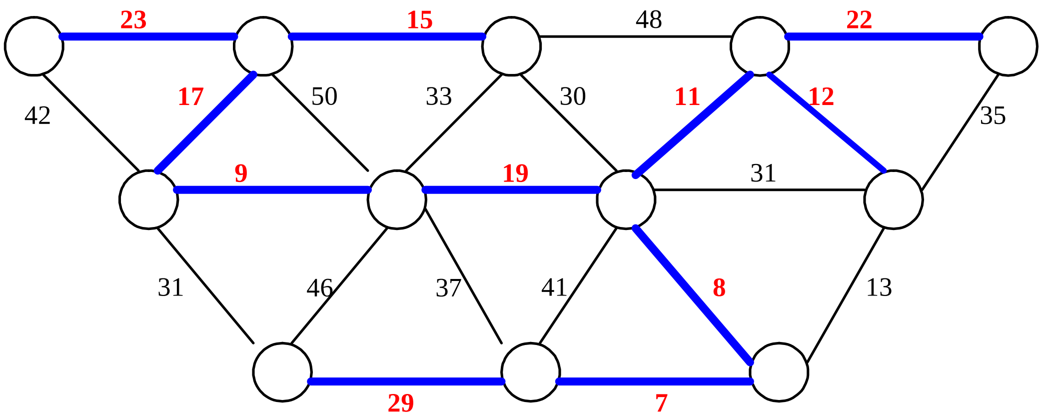
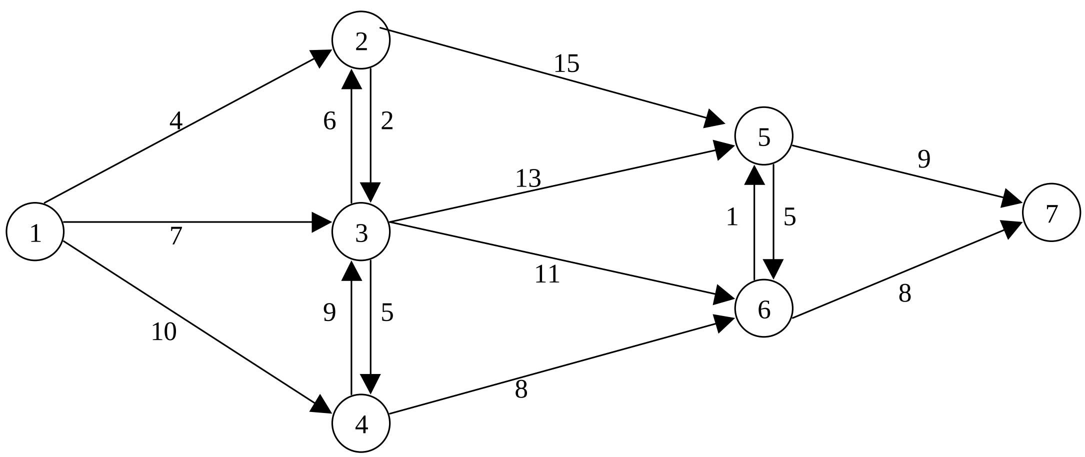
I.E. 2001 OPERATIONS RESEARCH
(Solutions to Assignment 10)

Question 1:

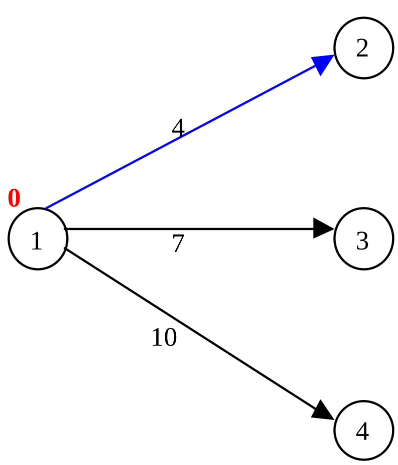


Length of MST = 23+15+22+17+11+12+9+19+8+29+7 **= 172**

Question 2:



