I.E.2001 OPERATIONS RESEARCH

(*Homework Assignment 7: Due Thursday, Mar. 19, 2020*)

Question 1:

Construct the duals for each the following LP problems – do not try solving anything!

$$\begin{array}{ll} \text{Minimize} & Z = 4x_1 + 3x_2 \\ \text{st} & 2x_1 + x_2 \geq 25 \\ & -3x_1 + 2x_2 \geq 15 \\ & x_1 + x_2 \geq 15 \\ & x_1, x_2 \geq 0 \end{array}$$

$$\begin{array}{ll} \text{Maximize} & Z = -2x_1 + x_2 - 4x_3 + 3x_4 \\ \text{st} & x_1 + x_2 + 3x_3 + 2x_4 \geq 10 \\ & x_1 + x_2 + 3x_3 + 2x_4 \leq 40 \\ & -x_1 + x_3 - x_4 \leq 10 \\ & 2x_1 + x_2 \leq 20 \\ & x_1 + 2x_2 + x_3 + 2x_4 = 20 \\ & x_2, x_3, x_4 \geq 0;\ x_1 \text{ UNR} \end{array}$$

Question 2: Consider the following (primal) linear programs:

i)

$$\begin{array}{ll} \text{Maximize} & Z = 3x_1 + 2x_2 \\ \text{st} & 5x_1 + 4x_2 \leq 20 \\ & 2x_1 + 4x_2 \leq 16 \\ & x_1, x_2 \geq 0 \end{array}$$

ii)

$$\begin{array}{ll} \text{Minimize} & Z = -3x_1 + 4x_2 \\ \text{st} & -x_1 + x_2 \geq 2 \\ & -x_1 - 2x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{array}$$

iii)

$$\begin{array}{ll} \text{Maximize} & Z = x_1 + x_2 \\ \text{st} & -x_1 + x_2 \geq 1 \\ & x_1 - x_2 \geq 1 \\ & x_1, x_2 \geq 0 \end{array}$$

For each of the above problems

1) Write out the corresponding dual problem
2) Solve the **dual** (either graphically or using `LINDO` or `Excel-Solver`)
3) Based on your solution and duality theory, what can you say about the **optimal value** of the original (**primal**) problem?

Question 3: A company that makes and distributes cookies, cakes and crackers has three plants that produce and ship its products out to five regional warehouses for distribution to retail outlets. For distribution purposes, the company uses CWT (1 CWT = 100 lbs.) as the unit of measurement. The plants which are located at Pittsburgh, Memphis and Omaha have monthly supplies of 180, 250 and 150 CWT respectively, while the monthly demands at the warehouses which are located at Newark, Chicago, Atlanta, Dallas and Los Angeles are 120, 100, 160, 80 and 150 CWT respectively. The unit freight costs between all plant-warehouse pairs (in $/CWT) are given below:

	Newark	Chicago	Atlanta	Dallas	Los Angeles
Pittsburgh	4	6	5	12	19
Memphis	10	4	8	5	14
Omaha	13	9	3	6	10

The company incurs a penalty at each warehouse that does not have all of its monthly demand satisfied and these are given by \$0.15, \$0.10, \$0.25, \$0.20 and \$0.05 per CWT of unsatisfied demand at Newark, Chicago, Atlanta, Dallas and Los Angeles respectively. On the other hand, any unused supply at Pittsburgh, Memphis and Omaha incur storage costs of \$0.15, \$0.20 and \$0.10 per CWT respectively.

1. Formulate this as a **<u>balanced</u>** transportation problem. Then use `Excel Solver` to solve the LP. State **in words** what your plan is.
2. Suppose supply at Pittsburgh increases to 230 CWT. Further, the unit production costs vary at the plants due to differences in labor costs, and are \$0.90, \$0.75 and \$0.80 per CWT at Pittsburgh, Memphis and Omaha respectively. Reformulate the problem as a **<u>balanced</u>** transportation problem and find the new solution using `Excel Solver`.

