HW # 2 Answers

1. $\psi_I = A\sin kx$ $\qquad k = \sqrt{2mE}/\hbar$

$\psi_{II} = Be^{-\kappa x} + Ce^{\kappa x}$ $\qquad \kappa = \sqrt{2m(V_0 - E)}/\hbar$

$\psi_{III} = De^{ikx} + Ee^{-ikx}$

Note why we have two terms in Region II

Note $|D| = |E|$. If we shoot in a particle from the right, it must be reflected back.

So we can write $D = -e^{2i\delta}E$ {the reason for the "2" will become clear below}

$$\psi_{III} = E\left(-e^{2i\delta}e^{ikx} + e^{-ikx}\right)$$

$$\psi_{III} = -Ee^{i\delta}\left[e^{i(kx+\delta)} + e^{-i(kx+\delta)}\right] = 2ie^{i\delta}\sin(kx+\delta)$$

δ is a phase shift

If one simply had an infinite potential at $x=0$, $\psi = \sin(kx)$. The potential well and barrier cause a phase shift. Everything about the scattering process is contained in δ.

The cross section is $\frac{2\pi}{\sqrt{E}}\sin^2\delta = \sigma$

Note as δ goes from 0 to $\frac{\pi}{2} + \pi$, there is a peak in σ. This is due to a resonance, where the particle is temporarily trapped behind the barrier.

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2. In atomic units $E = \left(\frac{\pi^2}{2}\right)\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{a^2} + \frac{4n_z^2}{a^2}\right)$

So $E_{111} = 6\left(\frac{\pi^2}{2a^2}\right)$

and $E_{211} = E_{121} = 9\left(\frac{\pi^2}{2a^2}\right)$

Now consider the distortion

ground state: $E = \frac{\pi^2}{2}\left[\frac{1}{(a+\delta)^2} + \frac{1}{(a-\delta)^2} + \frac{4}{a^2}\right]$

$$E \approx \frac{\pi^2}{2a^2}\left[1 - \frac{2\delta}{a} + 3\left(\frac{\delta}{a}\right)^2 + \dots + 1 + \frac{2\delta}{a} + 3\left(\frac{\delta}{a}\right)^2 + \dots + 4\right]$$

$$E = \frac{\pi^2}{2a^2}\left[6 + 6\left(\frac{\delta}{a}\right)^2 + \dots\right]$$

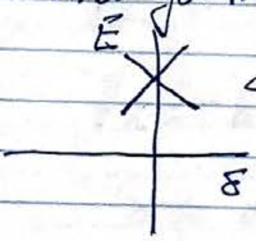
So for the ground state there is no linear term in the energy dependence, and the distortion > the energy.

Now consider the (2,1,1) state

$$E = \frac{\pi^2}{2}\left[\frac{4}{(a+\delta)^2} + \frac{1}{(a-\delta)^2} + \frac{4}{a^2}\right] \approx \frac{\pi^2}{2a^2}\left[4 - \frac{8\delta}{a} + 12\left(\frac{\delta}{a}\right)^2 + \dots + 1 + \frac{2\delta}{a} + 3\left(\frac{\delta}{a}\right)^2 + \dots +\right.$$

$$= \frac{\pi^2}{2a^2}\left[9 - 6\left(\frac{\delta}{a}\right) + 15\left(\frac{\delta}{a}\right)^2 + \dots\right]$$

And for the (1,2,1) state $E = \frac{\pi^2}{2a}\left[9 + 6\left(\frac{\delta}{a}\right) + 15\left(\frac{\delta}{a}\right)^2 + \dots\right]$



← How the energy varies for small distortions.