the next be no more than 10%. On average, a dishwasher requires 1.5 labor-hours and a washing machine requires 2 labor-hours. The accountants figure that exclusive of labor, each dishwasher costs the company $125 to make and each washing machine $95. The inventory holding costs per unit per quarter are $4.50 for dishwashers and $3.80 for washing machines. On average, each labor hour costs $6.50. What is the best way to plan production over the next year in order to meet the sales targets ?

## 7. *A REFINERY OPERATIONS PLANNING PROBLEM*

Olé Oil produces three products*: heating oil, gasoline and jet fuel*. The average octane levels must be at least 4.5 for heating oil, 8.5 for gas and 7.0 for jet fuel.

To produce these products Olé purchases two types of crude oil: Crude 1 (at $12 per barrel) and Crude 2 (at $10 per barrel). Each day, at most 10,000 barrels of each type of oil can be purchased.

Before crude can be used to produce products for sale, it must be distilled. Each day, at most 15,000 barrels of oil can be distilled. It costs 10 cents to distill a barrel of oil. The result of distillation is as follows:

1. Each barrel of Crude 1 yields 0.6 barrels of naphtha, 0.3 barrels of distilled 1, 0.1 barrels of distilled 2.
2. Each barrel of Crude 2 yields 0.4 barrels of naphtha, 0.2 barrels of distilled 1, 0.4 barrels of distilled 2.

Distilled naphtha can be used to produce only jet fuel or gasoline. Distilled oil can be used to produce heating oil, or it can be sent through a catalytic cracker (at a cost of 15 cents per barrel). Each day, at most 5000 barrels of distilled oil can be sent through the cracker.

1. Each barrel of distilled 1 sent through the cracker yields 0.8 barrels of cracked 1, and 0.2 barrels of cracked 2.
2. Each barrel of distilled 2 sent through the cracker yields 0.7 barrels of cracked 1, and 0.3 barrels of cracked 2.

Cracked oil can be used to produce gasoline and jet fuel but not to produce heating oil.

The octane levels of each type of oil is as follows: naphtha = 8; distilled 1= 4; distilled 2 = 5; cracked 1 = 9; cracked 2 = 6.

A schematic diagram of the entire process is shown in the accompanying figure.

All heating oil produced can be sold at $14 per barrel; all gasoline produced at $18 per barrel; and all jet fuel produced at $16 per barrel. Marketing considerations dictate that at least 3000 barrels of each product must be produced daily.

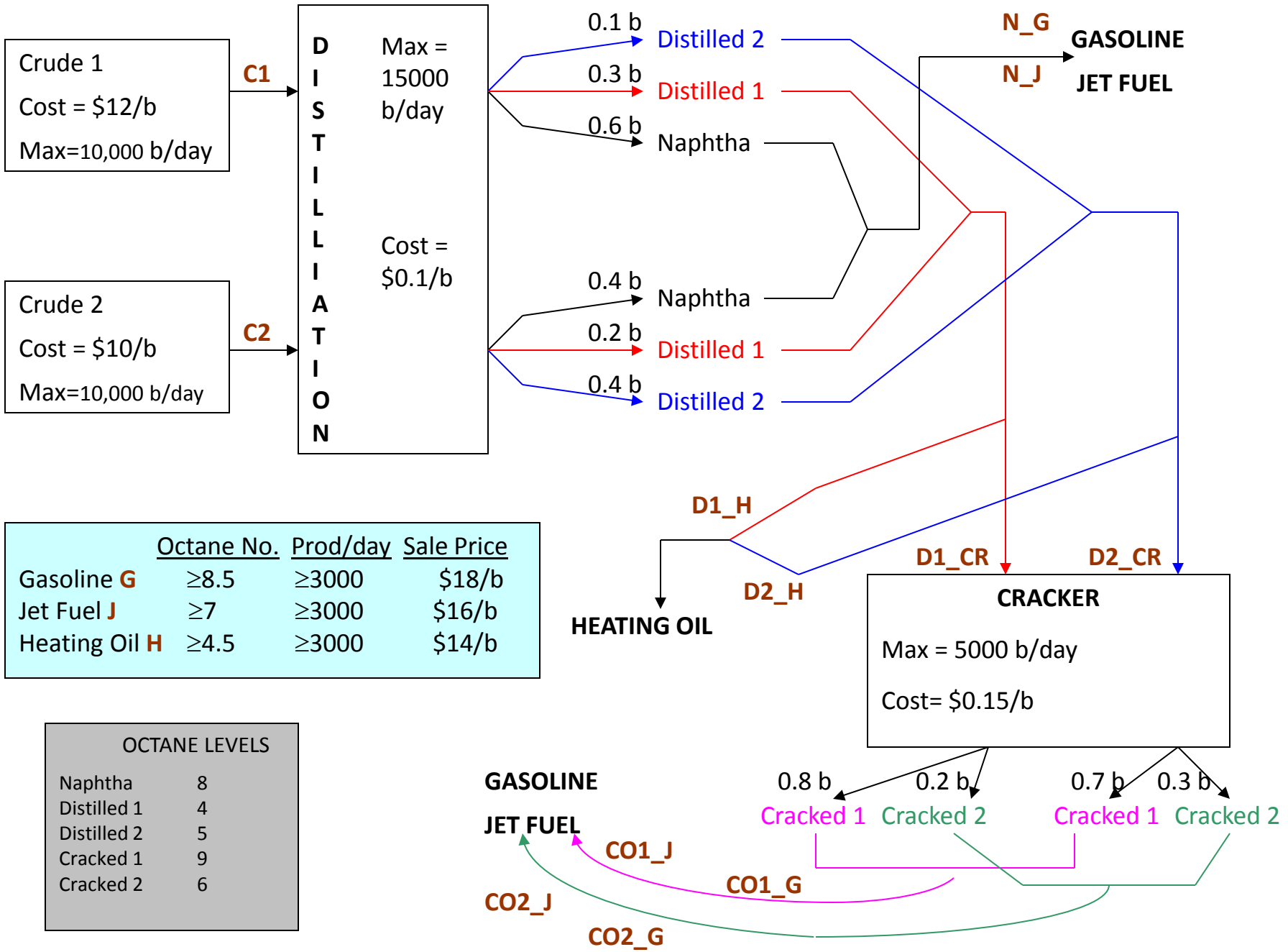
Formulate an LP to maximize Olé's daily profit.

## 8. *A FINANCIAL PLANNING PROBLEM*

The investments manager for a company has just signed a contract to buy $750,000 worth of new equipment. The contract calls for $150,000 to be paid 2 months from now, and the remainder to be paid on delivery 6 months from now. To meet this schedule of payments, the manager has decided to set up a sinking investment fund. A total of 12 different investment opportunities are available over the next 6 months as summarized below:

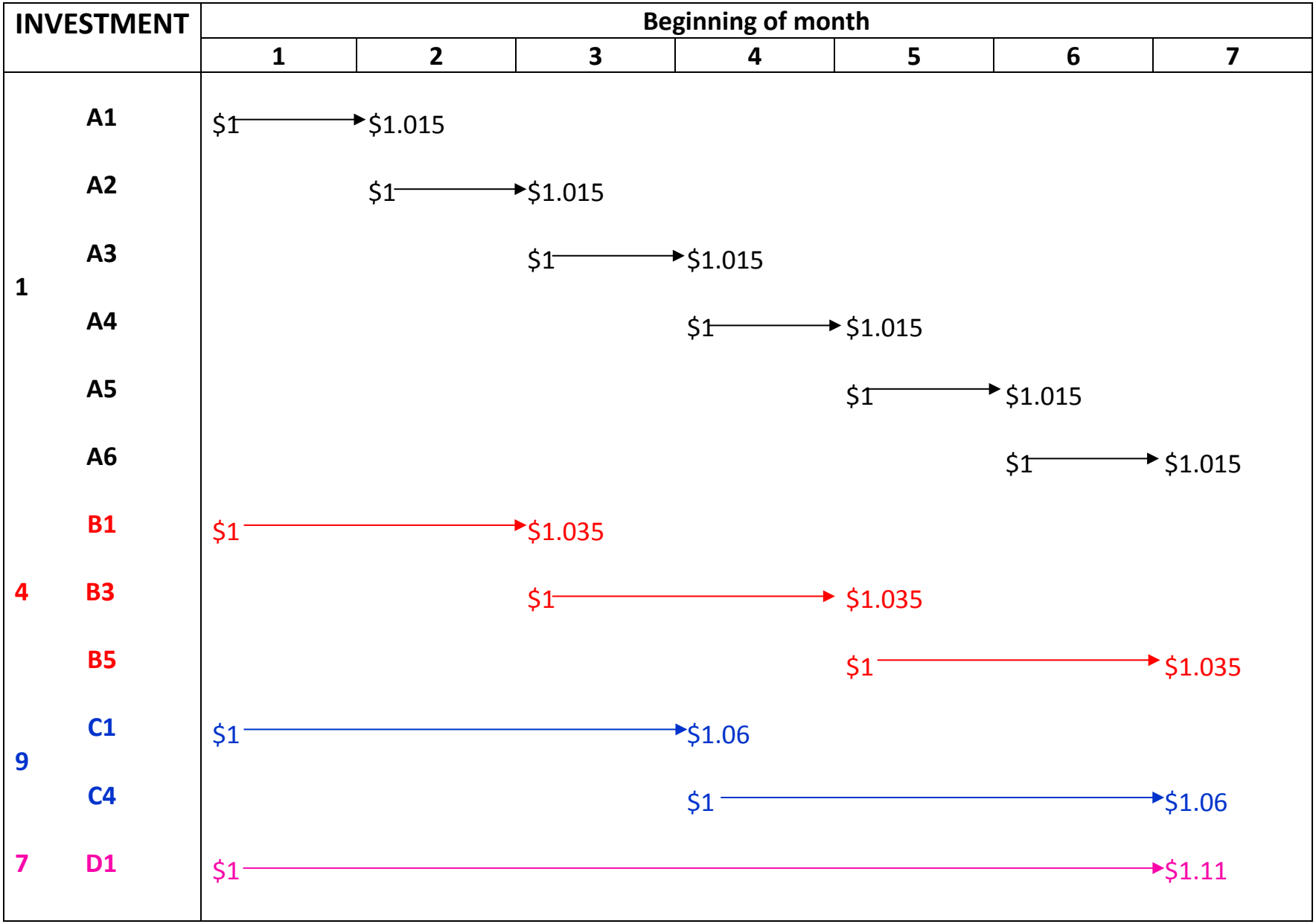
| **Investment Opportunity** | **Available at the beginning of ...** | **Months to Maturity** | **Yield at Maturity** | **Risk Index** |
|---|---|---|---|---|
| A | Months 1,2,3,4,5 and 6 | 1 | 1.5% | 1 |
| B | Months 1,3 and 5 | 2 | 3.5% | 4 |
| C | Months 1 and 4 | 3 | 6.0% | 9 |
| D | Month 1 | 6 | 11.0% | 7 |

The risk index is a subjective evaluation of an investments risk and runs on a scale of 1 to 10, with 10 being the most and 1 being the least risky. Two self-imposed guidelines are to be followed: (1) during any month the average risk index of invested funds cannot exceed 6, and (2) at the beginning of any month (after any new investments have been made), the average months to maturity of invested funds cannot exceed 2.5 months. The goal is to use the above data and determine the minimum amount to place in the fund so as to meet the payment schedule.




| | Octane No. | Prod/day | Sale Price |
|---|---|---|---|
| Gasoline G | ≥8.5 | ≥3000 | $18/b |
| Jet Fuel J | ≥7 | ≥3000 | $16/b |
| Heating Oil H | ≥4.5 | ≥3000 | $14/b |

| OCTANE LEVELS | |
|---|---|
| Naphtha | 8 |
| Distilled 1 | 4 |
| Distilled 2 | 5 |
| Cracked 1 | 9 |
| Cracked 2 | 6 |


INVESTMENT
Beginning of month
1
2
3
4
5
6
7
A1
A2
A3
1
A4
A5
A6
B1
4
B3
B5
C1
9
C4
7
D1
$1
$1.015
$1.035
$1.06
$1.11

# Graphical Solution of Linear Programs

Chemco, Inc. has two major product lines-- fertilizers and pesticides. Both products have to go through two common departments-- manufacture and packaging. Assuming that all products made can be sold, the management wants to know how much of each product is to be produced next month. The following data is available:

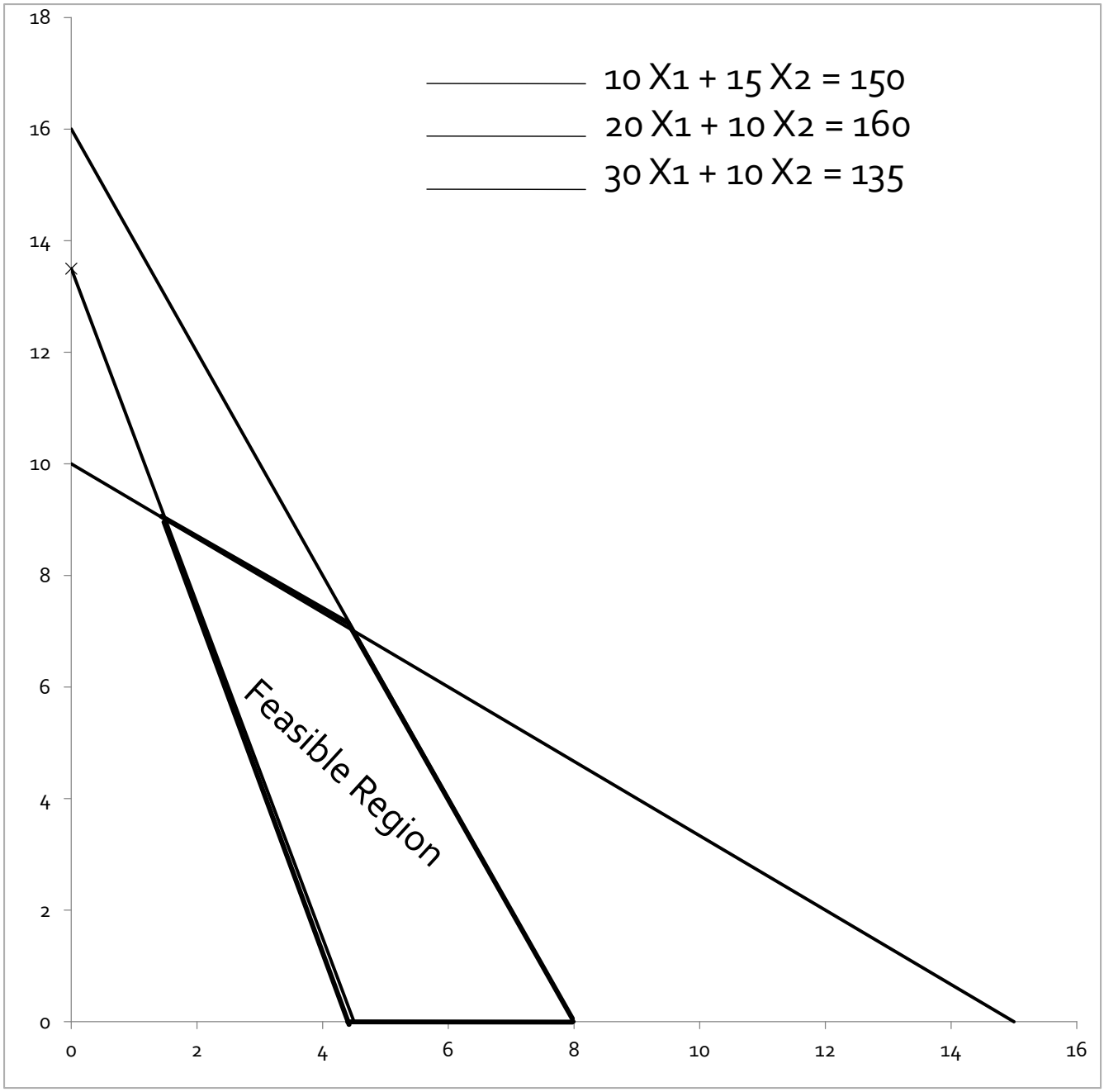
| Product | Profit/unit($) | Hours Required | |
|---|---|---|---|
| | | Manufacture | Packaging |
| Fertilizers | 5000 | 10 | 20 |
| Pesticides | 4000 | 15 | 10 |

Total available hours are 150 hrs in the manufacturing area and 160 hrs in the packaging area.