<u>Iteration 1</u>



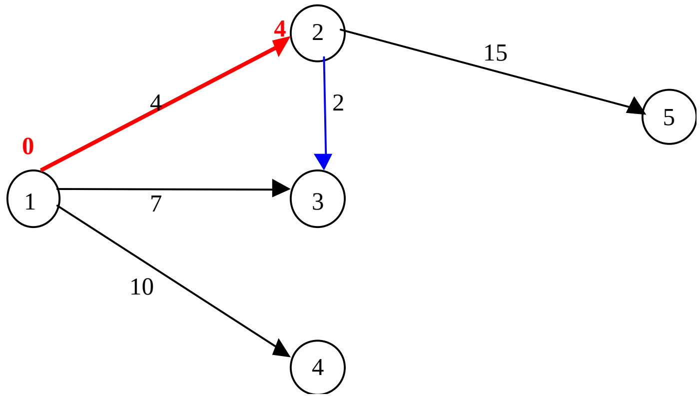
P={1}, T={2,3,4,5,6,7}, Ω={2,3,4}

D_2=Min{0+L_{12}}=**4**
D_3=Min{0+L_{13}}=7
D_4=Min{0+L_{14}}=10

Thus D^*=4 corresponding to D_2

Closest Node is {2}

Iteration 2



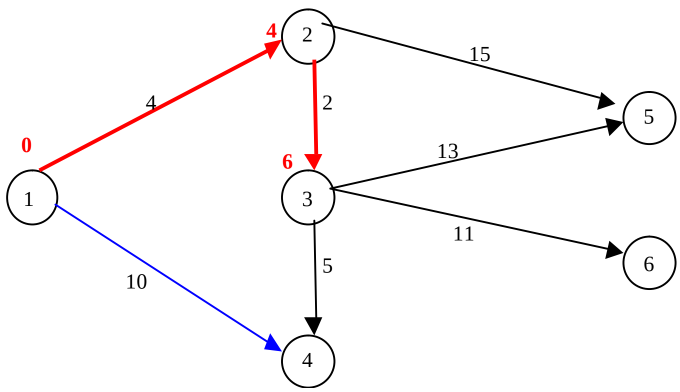
P={1,2}, T={3,4,5,6,7}, Ω={3,4,5}

D_3=Min{0+L_{13}, **4+L_{23}**}=**6**
D_4=Min{0+L_{14}}=10
D_5=Min{4+L_{25}}=19

Thus D^*=3 corresponding to D_3

2nd Closest Node is {3}

Iteration 3



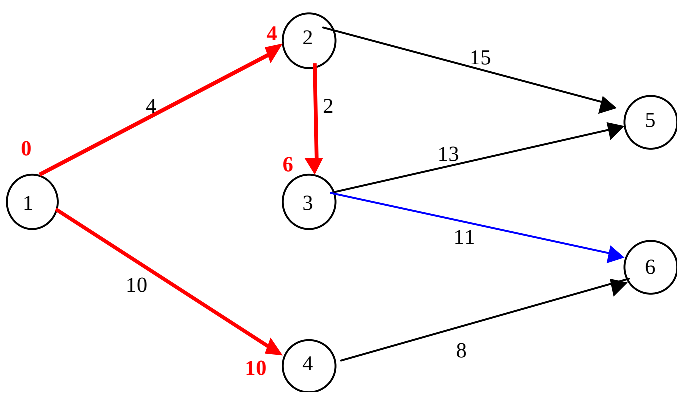
P={1,2,3}, T={4,5,6,7}, Ω={4,5,6}

D_4=Min{**0+L_{14}**, 6+L_{34}}=**10**
D_5=Min{4+L_{25}, 6+L_{35}}=19
D_6=Min{6+L_{36}}=17

