I.E. 2001 OPERATIONS RESEARCH

(Solutions to Assignments 7)

Question 1

1) Minimize
$$Z = 4x_1 + 3x_2$$

st $2x_1 + x_2 \ge -3x_1 + 2x_2$
 $x_1 + x_2 \ge -3x_1 + 2x_2$

 $x_1, x_2 \ge 0$ All inequality constraints are "normal" (\geq for a Min problem) so we can proceed... The dual is

 $2x_1 + x_2 \ge 25$ $-3x_1 + 2x_2 \ge 15$ $x_1 + x_2 \ge 15$

Maximize
$$W = 25y_1 + 15y_2 + 15y_3$$

st $2y_1 - 3y_2 + y_3 \le 4$
 $y_1 + 2y_2 + y_3 \le 3$
 $y_1, y_2, y_3 \ge 0$

2)

Maximize
$$Z = -2x_1 + x_2 - 4x_3 + 3x_4$$

st $x_1 + x_2 + 3x_3 + 2x_4 \ge 10$
 $x_1 + x_2 + 3x_3 + 2x_4 \le 40$
 $-x_1 + x_3 - x_4 \le 10$
 $2x_1 + x_2 \le 20$
 $x_1 + 2x_2 + x_3 + 2x_4 = 20$
 $x_2, x_3, x_4 \ge 0; x_1 UNR.$

Before finding the dual ensure that all inequality constraints are "normal" (i.e., \leq for a Max problem)

Maximize
$$Z = -2x_1 + x_2 - 4x_3 + 3x_4$$

st $-x_1 - x_2 - 3x_3 - 2x_4 \le -10$
 $x_1 + x_2 + 3x_3 + 2x_4 \le 40$
 $-x_1 + x_3 - x_4 \le 10$
 $2x_1 + x_2 \le 20$
 $x_1 + 2x_2 + x_3 + 2x_4 = 20$
 $x_2, x_3, x_4 \ge 0; x_1 \text{ UNR.}$
The dual is
Minimize $W = -10y_1 + 40y_2 + 10y_3 + 20y_4 + 20y_5$
st $-y_1 + y_2 - y_3 + 2y_4 + y_5 = -2$ ("=" since it corresponds to a UNR variable x_1)
 $-y_1 + y_2 - y_3 + 2y_4 + y_5 \ge -4$
 $-3y_1 + 3y_2 + y_3 - y_5 \ge -4$
 $-2y_1 + 2y_2 - y_3 - 2y_5 \ge 3$
 $y_1, y_2, y_3, y_4 \ge 0, y_5 \text{ UNR.}$ ("UNR" since it corresponds to an "=" constr.)

Question 2

i)

Maximize $Z = 3x_1 + 2x_2$ Dual isMinimize $W = 20y_1 + 16y_2$ st $5x_1 + 4x_2 \le 20$ st $5y_1 + 2y_2 \ge 3$ $2x_1 + 4x_2 \le 16$ $4y_1 + 4y_2 \ge 2$ $x_1, x_2 \ge 0$ $y_1, y_2 \ge 0$

The optimum dual solution is y_1 *=0.6, y_2 *=0, W*=12 (from LINDO). By the strong duality theorem, since the dual is feasible and has a finite optimum, so does the original (primal). Moreover, the optimum value of the primal will also be equal to 12.

Minimize $Z = -3x_1 + 4x_2$ Dual isMaximize $W = 2y_1 - 3y_2$ st $-x_1 + x_2 \ge 2$ st $-y_1 + y_2 \le -3$ $-x_1 - 2x_2 \le 3$ (i.e., $x_1 + 2x_2 \ge -3$) $y_1 + 2y_2 \le 4$ $x_1, x_2 \ge 0$ $y_1, y_2 \ge 0$

When we attempt to solve the dual we find that its optimum value is 8. Therefore as a consequence of the strong duality theorem, the original (primal) problem also has the same optimal value.

iii)

ii)

Maximize $Z = x_1 + x_2$ Dual isMinimize $W = -y_1 - y_2$ st $-x_1 + x_2 \ge 1$ $(x_1 - x_2 \le -1)$ st $y_1 - y_2 \ge 1$ $x_1 - x_2 \ge 1$ $(-x_1 + x_2 \le -1)$ $-y_1 + y_2 \ge 1$ $x_1, x_2 \ge 0$ $y_1, y_2 \ge 0$

When we attempt to solve the dual we find that it is infeasible. Therefore, the original (primal) problem is **either infeasible or unbounded** and has no optimum solution.