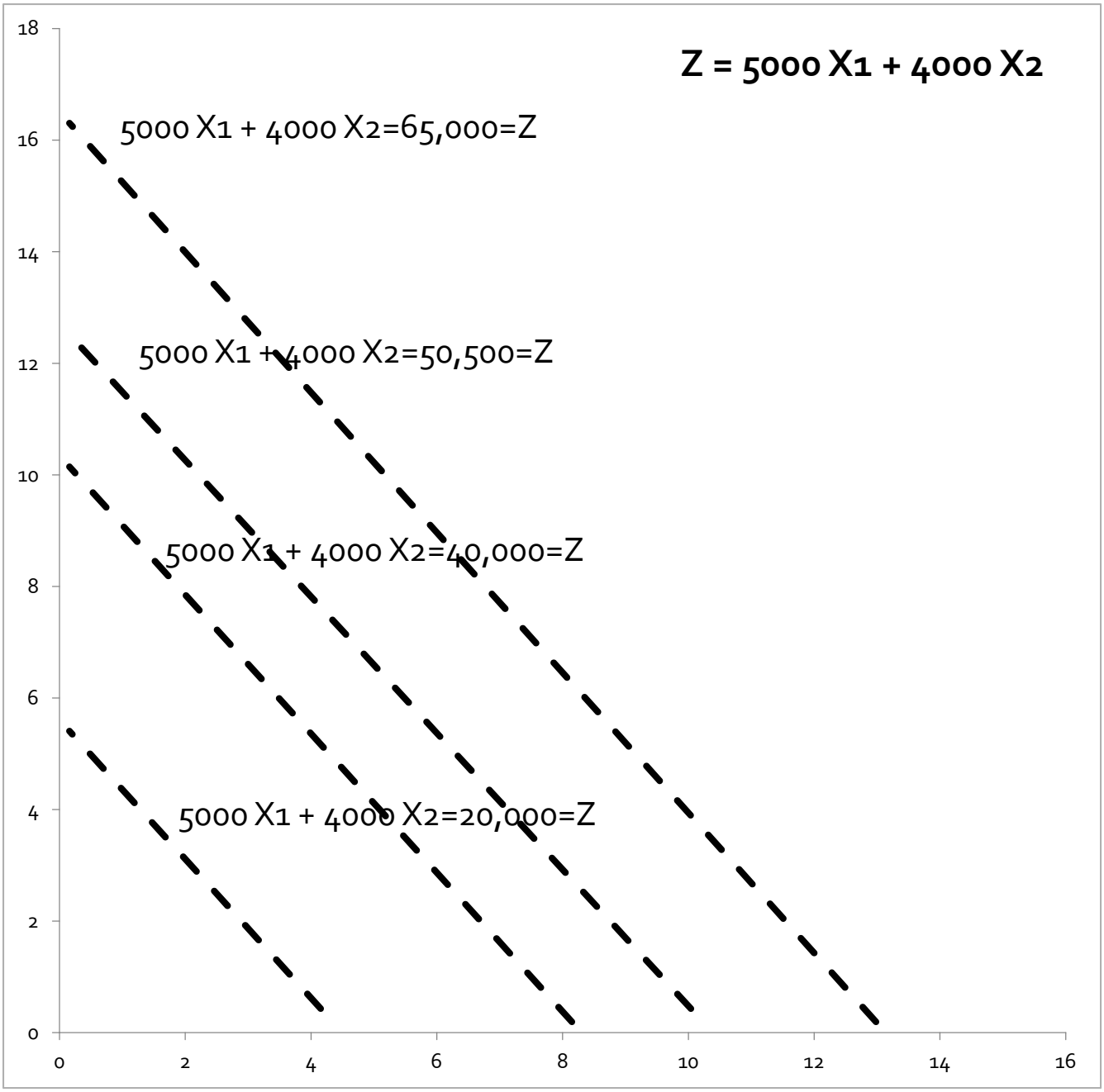
In addition the QC department requires that a total of at least 135 hours must be spent in final testing next month, where each unit of fertilizer requires 30 hrs and each unit of pesticide requires 10 hrs to test.

Formulate as a linear program to solve management's problem.