Thus D^*=10 corresponding to D_4

3rd Closest Node is {4}

Iteration 4



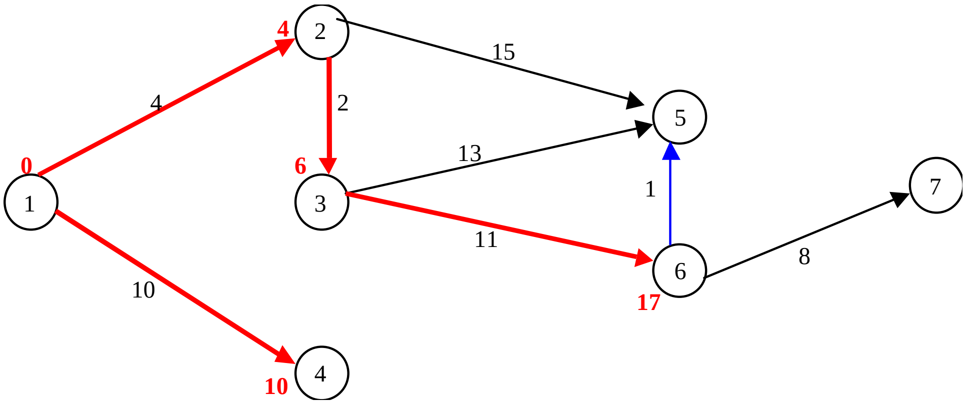
P={1,2,3,4}, T={5,6,7}, Ω={5,6}

D_5=Min{4+L_{25}, 6+L_{35}}=19
D_6=Min{**6+L_{36}**, 10+L_{46}}=**17**

Thus D^*=17 corresponding to D_6

4th Closest Node is {6}

Iteration 5



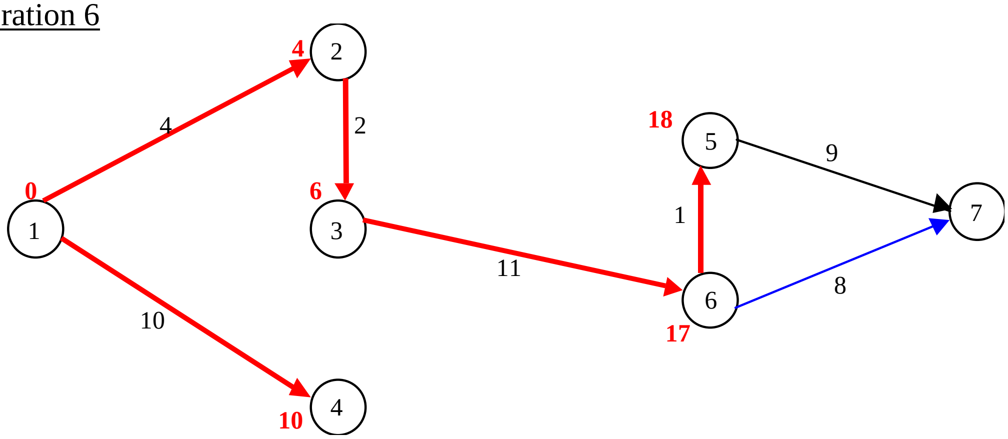
P={1,2,3,4,6}, T={5,7}, Ω={5,7}

D_5=Min{4+L_{25}, 6+L_{35}, **17+L_{65}**}=**18**

D_7=Min{17+L_{67}}=25

