I.E.2001 OPERATIONS RESEARCH

(Homework Assignment 8: Solutions)

Question 1:

Question 1, page 398 of the text (five employees & four jobs...)

After adding a dummy job column and "M" to denote the cost of disallowed assignments we obtain the following cost matrix on the left:

22	18	30	18	0
18	M	27	22	0
26	20	28	28	0
16	22	M	14	0
21	M	25	28	0

6	0	5	4	0
2	M	2	8	0
10	2	3	14	0
0	4	M	0	0
5	M	0	14	0

6	0	5	4	2
0	M	0	6	0
8	0	1	12	0
0	4	M	0	2
5	M	0	14	2

Since the matrix is already row reduced, we column reduce it to obtain the second matrix (note that one could use M-18 in column 2 and M-25 in column 3 but assuming that M is "very" large, one could just as well us "M" instead). Since all zeroes can be covered using four lines as shown, we're not done. The smallest uncovered cost is 2 and subtracting this from other uncrossed entries and adding it to all twice crossed entries we obtain the third matrix. Five lines are needed to cover all zeroes here.

We're done and the optimal assignment can be extracted from the table with candidate assignments where the 0's are located (shown below to the left): Person 1 does job 2, Person 2 does job 1, Person 3 is idle (job 5 is a dummy job), Person 4 does job 4, and Person 5 does job 3.

X		X	X	X
	X		X	
X		X	X	
	X	X		X
X	X		X	X

22	18	30	18	0
18	M	27	22	0
26	20	28	28	0
16	22	M	14	0
21	M	25	28	0

Total time = 18 + 18 + 0 + 14 + 25 = **75** hours.

Question 2.

Question 28, page 411 of the text (professors...)

Since each professor must teach two sections we make an extra "copy" of each professor, for a total of 6 agents. Then to convert the problem to a minimization, we multiply all the rankings by -1. The "cost" matrix is given by the following

Section / Professor	9 AM	10 AM	11 AM	1 PM	2 PM	3 PM
1	**-8**	**-7**	**-6**	**-5**	**-7**	**-6**
1′	**-8**	**-7**	**-6**	**-5**	**-7**	**-6**
2	**-9**	**-9**	**-8**	**-8**	**-4**	**-4**
2'	**-9**	**-9**	**-8**	**-8**	**-4**	**-4**
3	**-7**	**-6**	**-9**	**-6**	**-9**	**-9**
3'	**-7**	**-6**	**-9**	**-6**	**-9**	**-9**

Row reduction followed by column reduction yield the first and second matrices below:

0	1	2	3	1	2
0	1	2	3	1	2
0	0	1	1	5	5
0	0	1	1	5	5
2	3	0	3	0	0
2	3	0	3	0	0

0	1	2	2	1	2
0	1	2	2	1	2
0	0	1	0	5	5
0	0	1	0	5	5
2	3	0	2	0	0
2	3	0	2	0	0

$\underline{0}$	1	1	2	0	1
0	1	1	2	$\underline{0}$	1
0	$\underline{0}$	0	0	4	4
0	0	0	$\underline{0}$	4	4
3	4	$\underline{0}$	3	0	0
3	4	0	3	0	$\underline{0}$

Since the second matrix can be covered with 5 lines as shown above, we proceed with the next step (smallest uncovered entry = 1).

This yields the optimal tableau and the table with candidate assignments where the 0's are located (shown below to the left)assignments shown: Professor 1 teaches 9 AM and 2 PM sections, Professor 2 teaches 10 AM and 1 PM sections, and Professor 3 teaches 11 AM and 3 PM sections.

	X	X	X		X
	X	X	X		X
				X	X
				X	X
X	X		X		
X	X		X		

8	**7**	**6**	**5**	**7**	**6**
8	**7**	**6**	**5**	**7**	**6**
9	**9**	**8**	**8**	**4**	**4**
9	**9**	**8**	**8**	**4**	**4**
7	**6**	**9**	**6**	**9**	**9**
7	**6**	**9**	**6**	**9**	**9**

Maximum total ranking = 8+7+9+8+9+9 = **50**.

Question 3.

Question 1, at the end of Section 9.2 on page 502 of the text (Coach Night)

Define x_j = 1 if player j starts; j=1,2,3,4,5,6,7

0 otherwise.

Then the appropriate IP is as follows:

Maximize Z = $3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + x_7$

st

1. $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 5$ (five on a team)

   $x_1+x_3+x_5+x_7 \geq 4$ (or =4) (guards)

   $x_3+x_4+x_5+x_6+x_7 \geq 2$ (forwards)

   $x_2+x_4+x_6 \geq 1$ (center)

2. $(3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + 3x_7) \div 5 \geq 2$, i.e.,
   $3x_1 + 2x_2 + 2x_3 + x_4 + 3x_5 + 3x_6 + 3x_7 \geq 10$ (avg. ball handling)

   $(3x_1 + x_2 + 3x_3 + 3x_4 + 3x_5 + x_6 + 2x_7) \div 5 \geq 2$, i.e.,
   $3x_1 + x_2 + 3x_3 + 3x_4 + 3x_5 + x_6 + 2x_7 \geq 10$ (avg. shooting)

   $(x_1 + 3x_2 + 2x_3 + 3x_4 + x_5 + 2x_6 + 2x_7) \div 5 \geq 2$, i.e.
   $x_1 + 3x_2 + 2x_3 + 3x_4 + x_5 + 2x_6 + 2x_7 \geq 10$ (avg. rebounding)

5. $x_2 + x_3 \geq 1$ (either 2 or 3 must start)

All x_{ij} = 0 or 1

Question 4.

Question 3, pages 502-503 of the text (manufacturer...)

Define x_1 = units of product 1 produced,
x_2 = units of product 2 produced,
y_1=1 if **any** product 1 is produced; 0 otherwise.
y_2=1 if **any** product 2 is produced; 0 otherwise

Then the problem formulated is as follows (note that this is a *fixed-charge* problem where fixed charges are incurred for each product whenever *any* amount of that product is produced):

Maximize $Z = 2x_1 + 5x_2 - 10y_1 - 20y_2$
st
(1) $3x_1 + 6x_2 \le 120$ (raw material availability)
(2) $x_1 \le \mathbf{M}y_1$ (if y_1=0 then x_1 must be = 0, if y_1=1 then x_1 is limited only by constraint 1)
(3) $x_2 \le \boldsymbol{M}y_2$ (if y_2=0 then x_2 must be = 0, if y_2=1 then x_2 is limited only by constraint 1)
$x_1, x_2 \ge 0$; y_1, y_2 = 0 or 1

__**Note**__: While any "large" value for ***M*** would be okay, it would suffice to use 120÷3=40 for ***M*** in (2) and 120÷6=20 for ***M*** in (3).

Question 5.

Question 13, page 503 of the text (Glueco...)

Define y_j = 1 if production line j is used and 0 otherwise, j=1, 2
x_j = No. of workers assigned to line j, j=1,2
Then the formulation would be to minimize

Minimize $Z = 500x_1 + 900x_2 + 1000y_1 + 2000y_2$ (weekly costs)
st
$20x_1 + 50x_2 \ge 120$
$30x_1 + 35x_2 \ge 150$
$40x_1 + 45x_2 \ge 200$ (production requirements for the 3 glue types)
$x_1 \le 7y_1$
$x_2 \le 7y_2$ (if line j is not used x_j=0, and if it is used then $x_j \le 7$)
$x_1, x_2 \ge 0$ and integer, y_1, y_2 = 0 or 1.