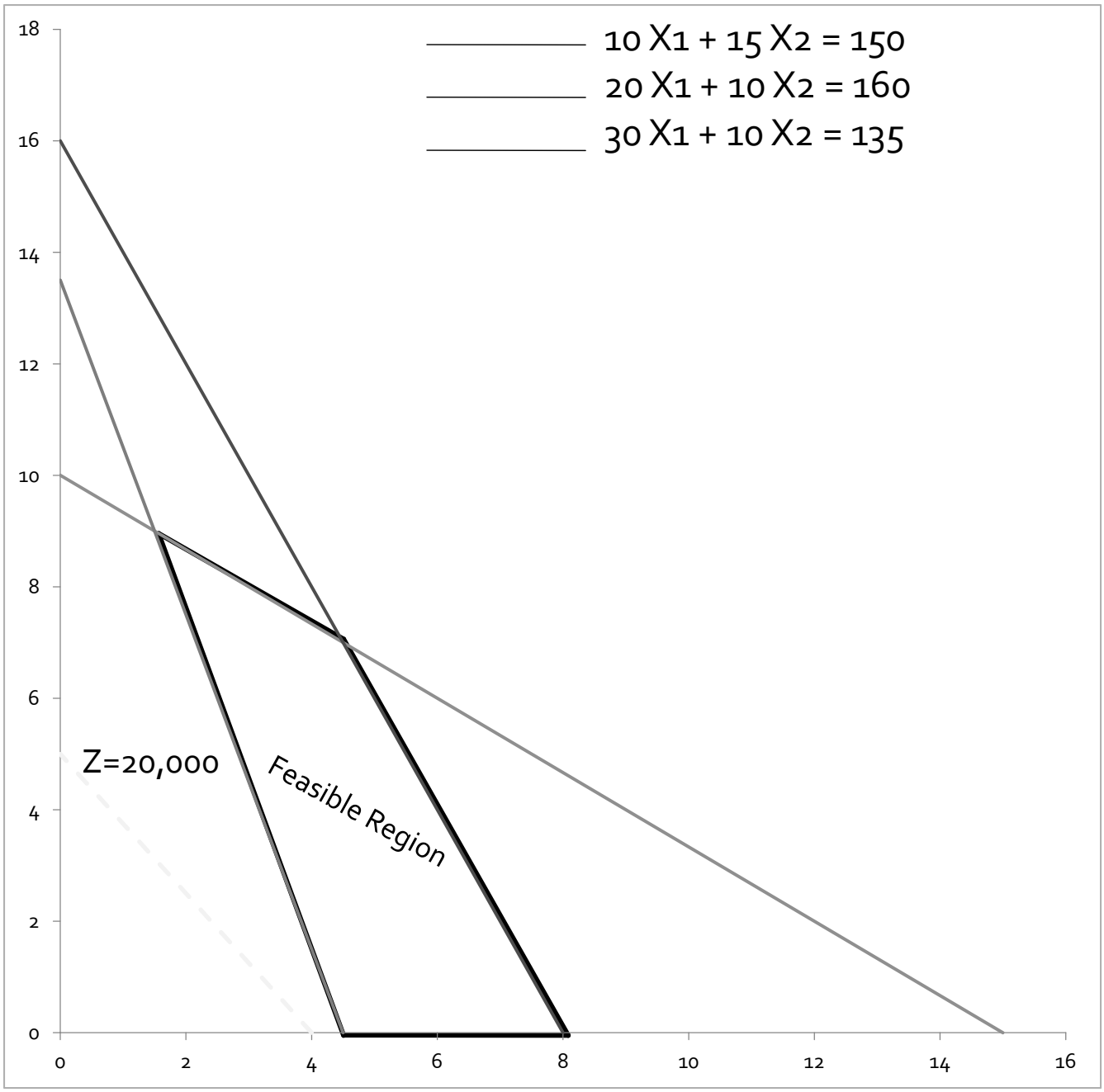
# Graphing the Feasible Region



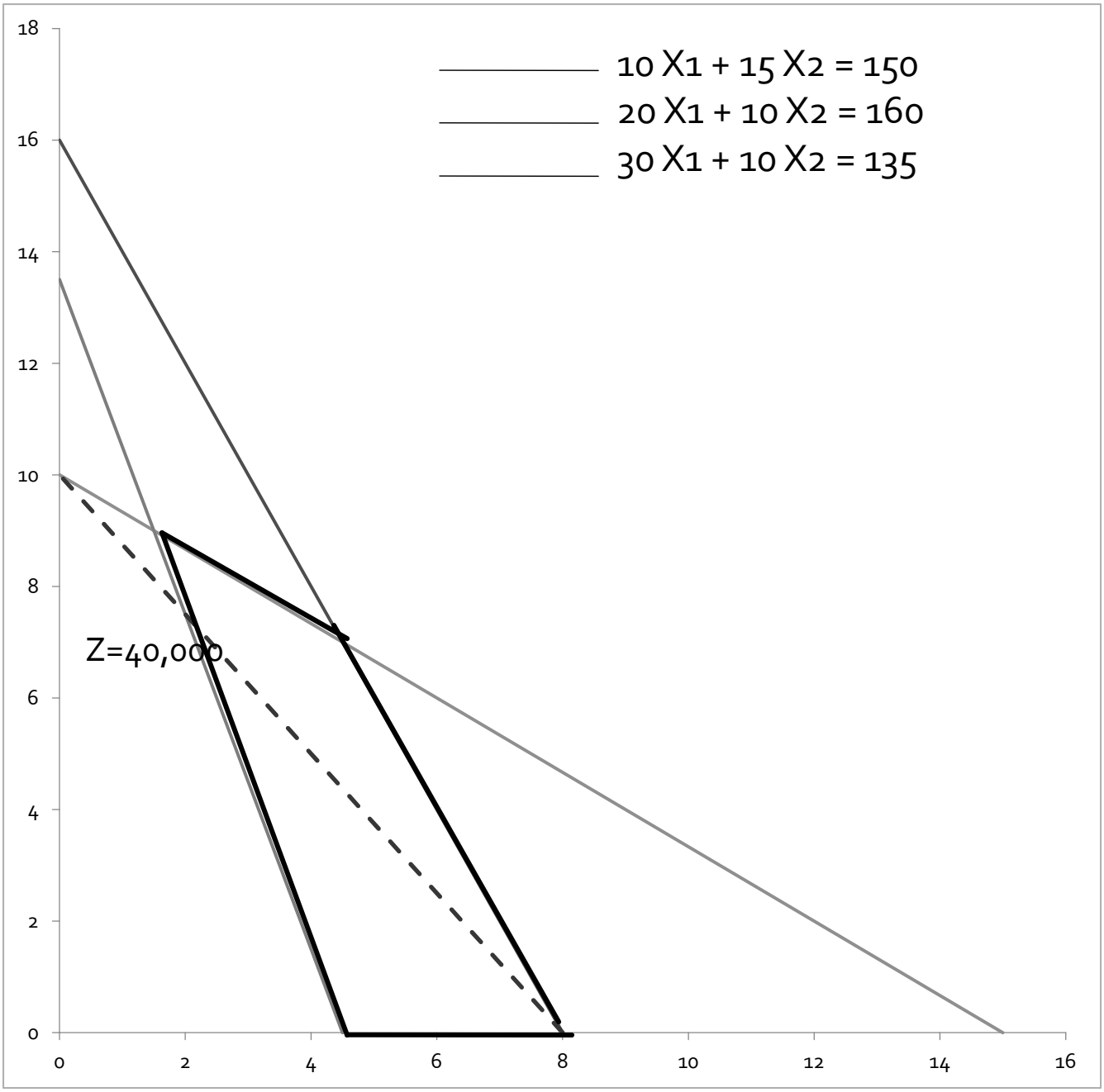
# Isocost (Isoprofit) Lines, aka Objective Function Contours



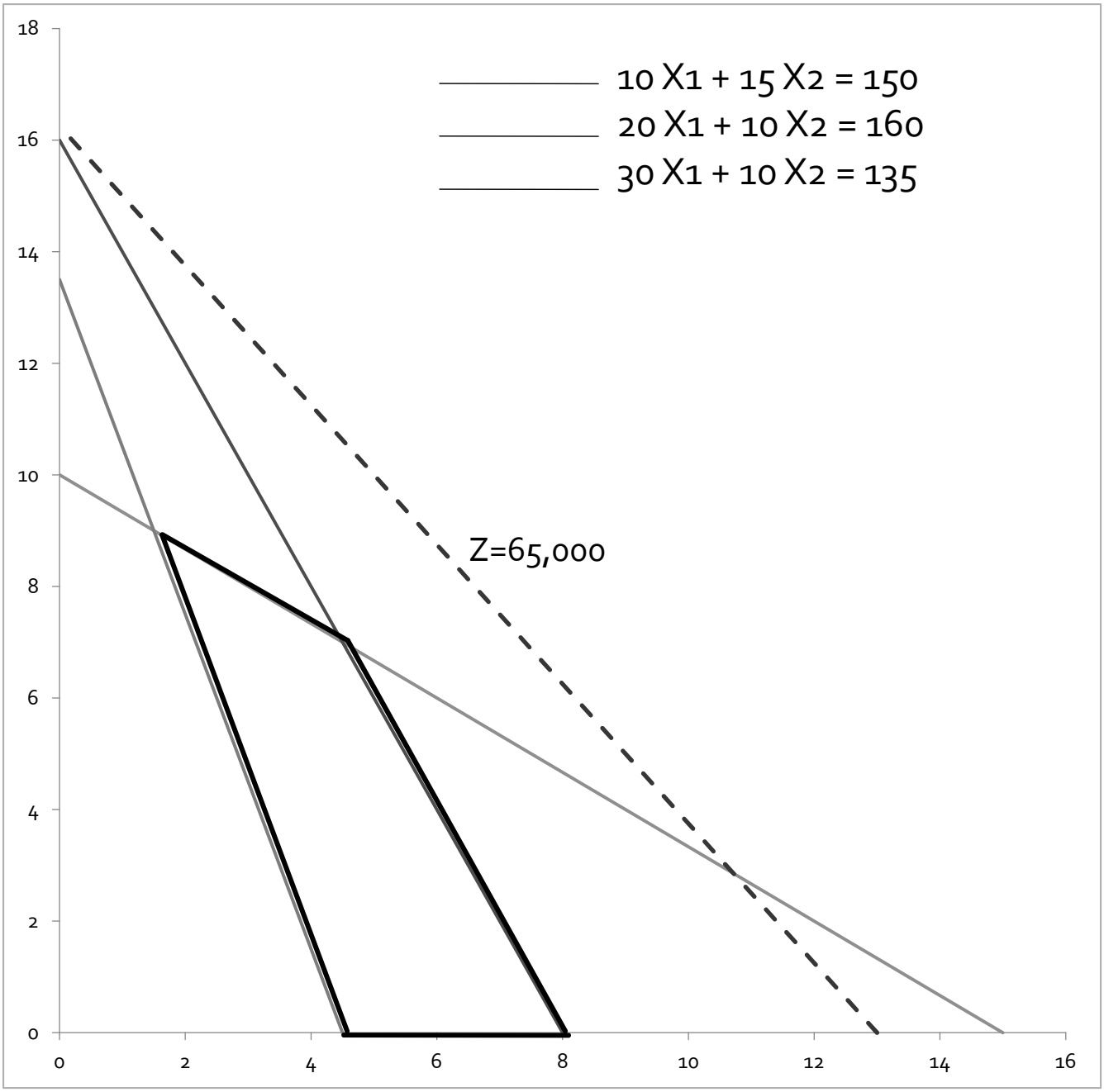
# The Feasible Region with an O.F. contour