Thus D^*=18 corresponding to D_5 and the 5th Closest Node is {5}

Iteration 6

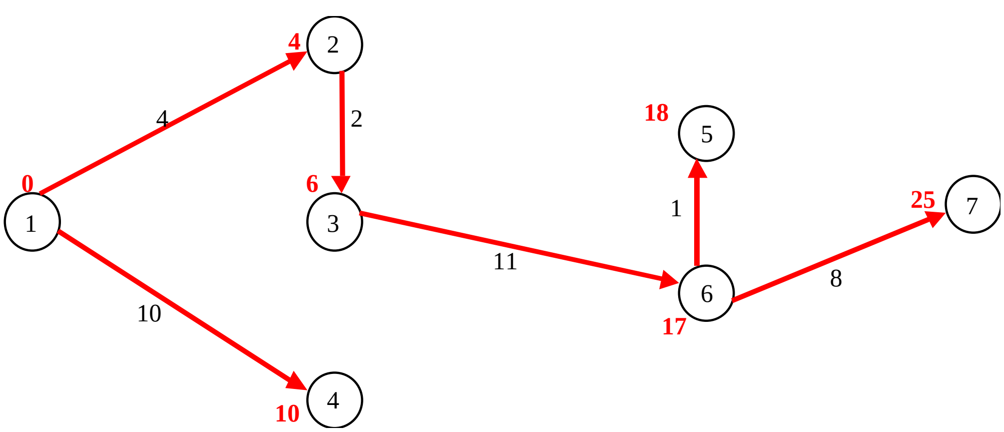


P={1,2,3,4,5,6}, T={7}, Ω={7}

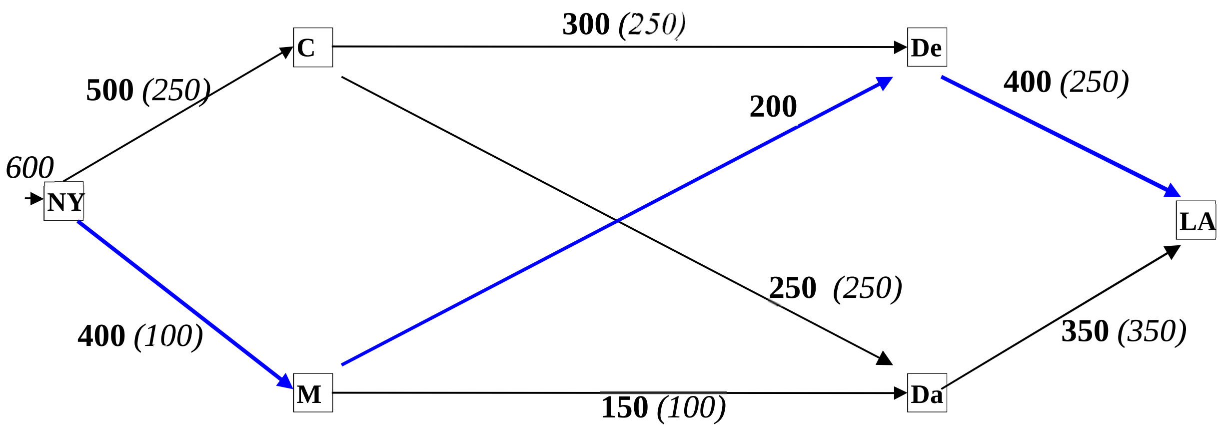
D_7=Min{**17+L_{67}**,18+L_{57}}=**25**

Thus D^*=25 corresponding to D_7 and the 6th Closest Node is {7}

The final shortest path network is as shown below:

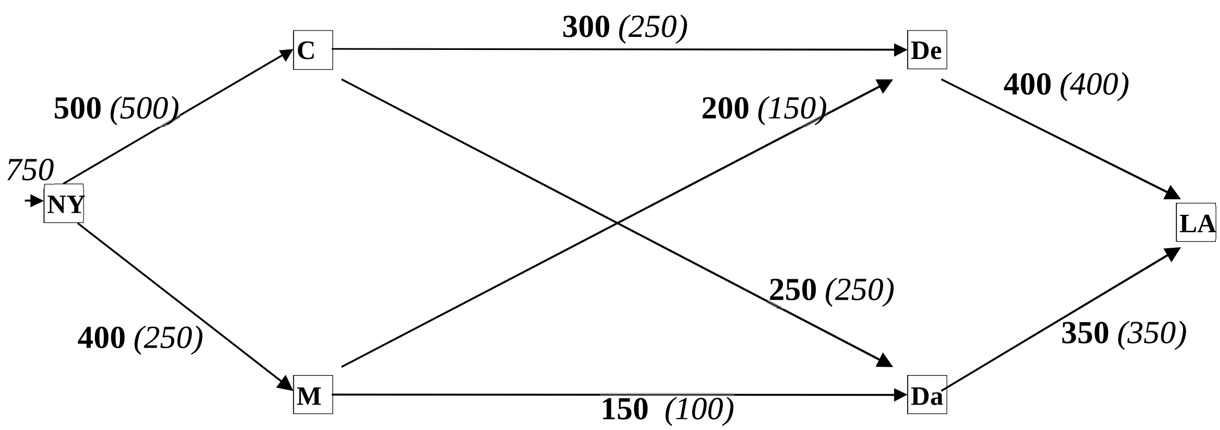


Question 3. (p. 472, No. 2)
The network is drawn below with the current flow:



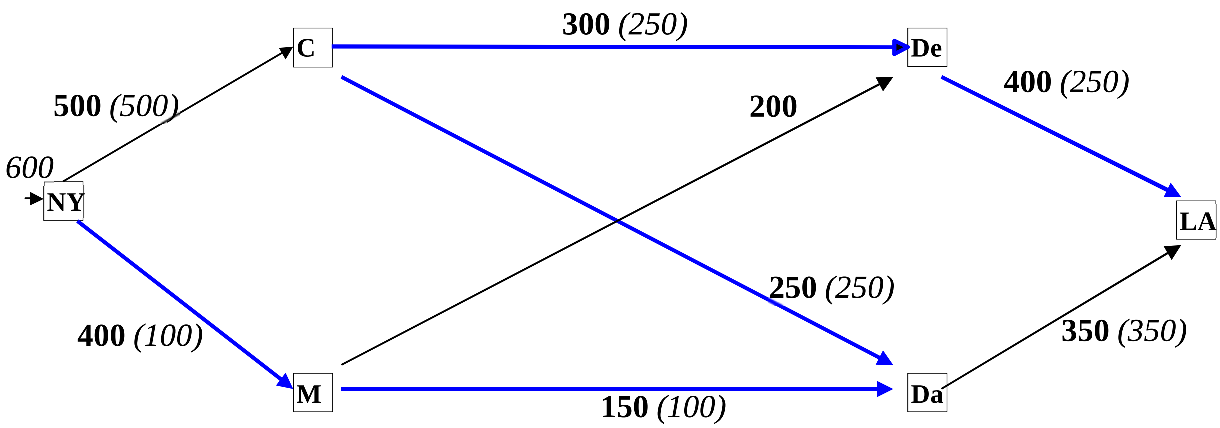
Ford-Fulkerson algorithm (one possible sequence of steps...):

Label the sink via the chain (NY-M)-(M-De)-(De-LA) and add a flow of Min (400-100, 200-0, 400-250) = 150 along each arc on the chain (since they're all forward arcs). This yields the optimal flow of 750 units shown below.

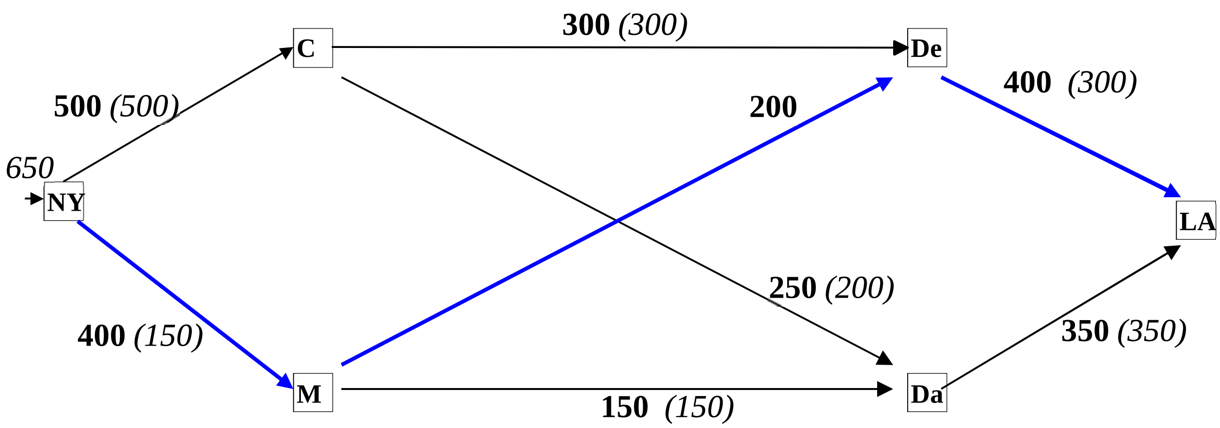


Another way might be as follows:

