I.E. 2001 OPERATIONS RESEARCH (Spring 2012)

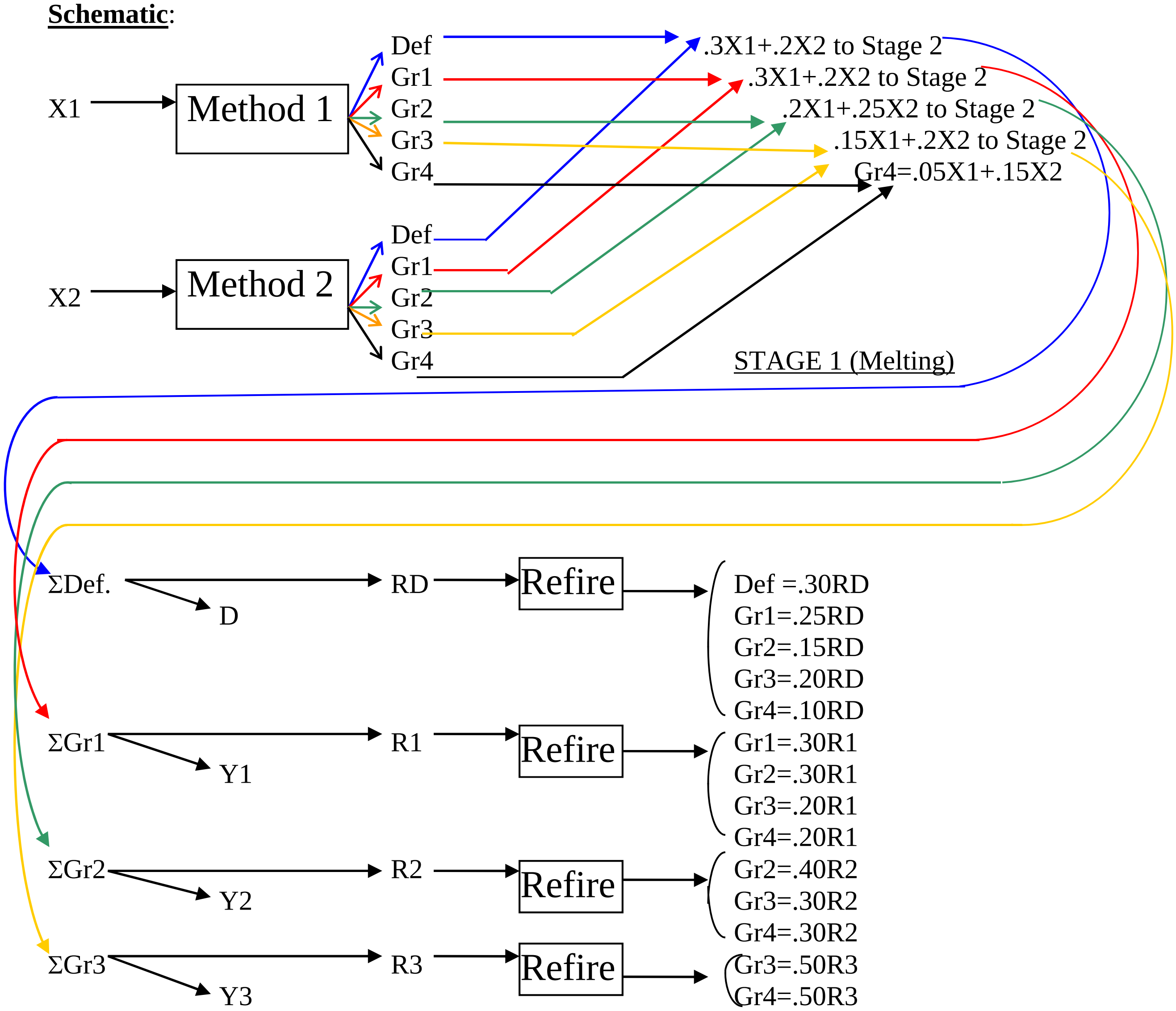
(Solutions to Assignment 2)