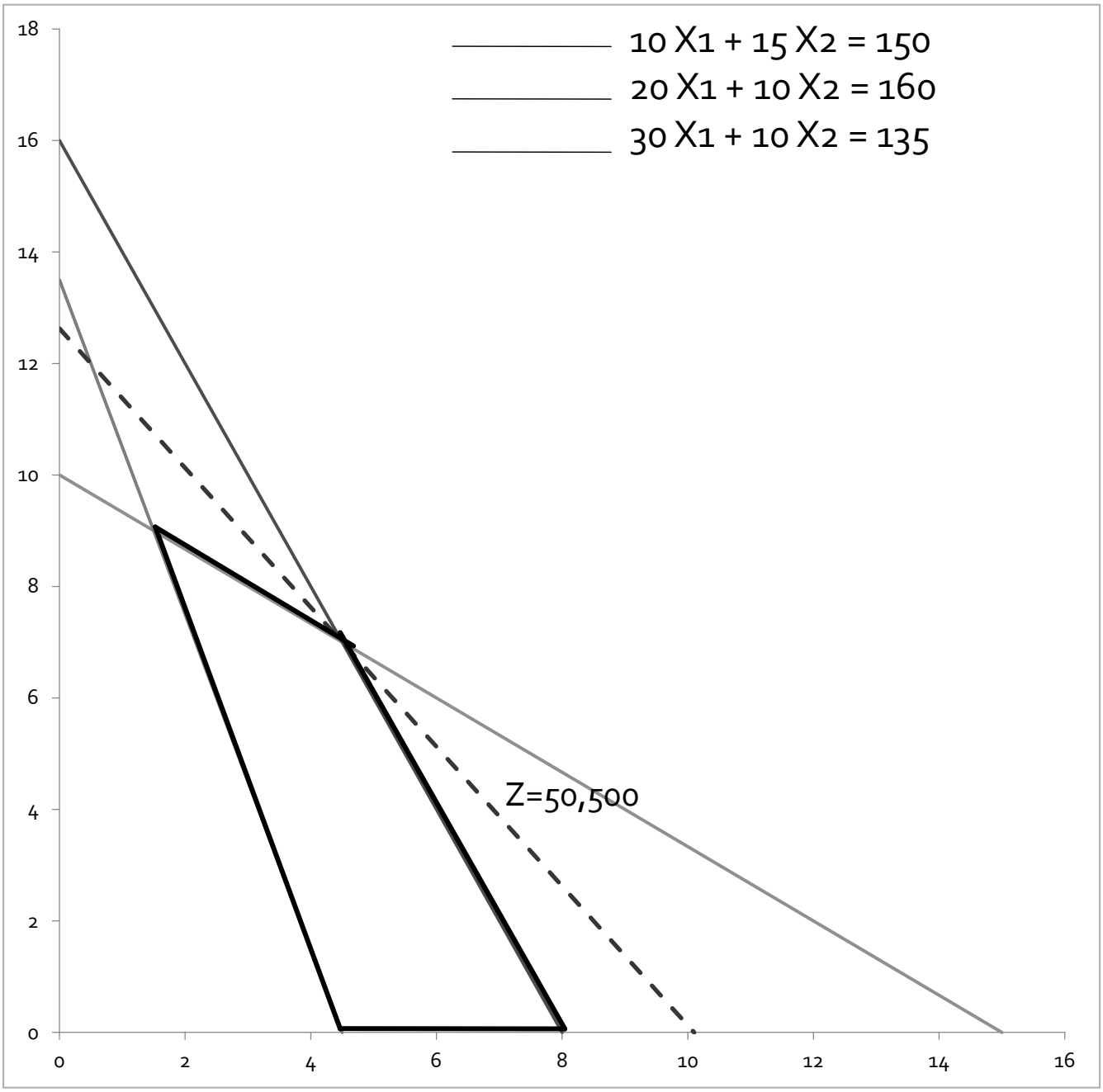
# The Feasible Region with another O.F. contour



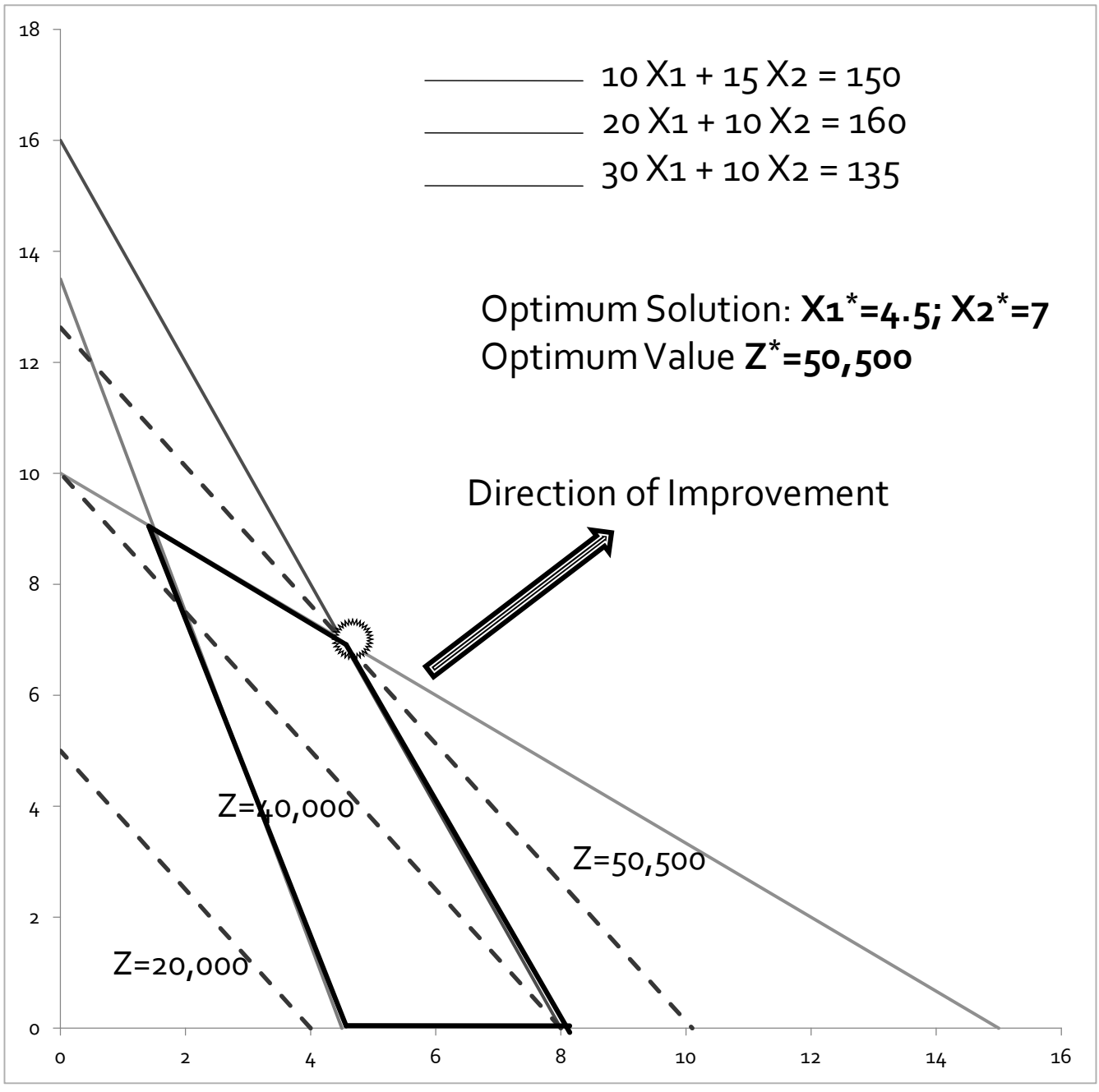
# The Feasible Region with another O.F. contour