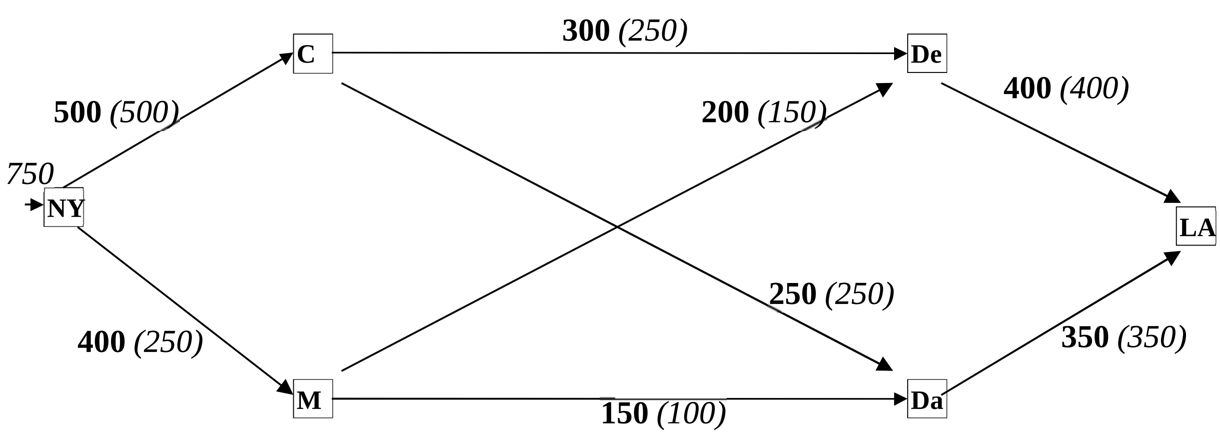
Label the sink via the chain (NY-M)-(M-Da)-(C-Da)-(C-De)-(De-LA) and compute Min (400-100, 150-100, 250-0, 300-250, 400-250) = 50 so that we add 50 along all of the above arcs except the reverse arc (C-Da) where we subtract 50 units (this being a reverse arc).



This gives the flow below:



Now label the sink via the chain (NY-M)-(M-De)-(De-LA) and add a flow of Min (400-150, 200-0, 400-300) = 100 along each arc on the chain (all are forward arcs). This yields the same optimal flow of 750 units as before:

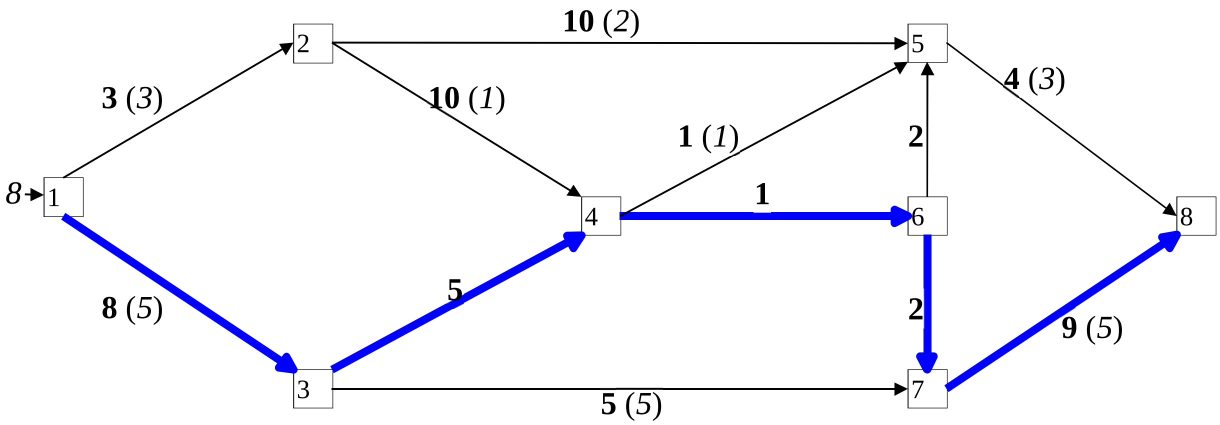


To verify this consider the labeling procedure that gets as far as NY-M, M-Da, C-Da, C-De before we can go no further. So V_2={NY, M, Da, De, C}, V_1={LA} and the cut set is { Da-LA, De-LA}. This cut set has capacity 400+350=750. So this must be the optimal flow.