Question 51, p. 122

Let

X1 = number of transistors' worth of germanium melted by method 1
X2 = number of transistors' worth of germanium melted by method 2
RD = number of defective transistors' worth of germanium refired
R1 = number of grade 1 transistors' worth of germanium refired
R2 = number of grade 2 transistors' worth of germanium refired
R3 = number of grade 3 transistors' worth of germanium refired
D = number of defective transistors' worth of germanium not refired
Y1 = number of grade 1 transistors' worth of germanium not refired
Y2 = number of grade 2 transistors' worth of germanium not refired
Y3 = number of grade 3 transistors' worth of germanium not refired

Schematic:



STAGE 2 (possible refiring)

Then the appropriate LP is

Min z = (50X1 + 70X2) + (25RD + 25R1 + 25R2 + 25R3) Costs
(Melting) (Refiring)

s.t.

0.3X1 + 0.2X2 - RD -D = 0
0.3X1 + 0.2X2 - R1 - Y1 = 0
0.2X1 + 0.25X2 - R2 - Y2 = 0
0.15X1 + 0.20X2 -R3 - Y3 = 0 **Material Balance - First Stage (Melting)**

0.25RD + 0.30R1 + Y1 ≥ 3000 Grade 1 demand
0.15RD + 0.30R1 + 0.40R2 + Y2 ≥ 3000 Grade 2 demand
0.20RD + 0.20R1 + 0.30R2 + 0.50R3 + Y3 ≥ 2000 Grade 3 demand
0.05X1 + 0.15X2 + 0.10RD + 0.20R1 + 0.30R2 + 0.50R3 ≥ 1000 Grade 4 demand
Second Stage (Refire)

X1 + X2 + RD + R1 + R2 + R3 ≤ 20000 (capacity)

All variables ≥ 0

Question 52, p. 122

Let

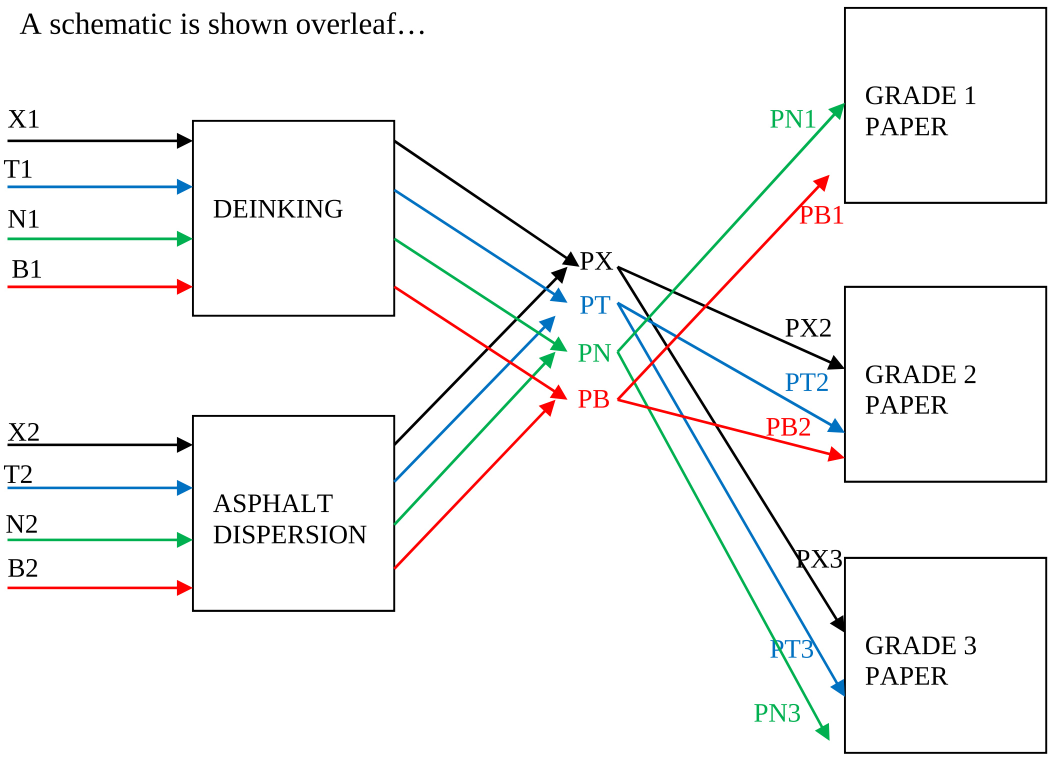
X1 = Tons of purchased boxboard sent through deinking
T1 = Tons of purchased tissue sent through deinking
N1 = Tons of purchased newsprint sent through deinking
B1 = Tons of purchased book paper sent through deinking