Question 3

1. Since the total supply (=580) is less than total demand (=610), add a dummy source (*i*=4) to "supply" the excess demand. Define X_{ij} as the no. of CWT shipped from plant *i* to warehouse *j* for *i*=1,2,3,4 and *j*=1,2,3,4,5. C_{ij} as the cost to ship from plant *i* to warehouse *j* for *i*=1,2,3,4 and *j*=1,2,3,4,5. The LP is:

 $\begin{array}{rcl} \text{Min } 4X_{I1}+5X_{I2}+...+6X_{34}+10X_{35}+0.15X_{41}+0.10X_{42}+0.25X_{43}+0.20X_{44}+0.05X_{45} \\ \text{st} & \leftarrow & (\text{from dummy source}) & \rightarrow \\ & \sum_{j}X_{Ij}=180, \quad \sum_{j}X_{2j}=250, \quad \sum_{j}X_{3j}=150, \quad \sum_{j}X_{4j}=30 \\ & \sum_{i}X_{i1}=120, \quad \sum_{i}X_{i2}=100, \quad \sum_{i}X_{i3}=160, \quad \sum_{i}X_{i4}=80, \quad \sum_{i}X_{i5}=150 \\ & \text{All } X_{ii} \text{ nonnengative.} \end{array}$

The Excel worksheet with the cost-requirements tableau for this is as shown below.

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5	Omaha	13	9	3	6	10						
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Solving the problem using Excel-Solver yields the following optimal shipping plan, with cost = **\$3361.50**

	NY	Chi	Atl	Dal	LA	SUPPLY
Pgh	120	0	60	0	0	180
Memphis	0	100	0	80	70	250
Omaha	0	0	100	0	50	150
Dummy	0	0	0	0	30	30
DEMAND	120	100	160	80	150	

The optimal plan calls for shipments as shown above. Note that the 30 units shipped from plant 4 (the Dummy) to Los Angeles signify that Los Angeles has an unsatisfied demand of 30 CWT since plant 4 is a fictitious source that does not actually exist - each plant ships out all available supply at the plant.

2. If supply at Pittsburgh increases to 230 CWT then total supply = 630, which now exceeds total demand (=610). So we add a dummy destination (*j*=6) to "absorb" the excess supply. Define X_{ij} as the no. of CWT shipped from plant *i* to warehouse *j* for *i*=1,2,3 and *j*=1,2,3,4,5,6, and C_{ij} as the cost to *produce and* ship from plant *i* to warehouse *j* for *i*=1,2,3 and *j*=1,2,3,4,5,6. The LP is:

Question 4

Denneng	of or annes or prote	act j maac at plan	the cost a	na requiremente ta		0.0
-	1	2	3	4 (Dummy)	Supply	
1	31	45	38	0	4,000	
2	29	41	35	0	6,000	
3	32	46	40	0	4,000	
4	28	42	Μ	0	6,000	
5	29	43	Μ	0	10,000	
Demand	6000	10,000	8,000	6,000	30,000	

Define X_{ij} = no. of units of product *j* made at plant *i*. The cost and requirements table is shown below:

The Excel worksheet is shown below:

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2		29	41		35	0	6000			
3		32	46		40	0	4000			
4		28	42		1000	0	6000			
5		29	43		1000	0	10000			
S	upply	6000	10000	0	8000	6000	30000			
1			Distri	bution Matrix	-					
		1	2		3	Dummy	Demand			
1							=SUM(B12:E12)			
2							=SUM(B14:E14)			
4							=SUM(B15:E15)			
5							=SUM(B16:E16)			
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The resulting optimal plan is as follows:

	1	2	3	4 (Dummy)	Supply
1	0	0	2000	2000	4,000
2	0	0	6000	0	6,000
3	0	0	0	4000	4,000
4	6000	0	0	0	6,000
5	0	10000	0	0	10,000
Demand	6000	10,000	8,000	6,000	30,000

The optimal value of the objective, **Z=\$884,000**. Note that the problem has multiple optima (for instance an alternative solution has X_{51} =6000 X_{42} =6000, X_{52} =4000; other values are the same).

Question 5:

The cost and requirements matrix is identical to the tableau given, with the exception that we have:

- (1) cost of M for all paths that do not exist and 0 for all paths from a node to itself,
- (2) supplies of 300 at all junctions and warehouses and 375, 425 and 400 at the three canneries, and
- (3) demands of 300 at all canneries and junctions and 380, 365, 370 and 385 at the four warehouses.

The solution with a cost of \$145,175 is shown below:

	C1	C2	C3	J1	J2	J3	J4	J5	W1	W2	W3	W4		
C1	300	0	0	0	75	0	0	0	0	0	0	0	375	75
C2	0	300	0	0	0	0	0	0	80	45	0	0	425	125
C3	0	0	300	0	0	0	0	30	0	0	70	0	400	100
J1	0	0	0	300	0	0	0	0	0	0	0	0	300	
J2	0	0	0	0	225	0	0	0	0	75	0	0	300	
J3	0	0	0	0	0	300	0	0	0	0	0	0	300	
J4	0	0	0	0	0	0	300	0	0	0	0	0	300	
J5	0	0	0	0	0	0	0	270	0	0	0	30	300	
W1	0	0	0	0	0	0	0	0	300	0	0	0	300	
W2	0	0	0	0	0	0	0	0	0	245	0	55	300	
W3	0	0	0	0	0	0	0	0	0	0	300	0	300	
W4	0	0	0	0	0	0	0	0	0	0	0	300	300	
	300	300	300	300	300	300	300	300	380	365	370	385		
									80	65	70	85		300