# The Feasible Region with the optimal O.F. contour



# Moving a contour parallel to itself in the improving direction



# Graphical Solution of LPs: Another Example

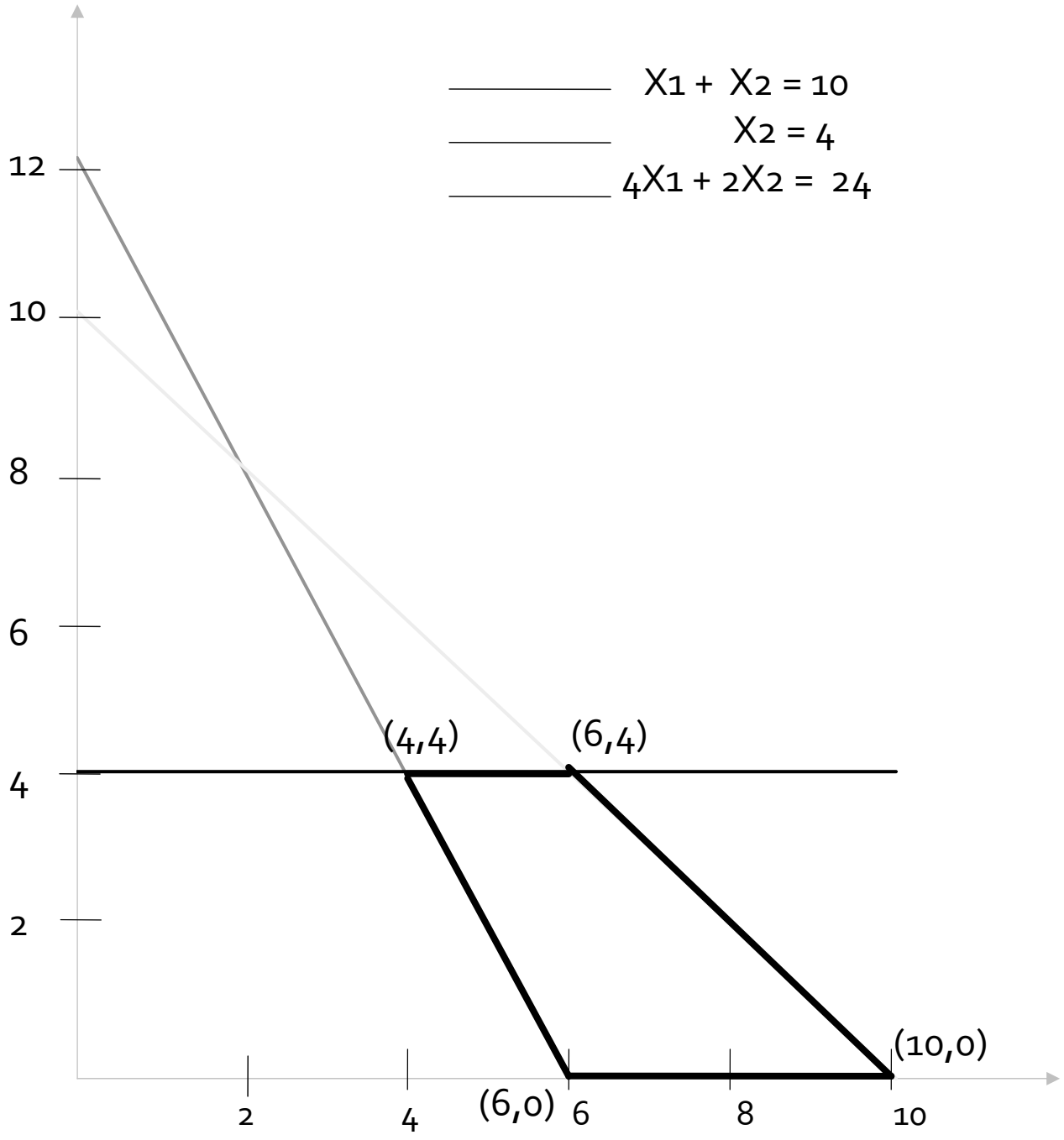
Consider the feasible region defined by:

$$X_1 + X_2 \leq 10$$
$$4X_1 + 2X_2 \geq 24$$
$$X_2 \leq 4$$
$$X_1, X_2 \geq 0$$

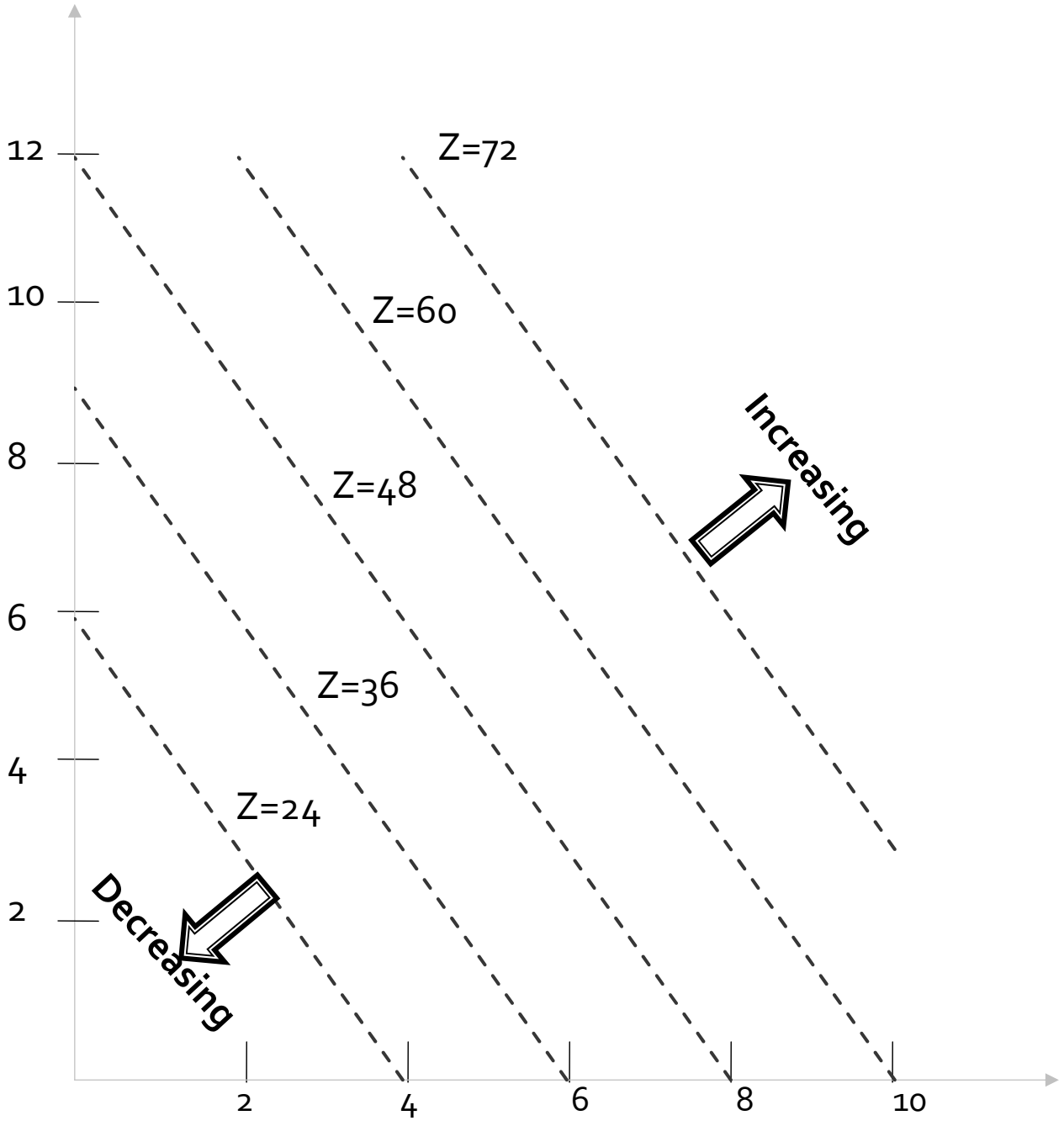
Let us find the optimum for each of the following objectives:

1. Minimize $6X_1+4X_2$
2. Maximize $6X_1+4X_2$
3. Minimize $8X_1+2X_2$
4. Maximize $8X_1+2X_2$
5. Minimize $X_1+X_2$
6. Maximize $X_1+X_2$