X2 = Tons of purchased boxboard sent through asphalt dispersion
T2 = Tons of purchased tissue sent through asphalt dispersion
N2 = Tons of purchased newsprint sent through asphalt dispersion
B2 = Tons of purchased book paper sent through asphalt dispersion

PX = Tons of available boxboard pulp
PT = Tons of available tissue pulp
PN = Tons of available newspaper pulp
PB = Tons of available book paper pulp

PX*i* = Tons of boxboard pulp used for grade *i* paper, *i*=2,3
PT*i* = Tons of tissue pulp used for grade *i* paper, *i*=2,3
PN*i* = Tons of newspaper pulp used for grade *i* paper, *i*=1,3
PB*i* = Tons of book paper pulp used for grade *i* paper, *i*=1,2

A schematic is shown overleaf…



Min Z = 5(X1 + X2) +6(T1+T2)+ 8(N1+N2) + 10(B1+B2) (raw materials cost)
+ 20(X1+T1+N1+B1) + 15(X2+T2+N2+B2) (processing cost)

st

0.9*0.15*X1 + 0.8*0.15*X2 = PX, i.e. 0.135X1 + 0.12X2 – PX = 0
0.9*0.20*T1 + 0.8*0.20*T2 = PT, i.e. 0.18T1 + 0.16T2 – PT = 0
0.9*0.30*N1 + 0.8*0.30*N2 = PN, i.e. 0.27N1 + 0.24N2 – PN = 0
0.9*0.40*B1 + 0.8*0.40*B2 = PB, i.e. 0.36B1 + 0.32B2 – PB = 0
(material balance for each pulp type)

PX2 + PX3 = PX, i.e. PX2 + PX3 – PX = 0
PT2 + PT3 = PT, i.e. PT2 + PT3 – PT = 0
PN1 + PN3 = PN, i.e. PN2 + PN3 – PN = 0
PB1 + PB2 = PB, i.e. PB2 + PB3 – PB = 0
(pulp used = pulp produced)

PN1 + PB1 ≥ 500
PX2 + PT2 + PB2 ≥ 500
PX3 + PT3 + PPN3 ≥ 600 (pulp requirements for paper grades 1, 2, 3)

X1 + T1 + N1 + B1 ≤ 3000
X2 + T2 + N2 + B2 ≤ 3000 (processing capacity)

All variables nonnegative

NOTE: You could eliminate the pulp balance constraints altogether and also eliminate the four variables PX, PT, PN, PB by replacing these in the first set of constraints with (PX2+PX3), (PT2+PT3), (PN1+PN3), (PB1+PB2), respectively …