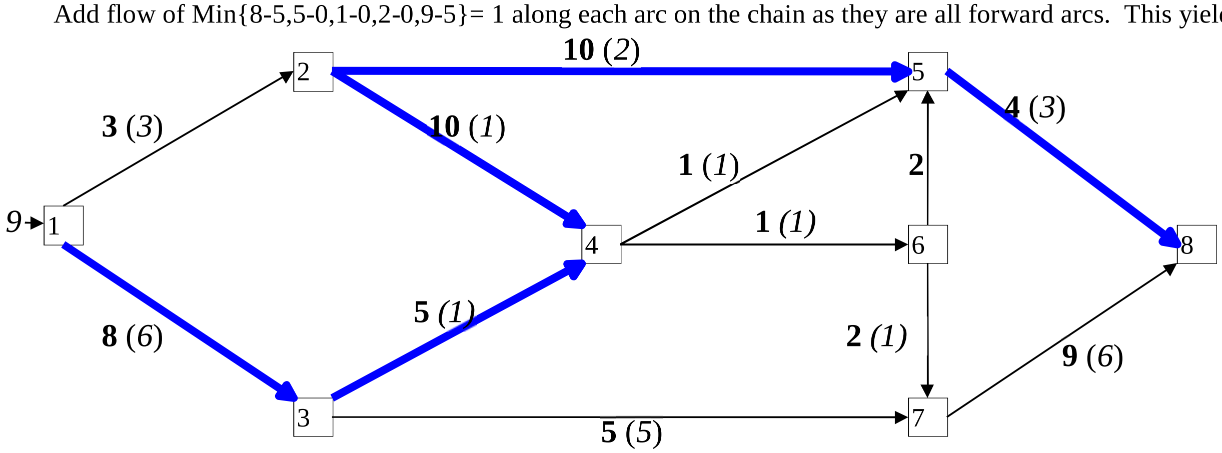
Question 4

Ford-Fulkerson algorithm (two possible sequence of steps...):

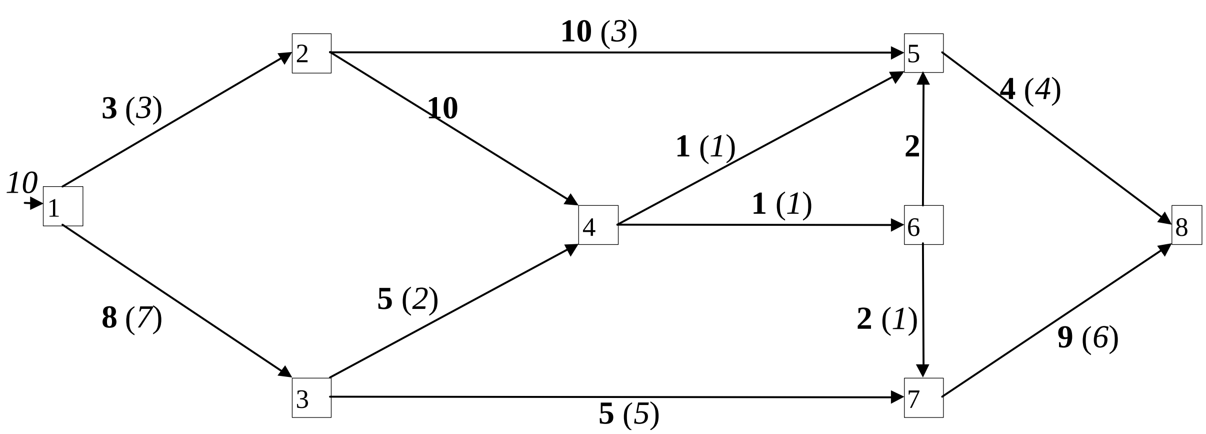
1. In the original graph label the sink via the chain (1-3) - (3-4) - (4-6) - (6-7) - (7-8)



Add flow of Min{8-5,5-0,1-0,2-0,9-5}= 1 along each arc on the chain as they are all forward arcs. This yields:

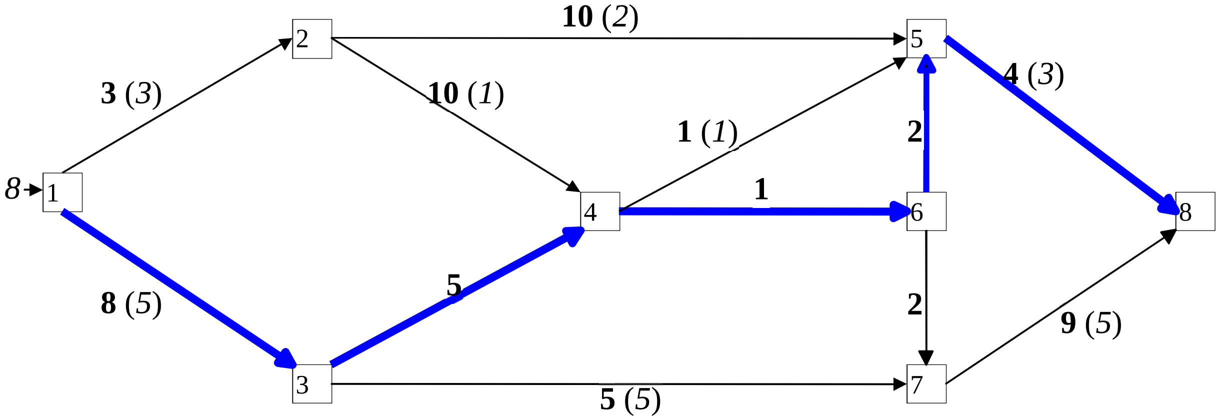


Now label the sink via the chain (1-3) - (3-4) - (2-4) - (2-5) - (5-8). Min{8-6,5-1,1-0,10-2,4-3}=1. So subtract a flow of 1 from arc (2-4) which is a backward arc and add a flow of 1 along all other arcs on the chain. This yields the optimal flow below:

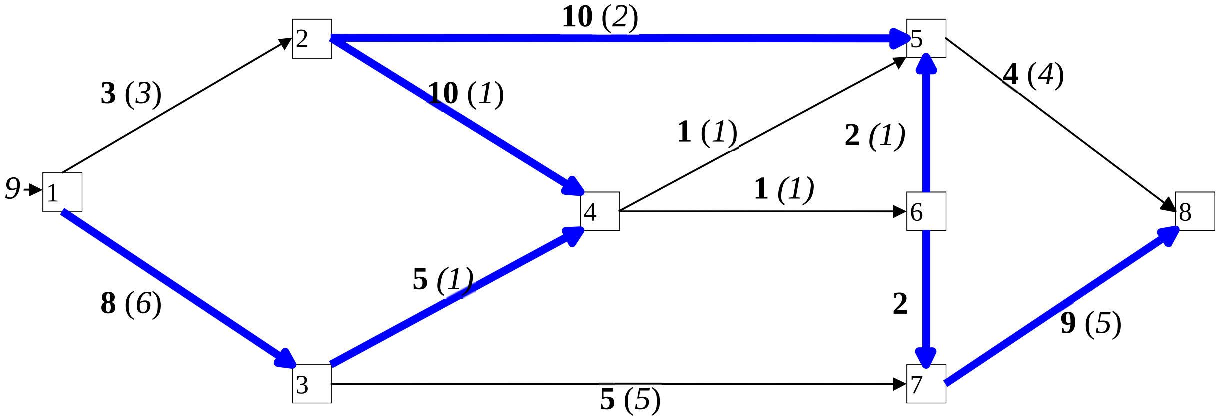


Alternatively,

2. In the original graph, label the sink via the chain (1-3) - (3-4) - (4-6) - (6-5) - (5-8)



Add flow of Min{8-5,5-0,1-0,2-0,4-3}= 1 along each arc on the chain since they are all forward arcs. This yields:



Now label the sink via the chain (1-3)-(3-4)-(2-4)-(2-5)-(6-5)-(6-7)-(7-8); Min{8-6,5-1,1-0,10-2,1-0,2-0,9-5}=1 subtract a flow of 1 from arcs (2-4) and (6-5) which are backward arcs and add a flow of 1 along all other arcs on the chain.

This yields the optimal (maximum) flow shown below (same as in 1.):