HW #6 Answers

1. $E_1 = \frac{1}{2}\hbar\omega - \frac{\varepsilon^2 Q^2 \langle 0|x|1\rangle^2}{\hbar\omega} = \frac{1}{2}\hbar\omega - \frac{1}{2}\frac{\varepsilon^2 Q^2}{k}$

$$E_2 = \frac{3}{2}\hbar\omega + \frac{\varepsilon^2 Q^2}{\hbar\omega}\left[-\langle 1|x|2\rangle^2 + \langle 1|x|0\rangle^2\right]$$
$$= \frac{3}{2}\hbar\omega - \frac{1}{2}\frac{\varepsilon^2 Q^2}{k}$$

So both levels are shifted by the same amount. This is a consequence of the fact that while the perturbation shifts the potential, it does not change the curvature. Note also that the correction does not depend on $\hbar$. Q^2/k is the polarizability, which is a classical entity.

2. $E_{00}^{(2)} = \frac{4}{R^6}\frac{|\langle 00|x_1 x_2|11\rangle|^2}{-2\hbar\omega} = \frac{-2}{R^6\hbar\omega}|\langle 0|x_1|1\rangle|^2|\langle 0|x_2|1\rangle|^2$

$$= \frac{-2}{R^6\hbar\omega}\left(\frac{\hbar}{2m\omega}\right)^2 = -\frac{1}{2R^6}\frac{\hbar}{m^2\omega^3}$$

Note for two interacting Drude oscillators, the perturbation should include a factor of $Q_1 Q_2$. Assume the two charges are equal to Q

$$E^{(2)} = -\frac{\hbar}{2R^6}\frac{Q^4\omega}{m^2\omega^4} = -\frac{\hbar\omega}{2R^6}\alpha^2.$$

Thus the dispersion energy is quantum mechanical in nature and depends on the product of the two polarizabilities