Question 4

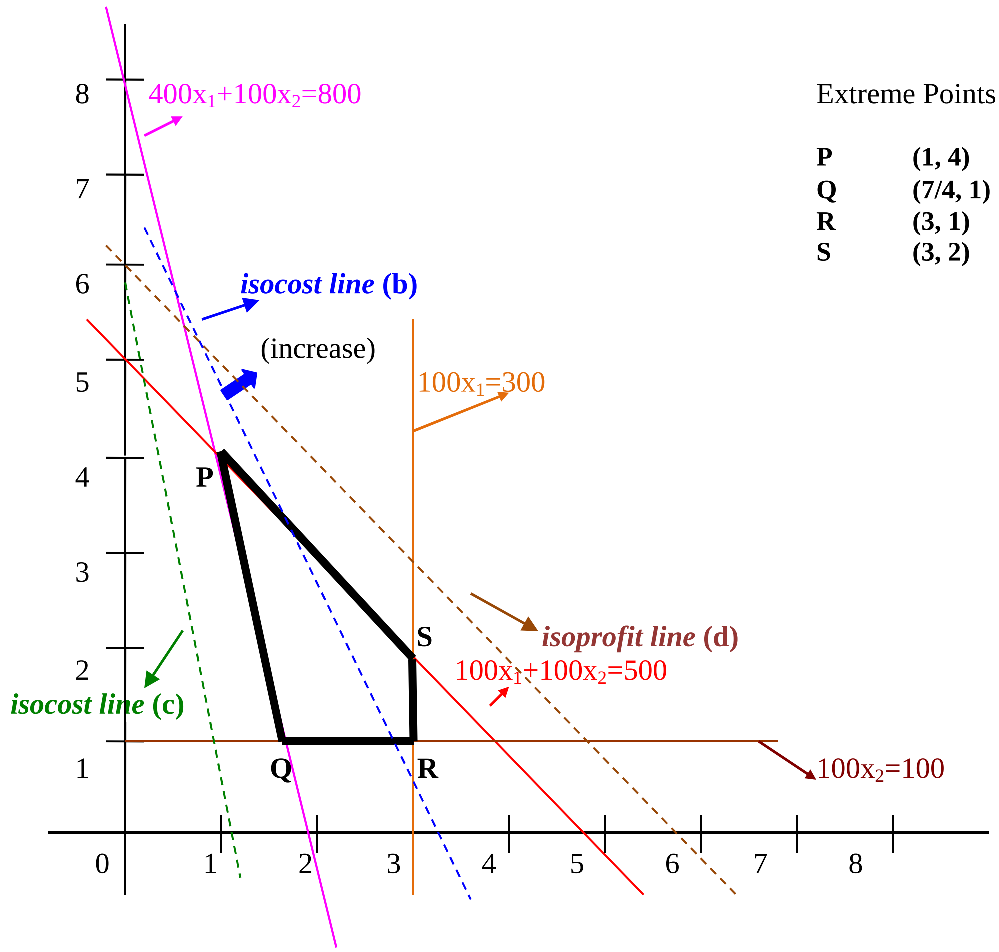
a) Define x_1 = No. of hours that Process 1 is run

x_2 = No. of hours that Process 2 is run.

Then the constraints of the LP are

$400x_1 + 100x_2 \geq 800$	(Chemical A)
$100x_1 + 100x_2 \leq 500$	(Chemical B)
$100x_1 \leq 300$	(Chemical C)
$100x_2 \geq 100$	(Chemical D)
$x_1, x_2, x_3 \geq 0$	(nonnegativity)

The feasible region and the extreme points are as shown below:



b) The objective is to minimize cost, $Z = 4x_1+2x_2$. An **isocost line** is sketched above. The optimum production plan occurs at the extreme point **Q** (minimization) and calls for $\boldsymbol{x_1=7/4}$, $\boldsymbol{x_2=1}$, with a value for the optimum objective given by **Z=9**. The constraints corresponding to Chemicals A and D are active. For the constraint corresponding to B, there is a slack of 500-(7/4*100+2*100) = 125 lbs, and for the constraint corresponding to C, there is a slack of 300-(7/4*100) = 125 lbs. Constraints corresponding to A and D are active and hence have no excess.

