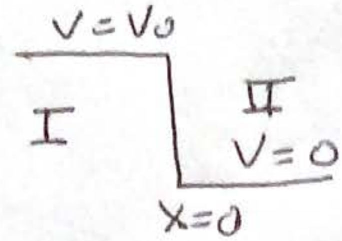
HW # 2.

(1)



$E < V_0$, $\psi_I = A e^{+Kx}$

$\psi_{II} = B e^{ikx} + C e^{-ikx}$

$k = \sqrt{2mE}/\hbar$

$K = \sqrt{2m(V_0 - E)}/\hbar$

$x=0 \quad A = B + C$

$+KA = ik(B-C) \Rightarrow K = \dfrac{ik(B-C)}{B+C}$

$(B+C)K = ik(B-C)$

$B(K-ik) = -C(K+ik) \Rightarrow B = C\left(\dfrac{ik+K}{ik-K}\right)$

$A = C + B = C\left[1 + \dfrac{ik+K}{ik-K}\right] = \dfrac{2ik}{ik-K}\,C$

Note $\left|\dfrac{B}{C}\right|^2 = 1$

$E > V_0$, $\psi_I = A e^{-iKx}$

$\psi_{II} = B e^{ikx} + C e^{-ikx}$

$K = \sqrt{2m(E-V_0)}/\hbar$

I have assumed that the particle is incident from the right.

$x=0 \quad A = B + C$

$-iKA = ik(B-C)$

$iK = ik\dfrac{(C-B)}{B+C} \Rightarrow K(B+C) = k(C-B)$

$B(K+k) = C(k-K)$

$B = \left(\dfrac{k-K}{k+K}\right)C$

$A = B + C = C\left[1 + \dfrac{k-K}{k+K}\right] = \dfrac{2k}{k+K}\,C$

$\left|\dfrac{B}{C}\right|^2 = \dfrac{k^2+K^2-2kK}{k^2+K^2+2kK}$

$\left|\dfrac{A}{C}\right|^2 = \dfrac{4k^2}{k^2+K^2+2kK}$

Read Wikipedia to see why $|A/C|^2$ picks up another factor of K/k.

Yes, there is partial reflection when $E > V_0$

(2) For the particle-in-box problem with ∞ potential outside the box, $\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$ which obviously commutes with $\hat{p}_x = \frac{\hbar}{i} d/dx$

(3) Particle in box problem with sloped $(-bx)$ bottom:

$$\langle 0|-bx|0\rangle = \frac{-2b}{a}\int_0^a \sin^2\left(\frac{\pi x}{a}\right) x\, dx = -\frac{2b}{a}\frac{a^2}{4}$$

$= -ab/2$

So the total energy is $\frac{\pi^2\hbar^2}{2ma^2} - \frac{ab}{2}$

(4) 2D particle in Box problem with

$0 \le x \le L$ and $0 \le y \le L/2$

So the energy is proportional to

$n_x^2 + 4n_y^2$

this $= 20$ for the combination $(4,1)$ as well as for $(2,2)$

So there are accidental degeneracies.