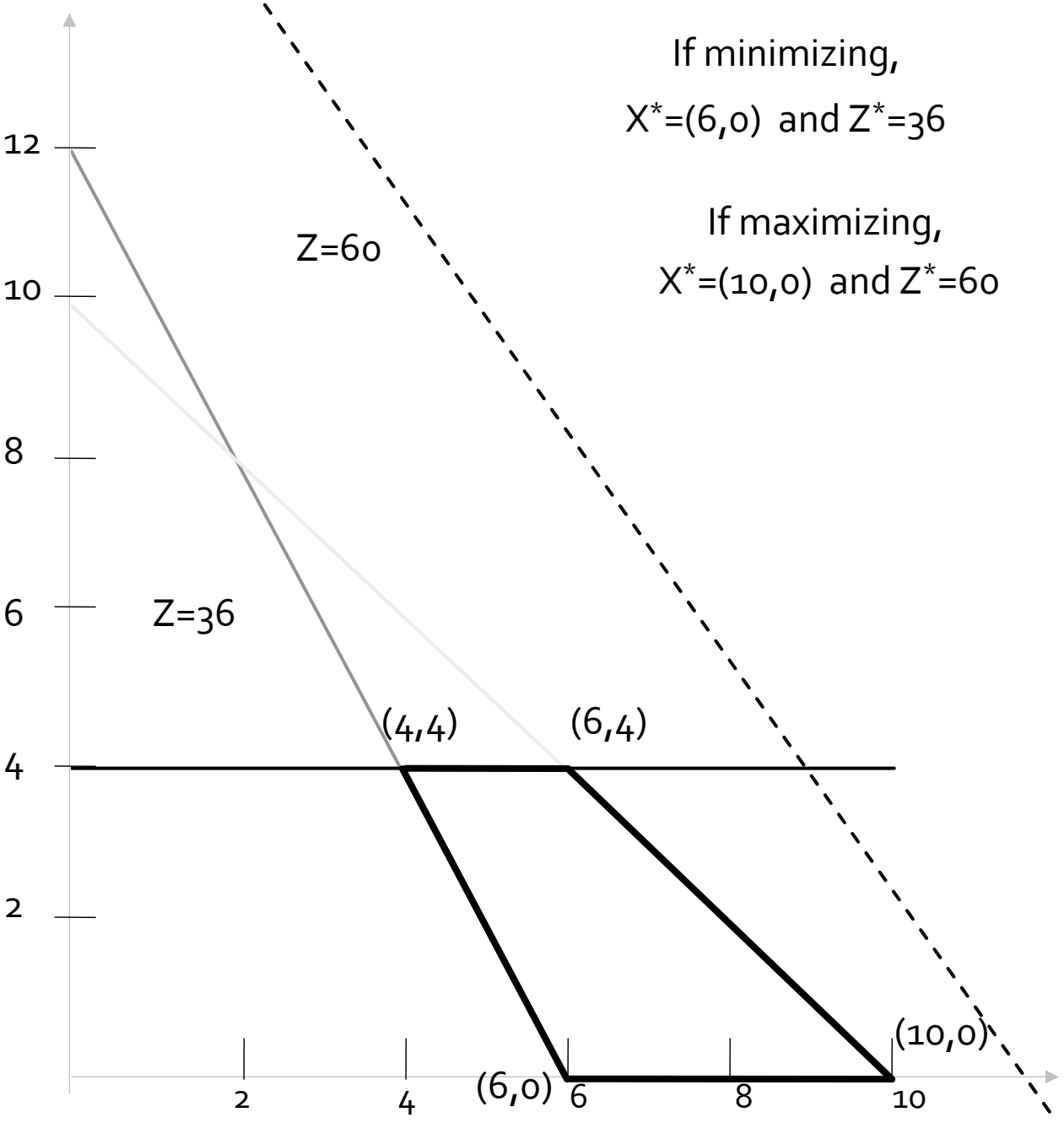
# The Feasible Region



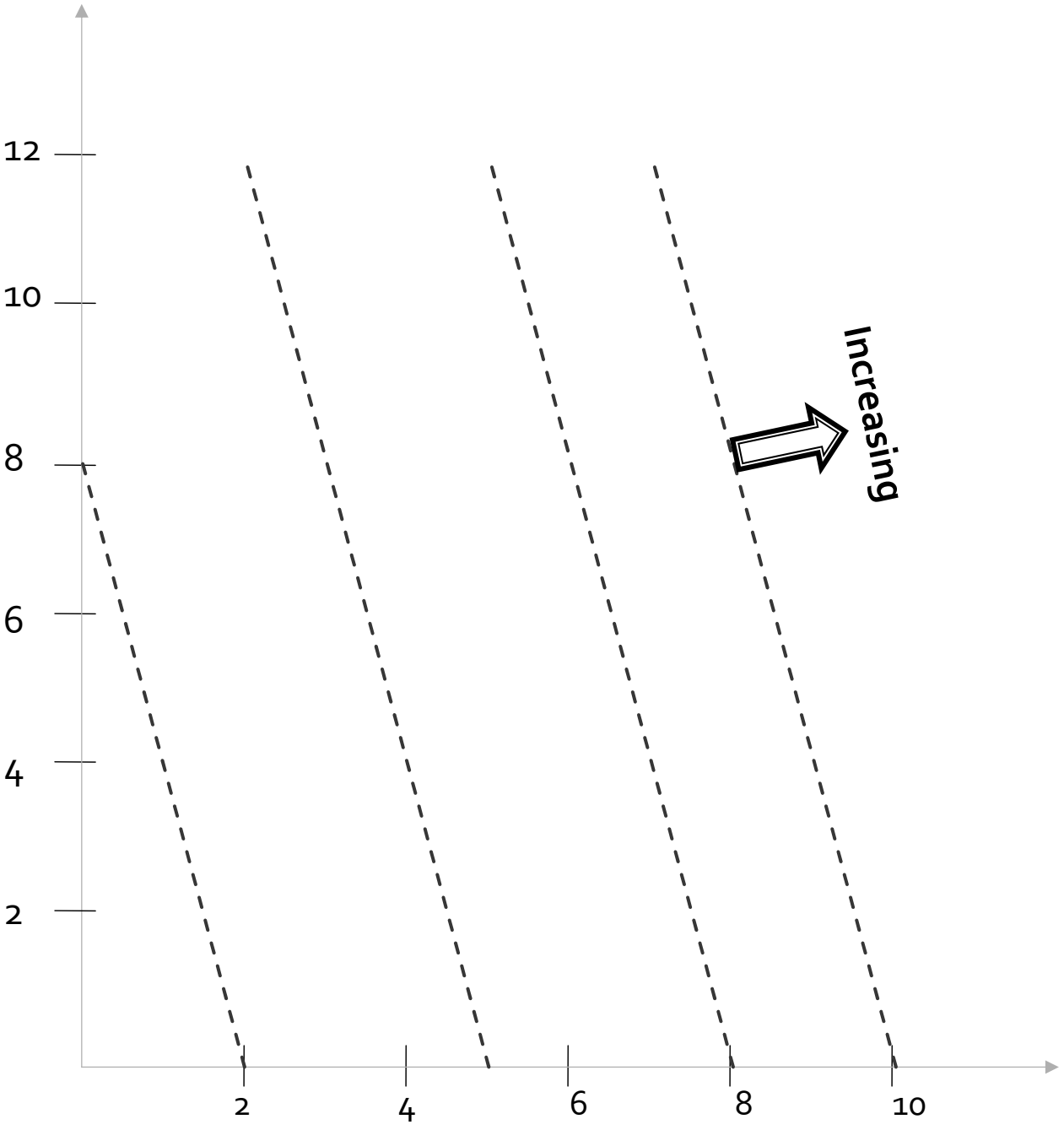
# Isocost Lines for the objective $Z=6X_1+4X_2$