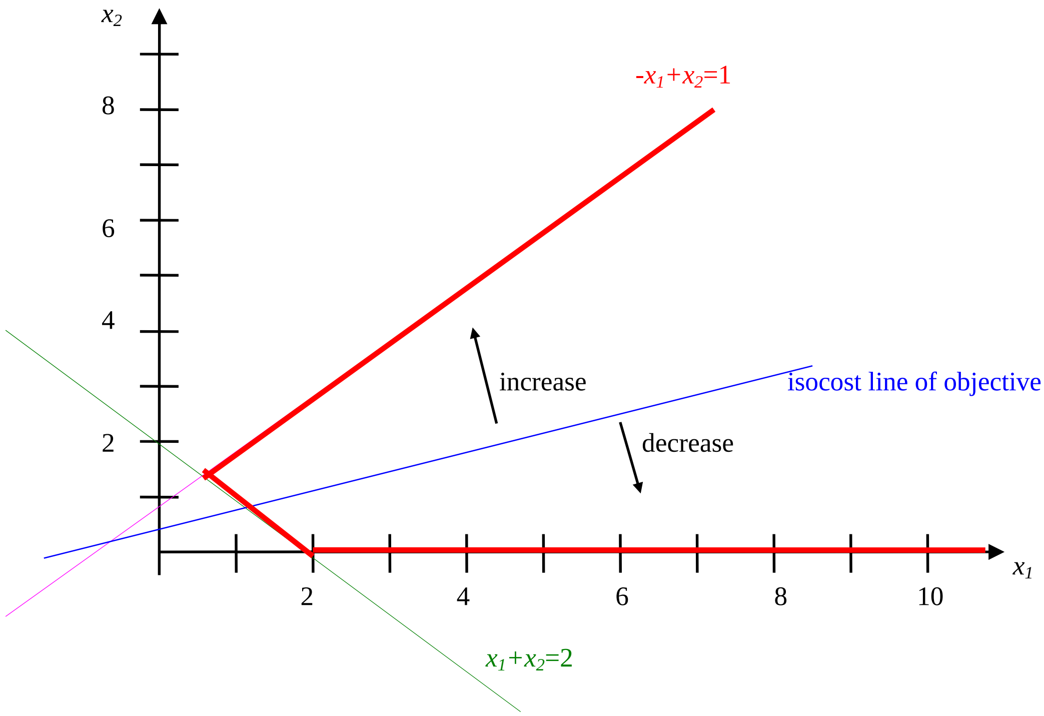
c) In this case, the objective is to minimize $Z = 5x_1+x_2$. The **isocost line** is sketched above. Here the optimum production plan is at the extreme points **P** with $x_1=1$, $x_2=4$ and a value for the optimum objective given by Z=9. The constraints corresponding to Chemicals A and B are active. For the constraint corresponding to C, there is a slack of 300-(1*100) = 200 lbs, and for the constraint corresponding to D, there is a slack of (4*100)-100 = 300 lbs. Constraints corresponding to A and B are active and hence have no excess.

d) In this case we would want to try and maximize profits. The total amounts of chemicals A, B, C and D that are produced by x_1 hours of Process 1 and x_2 hours of Process 2 are given (respectively) by $400x_1+100x_2$, $100x_1+100x_2$, $100x_1$ and $100x_2$. Assuming that everything that is produced can be sold, the revenues from these yield a total of $0.01(400x_1+100x_2) + 0.05(100x_1+100x_2) + 0.05(100x_1) + 0.04(100x_2) = 14x_1 + 10x_2$ dollars. Subtracting the production cost of $5x_1+x_2$ dollars yields a profit function of $Z = 9x_1+9x_2$ to be maximized. An **isoprofit line** is shown for this and the optimum occurs at all points joining **P** and **S**. There are two optimum extreme points: **P** ($x_1=1$, $x_2=4$) and **S** ($x_1=3$, $x_2=2$). The minimum cost in both cases is given by $45 (= 9*1+9*4 = 9*3+9*2). There are infinitely many optimum solutions to the LP and are given in general by

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 4 \end{bmatrix} + (1-\alpha)\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3-2\alpha \\ 2+2\alpha \end{bmatrix},$$

where α is a constant in the closed interval [0,1]. The objective is given by $9*(3-2\alpha) + 9*(2+2\alpha) = 45$.

Question 4



The feasible region is open and unbounded.

The problem has no optimal solution for minimizing $-2x_1 + 6x_2$, since the slope of the isocost line shown is such that it can be moved in the direction of decrease without ever completely leaving the feasible region: x_1 can be made arbitrarily large while keeping x_2 fixed at some small nonnegative value so that the objective goes to $-\infty$.

If the problem is a maximization of the objective, the problem is still unbounded because the isocost line can also be moved in the direction of increase without ever completely leaving the feasible region: x_2 can be made arbitrarily large while ensuring that $6x_2$ is always larger than $2x_1$ so that the objective goes to ∞.