HW #2.

(1) The van der Waals equation is given by

$$\frac{P}{RT} = \frac{\rho}{1-b\rho} - \frac{a\rho^2}{RT}$$

Show that there is a region in the T-ρ plane where this equation violates stability... Determine the boundary of this region.

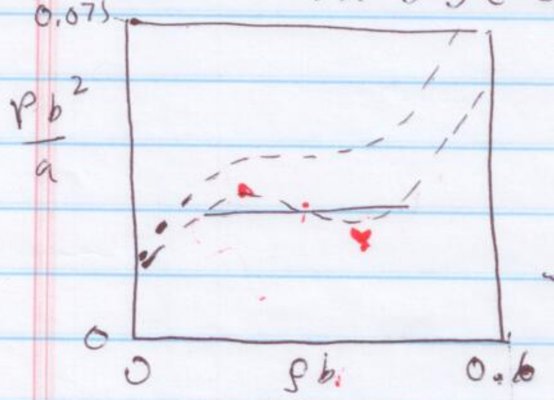
$$\left(\frac{\partial P}{\partial V}\right)_{T,n} = -\frac{V^2}{n}\left[\frac{RT}{(1-b\rho)^2} - 2a\rho\right]$$

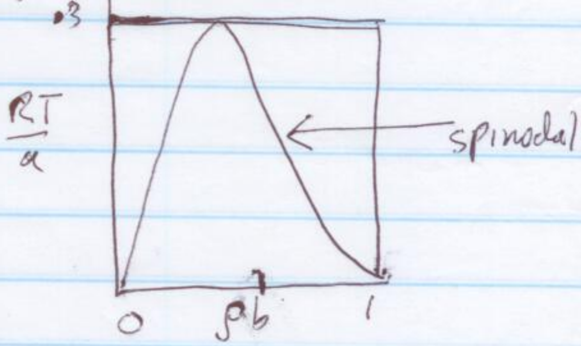
$$-\left(\frac{\partial P}{\partial V}\right)_{T,n} = \frac{V^2}{n}\left[\frac{RT}{(1-b\rho)^2} - 2a\rho\right]$$

this is unstable if $\frac{RT}{(1-b\rho)^2} - 2a\rho < 0$

The boundary of the unstable region is given by

$$TR - 2a\rho(1-b\rho)^2 = 0$$





picture showing two isotherms and maxwell construction

Homework # 2.

Comment on last problem.

There is a typo where the two energies are added. The correct energy is 3375 R. This, in turn causes a small error in T, which should be 321 K.

(2) Suppose two systems have the following equations of state.

$$\frac{1}{T^{(1)}} = \frac{3}{2} R \frac{N^{(1)}}{E^{(1)}} \qquad \frac{1}{T^{(2)}} = \frac{5}{2} \frac{R N^{(2)}}{E^{(2)}}$$

and $N^{(1)} = 2$ and $N^{(2)} = 3$. Suppose further that the two systems are brought into contact and heat can flow between them and the total energy is 2.5×10^3 J. What is the internal energy of each system once equilibrium is achieved?

$$E^{(1)} + E^{(2)} = 2.5 \times 10^3 \text{ J}$$

$$\frac{3}{2} \frac{R(2)}{E_1^{(1)}} = \frac{5}{2} \frac{R(3)}{E_2^{(2)}} \longrightarrow 3E_2^{(2)} = \frac{15}{2} E^{(1)} \longrightarrow E^{(2)} = \frac{5}{2} E^{(1)}$$

$$\frac{7}{2} E^{(1)} = 2.5 \times 10^3 \text{ J} \Rightarrow E^{(1)} = 714 \text{ J}$$

$$E^{(2)} = 1786 \text{ J}$$

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(3) Consider the same two systems as in problem (2) but suppose system 1 starts at $T^{(1)} = 250$K and system 2 starts at $T^{(2)} = 350$K. What is the temperature after equilibration?

$$E^{(1)} = \frac{3}{2} 2R(250) = 750R$$

$$E^{(2)} = \frac{5}{2} 3R(350) = 2625R$$

$$E^{(1)} + E^{(2)} = 3405R$$

$\frac{3}{2} 2RT + \frac{5}{2} 3RT = 3405R$ at equilibrium.

$T = 304$K.