$$\hat{L}_y^2 \hat{L}_x - \hat{L}_x \hat{L}_y^2$$

$$-\hat{L}_y \hat{L}_x + \hat{L}_x \hat{L}_y = i\hbar \hat{L}_z \Rightarrow \hat{L}_y \hat{L}_x = \hat{L}_x \hat{L}_y - i\hbar \hat{L}_z$$

$$\hat{L}_y^2 \hat{L}_x = \hat{L}_y (\hat{L}_x \hat{L}_y - i\hbar L_z) = (\hat{L}_y \hat{L}_x) \hat{L}_y - i\hbar \hat{L}_y \hat{L}_z$$

$$= (\hat{L}_x \hat{L}_y - i\hbar \hat{L}_z) L_y - i\hbar \hat{L}_y \hat{L}_z = \hat{L}_x \hat{L}_y^2 - i\hbar [\hat{L}_z \hat{L}_y + \hat{L}_y \hat{L}_z]$$

Now have to do the same thing with $L_z^2 \hat{L}_x - \hat{L}_x \hat{L}_z^2$

$\psi = c_1 \phi_1 + c_2 \phi_2 + c_3 \phi_3$ where c_1, c_2, c_3 are eigenfunctions of $\hat{H}$.

How do we find the coefficients?

Multiply both sides of the Eq by ϕ_1 and integrate

$\langle \phi_1 | \psi \rangle = c_1$ since $\langle \phi_i | \phi_j \rangle = \delta_{ij}$

Also $\langle \phi_2 | \psi \rangle = c_2$, $\langle \phi_3 | \psi \rangle = c_3$.

So the coefficients are just the overlap of ψ with our basis functions.

Recall $1 = \sum_{i=1}^{\infty} |\phi_i\rangle\langle\phi_i|$

$$\psi = \sum_{i=1}^{\infty} \langle \psi | \phi_i \rangle \langle \phi_i |$$