I.E. 2001 OPERATIONS RESEARCH (Spring 2012)
(Solutions to Assignment 2)

Question 6, p. 114

Let x_i = no. of tons of Alloy $\underline{i}$ used to produce **one** ton of steel, (i=1,2).
Then the LP is:

Min $z = 190x_1 + 200\,x_2$	(Costs)
st	
$x_1 + x_2 = 1$	(by definition)
$.02x_1 + .025x_2 \geq 0.018$	
$.02x_1 + .025x_2 \leq 0.025$	Silicon
$.01x_1 + .015x_2 \geq 0.009$	
$.01x_1 + .015x_2 \leq 0.012$	Nickel
$.03x_1 + .04x_2 \geq 0.032$	
$.03x_1 + .04x_2 \leq 0.035$	Carbon
$42{,}000x_1 + 50{,}000x_2 \geq 45{,}000$	Tensile Strength
$x_1, x_2 \geq 0$	

Note that by our definition, x_1 and x_2 also represent the fraction of alloys 1 and 2 (respectively) in the steel (i.e., $100x_i$ represents the percentage).

Question 17, p. 115

Define OT = No. of tables made of oak
OC = No. of chairs made of oak
PT = No. of tables made of pine
PC = No. of chairs made of pine

Maximize $Z = 40OT + 40PT + 15OC + 15PC$	(Revenues)
st	
$17OT + 5OC \leq 150$	(Oak usage can't exceed oak availability)
$30PT + 13PC \leq 210$	(Pine usage can't exceed pine availability)
$OT, PT, OC, PC \geq 0$	

Question 27, p. 117

Let P_{ij} = Number of trucks of Type i produced in year j, i=1,2 and j=1,2,3
S_{ij} = Number of trucks of Type i sold in year j, i=1,2 and j=1,2,3
I_{ij} = Number of trucks of Type i in inventory at the end of year j, i=1,2; j=1,2,3.

Then an LP formulation is

Max Z = $20{,}000\,(S_{11}+S_{12}+S_{13}) + 17{,}000\,(S_{21}+S_{22}+S_{23})$ (Profit)
(Sales Revenues)
$- 15{,}000\,(P_{11}+P_{12}+P_{13}) - 14{,}000\,(P_{21}+P_{22}+P_{23})$
(Prod. Costs)
$- 2{,}000\,(I_{11}+I_{12}+I_{13}) - 2{,}000\,(I_{21}+I_{22}+I_{23})$
(Inventory Holding costs)

st

$P_{11}-S_{11}-I_{11}=0$
$I_{11}+P_{12}-S_{12}-I_{12}=0$
$I_{12}+P_{13}-S_{13}-I_{13}=0$ (Truck Type 1: Material Balance Constraints)

$P_{21}-S_{21}-I_{21}=0$
$I_{21}+P_{22}-S_{22}-I_{22}=0$
$I_{22}+P_{23}-S_{23}-I_{23}=0$ (Truck Type 2: Material Balance Constraints)

$P_{11}+P_{21} \leq 320$
$P_{12}+P_{22} \leq 320$
$P_{13}+P_{23} \leq 320$ (Capacity Constraints; periods 1, 2, 3)

$(15P_{11}+15P_{12}+15P_{13}+5P_{21}+5P_{22}+5P_{23}) \div (P_{11}+P_{12}+P_{13}+P_{21}+P_{22}+P_{23}) \leq 10$
(average pollution emission per truck =
total grams of pollution produced /total no. of trucks produced ≤10)

i.e., $5P_{11}+5P_{12}+5P_{13}-5P_{21}-5P_{22}-5P_{23} \leq 0$

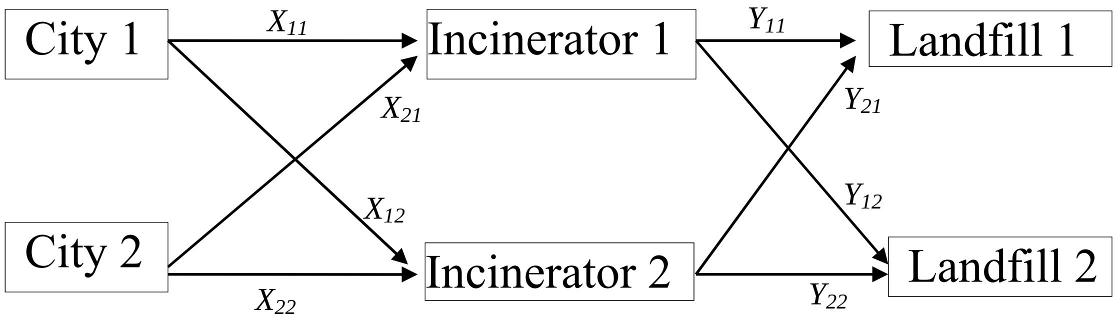
$S_{11}\leq 100$, $S_{12}\leq 200$, $S_{13}\leq 300$, $S_{21}\leq 200$, $S_{22}\leq 100$, $S_{23}\leq 150$
(Max. Sales)

$P_{ij}, I_{ij}, S_{ij} \geq 0$ for all i, j.

NOTE: The fact that $I_{ij}\geq 0$ ensures that sales never exceed what is available…

Question 50, p. 121-122

The problem may be represented schematically as follows:



Let X_{ij} = Tons of City i waste that is sent to Incinerator j; i=1,2; j=1,2.
Y_{ij} = Tons of debris sent from Incinerator i to Landfill j; i=1,2; j=1,2.

Then the appropriate LP is

$$\text{Min } Z = 40(X_{11}+X_{21}) + 30(X_{12}+X_{22}) + 3[30X_{11}+ 5X_{12}+ 36X_{21}+ 42X_{22} + 5Y_{11}+ 8Y_{12}+ 9Y_{21}+ 6Y_{22}]$$

s.t.

$X_{11} + X_{12} = 500$ (CITY 1 WASTE MATERIAL BALANCE)
$X_{21} + X_{22} = 400$ (CITY 2 WASTE MATERIAL BALANCE)

$Y_{11} + Y_{12} = .2(X_{11}+X_{21})$ (INCINERATOR 1 MATERIAL BALANCE)
$Y_{21} + Y_{22} = .2(X_{12}+X_{22})$ (INCINERATOR 2 MATERIAL BALANCE)

$Y_{11} + Y_{21} \leq 200$ (Landfill 1 capacity)
$Y_{12} + Y_{22} \leq 200$ (Landfill 2 capacity)

$X_{11} + X_{21} \leq 500$ (Inc. 1 limitation)
$X_{12} + X_{22} \leq 500$ (Inc. 2 limitation)

All $X_{ij} \geq 0$