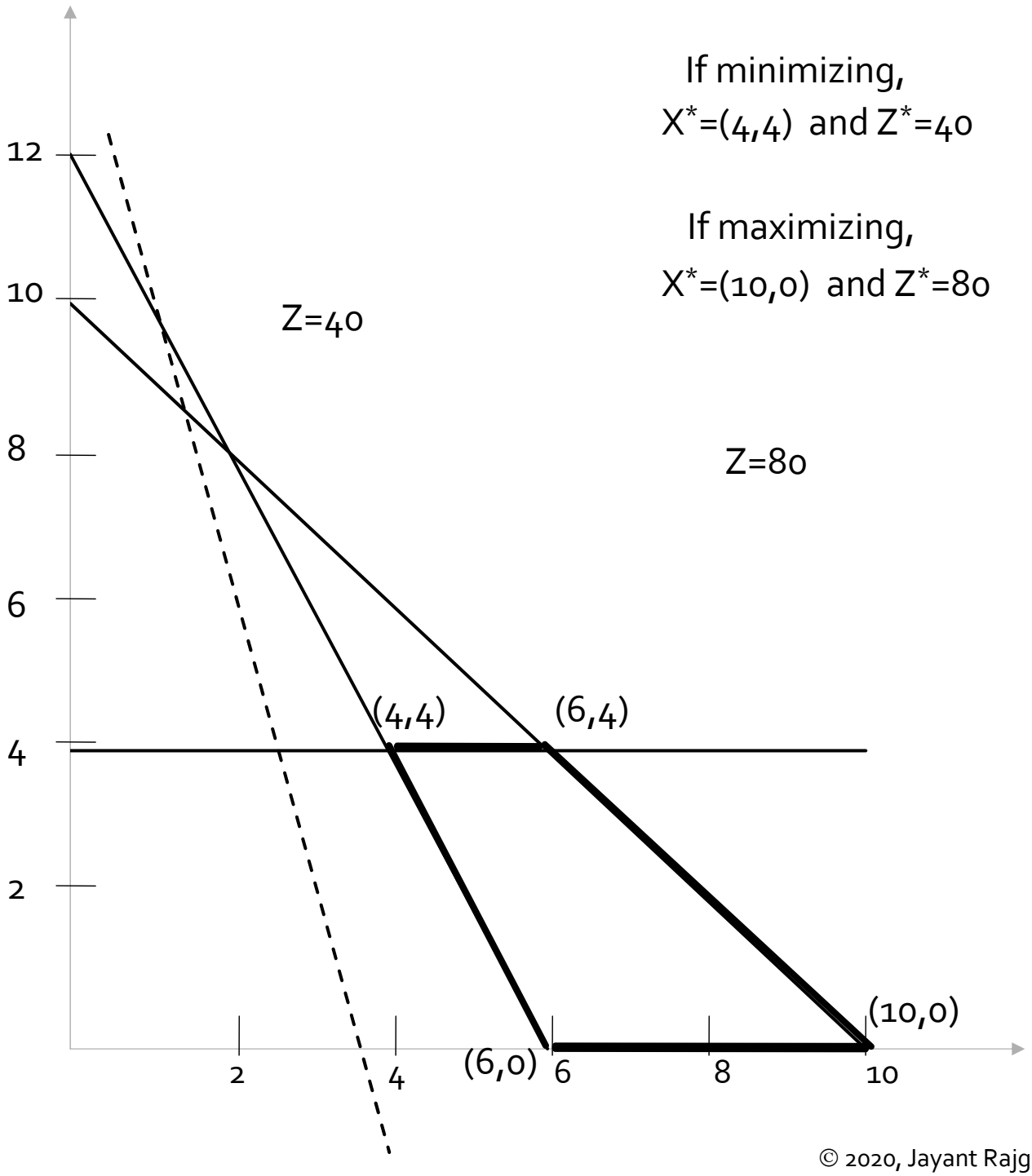
# The Feasible Region with contours of $6X_1+4X_2$



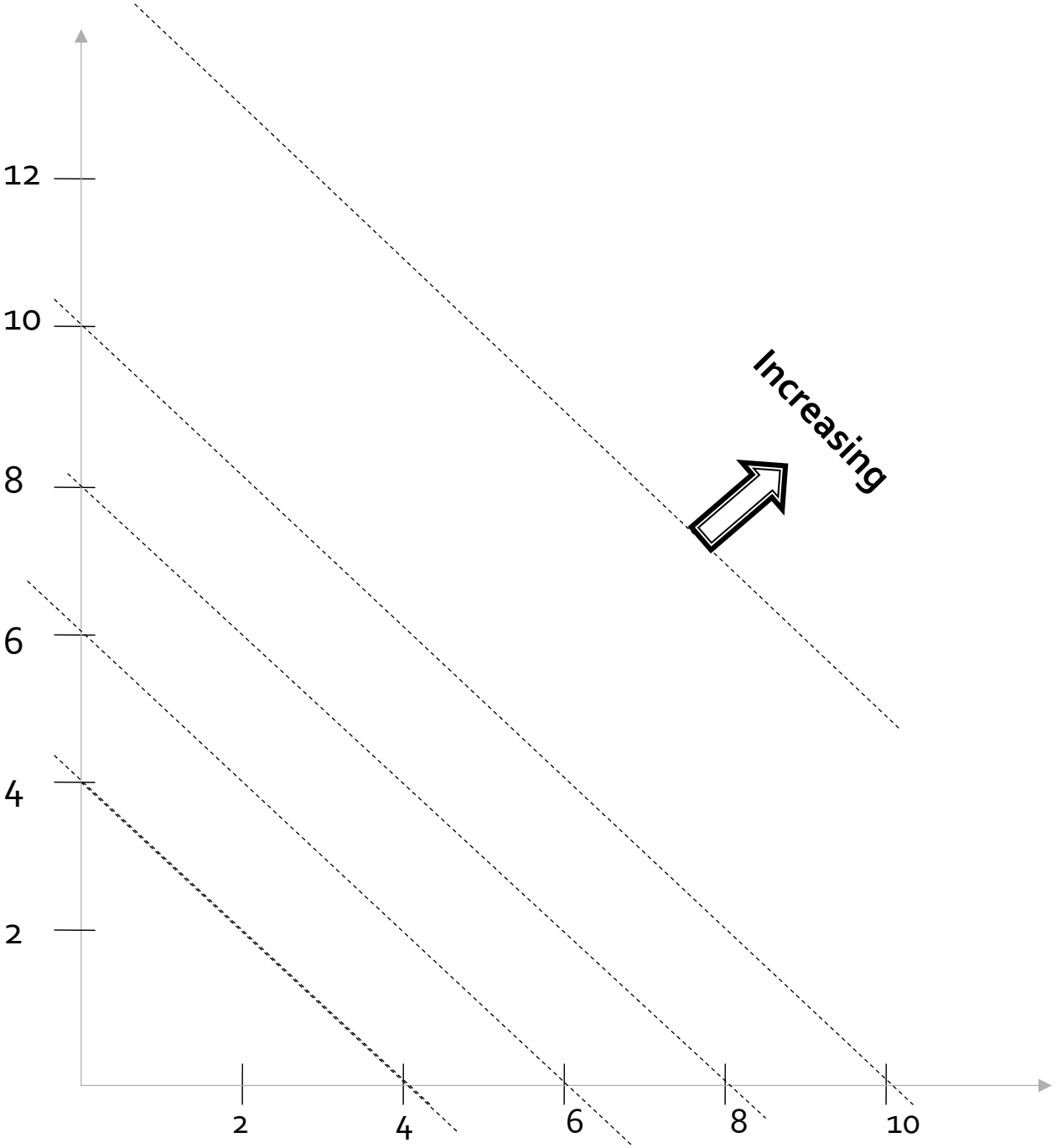
# Another Objective Function: Contours of $Z=8X_1+2X_2$



# The Feasible Region with contours of $8X_1+2X_2$



# A Third Objective Function: Contours of $Z=X_1+X_2$



# The Feasible Region with contours of $X_1+X_2$

If minimizing,
$X^*=(6,0)$ and $Z^*=6$

If maximizing, $Z^*=10$

With $X^*=(10,0)$ **OR** $(6,4)$ **OR ANYWHERE ON THE LINE** joining $(6,4)$ and $(10,0)$**!**