

I.E.2001 OPERATIONS RESEARCH
(Homework Assignment No. 4: Due Feb. 12, 2012)

I. Restate the following LP so that it is in the standard form - do NOT try solving it...

$$\begin{aligned} \text{Max} \quad & -3X_1 + X_2 - 2X_3 + X_4 \\ \text{st} \quad & -4X_1 + X_2 + X_3 \geq 4 \\ & 3X_1 - X_2 + 2X_3 \leq -6 \\ & X_2 + 4X_3 - X_4 = -1 \\ & 2X_1 - X_2 = -X_3 \\ & X_1 \text{ unrestricted; } X_2, X_3, X_4 \geq 0 \end{aligned}$$

II. Consider a linear program whose four technological constraints are

- 1) $X_1 + X_2 \leq 16$
- 2) $X_1 + 2X_2 \leq 28$
- 3) $12X_1 + 7X_2 \leq 168$
- 4) $X_2 \geq 2$

Also assume that $X_1, X_2 \geq 0$

- a) Graph (neatly...) the feasible region.
- b) Calculate the total number of basic solutions that could exist. How many of these actually exist? Explain any discrepancy.
- c) How many basic feasible solutions does the problem have?
- d) Mark the basic (both infeasible and feasible) solutions on the graph, and calculate the values of all variables (including slacks) at each of these basic solutions.

III. The simplex method was used to Maximize $Z = 4X_1 + 4X_2$ subject to the four constraints of Question (II) listed above, and the following tableau was obtained at some intermediate iteration. Show that the optimum value and solution are found at the next iteration.

Extra: Is there anything special or unusual about the optimal tableau?!

Basic	Z	X_1	X_2	S_1	S_2	S_3	S_4	RHS
Z	1	-2	0	0	2	0	0	56
S_1	0	0.5	0	1	-0.5	0	0	2
S_4	0	0.5	0	0	0.5	0	1	12
S_3	0	8.5	0	0	-3.5	1	0	70
X_2	0	0.5	1	0	0.5	0	0	14

IV. Use the simplex method to solve the following LP

$$\begin{aligned} \text{Min} \quad & -5X_1 - 4X_2 + X_3 - 3X_4 \\ \text{st} \quad & 3X_1 + 2X_2 - 3X_3 + X_4 \leq 24 \\ & 3X_1 + 3X_2 + X_3 + 3X_4 \leq 36 \\ & -X_1 + 2X_2 + 5X_3 \leq 3 \\ & X_1, X_2, X_3, X_4 \geq 0 \end{aligned}$$