HW #3 Answers

1. Evaluate $\langle 0|x^3|1\rangle$

$$\langle 0|x^3|1\rangle = \left(\frac{\hbar}{2m\omega}\right)^{3/2} \langle 0|(a^+ + a)^3|1\rangle$$

$$= \left(\frac{\hbar}{2m\omega}\right)^{3/2} \langle 0|aa^+a + aaa^+|1\rangle = 3\left(\frac{\hbar}{2m\omega}\right)^{3/2}$$

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2. What is $\Delta x \Delta p_x$ for the Harmonic oscillator?

$$\Delta x = \sqrt{\langle 0|x^2|0\rangle} = \sqrt{\frac{\hbar}{2m\omega}\langle 0|aa^+|0\rangle} = \sqrt{\frac{\hbar}{2m\omega}}$$

$$\Delta p_x = \sqrt{\langle 0|p_x^2|0\rangle} = \sqrt{\frac{\hbar m\omega}{2}\langle 0|aa^+|0\rangle} = \sqrt{\frac{\hbar m\omega}{2}}$$

$$(\Delta x)(\Delta p_x) = \frac{\hbar}{2}$$

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3. What is the vibrational energy in the harmonic approximation?

from the first derivative $R_e = 2^{1/6}\sigma$

$$\frac{d^2V}{dR^2} = \left[\frac{12(13)\sigma^{12}}{R^{14}} - \frac{6(7)\sigma^6}{R^8}\right]4\varepsilon$$

$$\left.\frac{d^2V}{dR^2}\right|_{R_e} = \frac{72\varepsilon}{2^{1/3}\sigma^2}$$

Remember $\omega = \sqrt{\frac{k}{\mu}}$

So $\omega = \sqrt{\frac{72\varepsilon}{2^{1/3}}}\frac{1}{\sigma}$