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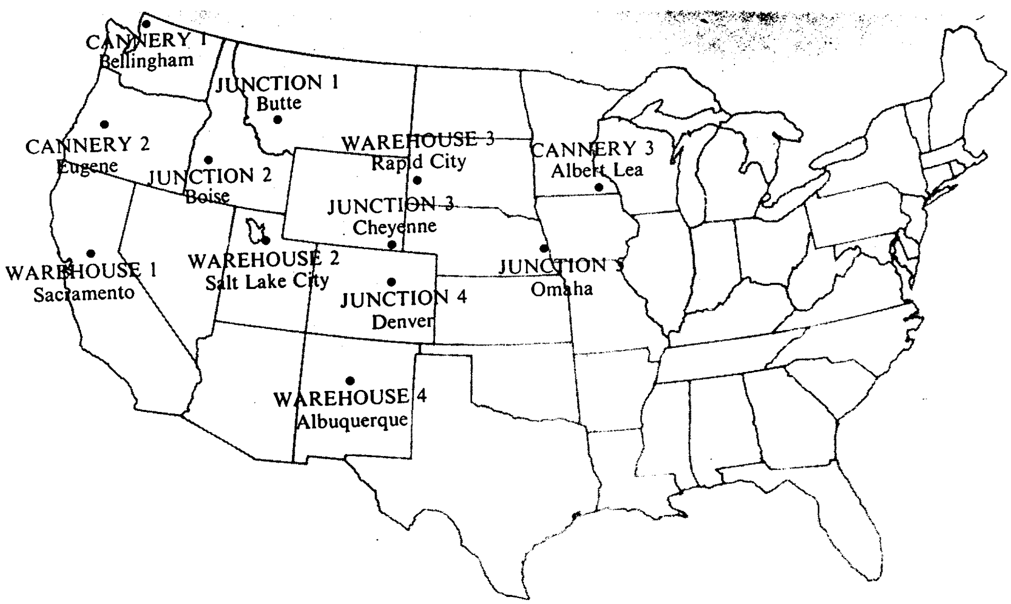
Question 4: A corporation has decided to produce three new products. Five existing plants are being considered as production centers. The unit manufacturing cost of the first product would be \$31, \$29, \$32, \$28, and \$29 in Plants 1, 2, 3, 4 and 5 respectively. The unit manufacturing costs for the second product would be \$45, \$41, \$46, \$42, and \$43 in Plants 1, 2, 3, 4 and 5 respectively. The unit manufacturing costs for the third product would be \$38, \$35, and \$40 at Plants 1, 2 and 3 respectively; Plants 4 and 5 do not have the technology to produce this third product. Sales forecasts indicate that 6000, 10000 and 8000 units of Products 1, 2 and 3 respectively should be produced per day. Plants 1, 2, 3, 4 and 5 have the capacity to produce 4000, 6000, 4000, 6000 and 10,000 total units daily, respectively, regardless of the product or combination of products involved. Assume that any plant having the capability and capacity to produce them can produce any combination of the products. Management wishes to know how to allocate the new products to the plants so as to minimize total manufacturing costs.

1. Construct the appropriate cost and requirements table for a formulation as a balanced transportation problem. What is an appropriate definition for X_{ij}?
2. Use `Excel Solver` to find the optimal production plan.

Question 5

One of the main products of a food company is canned peas. The peas are prepared at three canneries in Bellingham, WA, Eugene, OR and Albert Lea, MN. This season these three canneries will respectively produce 75, 125 and 100 truckloads of canned peas. The peas will need to be sent from these locations to four warehouses in Sacramento, CA, Salt Lake City, UT, Rapid City, SD, and Albuquerque, NM, where the demands are expected to be 80, 65, 70 and 85 truckloads respectively. The shipments will be made by using a network of different trucking companies with many truckloads being transferred to another truck along the way. These transfers can be made at other intermediate canneries, warehouses or at any of five junctions located at Butte, MT, Boise, ID, Cheyenne, WY, Denver, CO and Omaha, NE. The entire network is shown in the figure on the next page, and the table lists the shipping cost per truckload, where a dash (-) indicates that direct shipment is not possible. There are many shipment options. For example, from cannery 1 to warehouse 4, one option is a direct shipment at a cost of \$871. However, a cheaper option is to ship to junction 2 ($286), then transfer here to a truck bound for warehouse 2 ($207), then transfer here to a truck bound for warehouse 4 ($341) at a total cost of $286+$207+$341 = $834. Formulate this transshipment problem as a transportation problem by constructing the appropriate cost and requirements table and solve it using Excel SOLVER.



Location of canneries, warehouses, and junctions for the P & T Co.

Independent Trucking Data for P & T Co.

Shipping Cost Per Truckload

From \ To		*Cannery* 1	2	3	*Junction* 1	2	3	4	5	*Warehouse* 1	2	3	4	Output
Cannery	1		$146	—	$324	$286	—	—	—	$452	$505	—	$871	75
	2	$146		—	$373	$212	$570	$609	—	$335	$407	$688	$784	125
	3	—	—		$658	—	$405	$419	$158	—	$685	$359	$673	100
Junction	1	$322	$371	$656		$262	$398	$430	—	$503	$234	$329	—	
	2	$284	$210	—	$262		$406	$421	$644	$305	$207	$464	$558	
	3	—	$569	$403	$398	$406		$ 81	$272	$597	$253	$171	$282	
	4	—	$608	$418	$431	$422	$ 81		$287	$613	$280	$236	$229	
	5	—	—	$158	—	$647	$274	$288		$831	$501	$293	$482	
Warehouse	1	$453	$336	—	$505	$307	$599	$615	$831		$359	$706	$587	
	2	$505	$407	$683	$235	$208	$254	$281	$500	$357		$362	$341	
	3	—	$687	$357	$329	$464	$171	$236	$290	$705	$362		$457	
	4	$868	$781	$670	—	$558	$282	$229	$480	$587	$340	$457		
Allocation										80	65	70	85	