Chem 2430 Answers HW #1

1. $\psi = e^{-ax^2}$. We want to find N such that $N^2 \int_{-\infty}^{\infty} e^{-2ax^2} dx = 1$

$N^2 \sqrt{\frac{\pi}{2a}} = 1$. So $N = \left(\frac{2a}{\pi}\right)^{1/4}$

What is the probability of finding the particle between 0 and 1?

$Prob = \left(\frac{2a}{\pi}\right)^{1/2} \int_0^1 e^{-2ax^2} dx = \frac{\text{erf}(\sqrt{2a})}{2}$

2. Show that the $n=2$ and $n=4$ particle-in-box wavefunctions are orthogonal.

$$\int_0^L \sin\left(\frac{2\pi x}{L}\right) \sin\left(\frac{4\pi x}{L}\right) dx = 2\int_0^L \left[\sin\left(\frac{2\pi x}{L}\right)\right]^2 \cos\left(\frac{2\pi x}{L}\right) dx$$

$$= \frac{L}{3\pi} \sin^3\left(\frac{2\pi x}{L}\right)\Big|_0^L = 0$$

3. $\psi = c_1 e^{ikx} + c_2 e^{-ikx}$

$\psi(0) = 0 \Rightarrow c_2 = -c_1$

$\psi = c_1 (e^{ikx} - e^{-ikx}) = c \sin(kx)$

$E = \frac{\hbar^2 k^2}{2m}$

All positive values of E are possible