Chem. 2430: Exam # 1

(1) $\psi_I = A e^{ikx} + B e^{-ikx}$ | $k = \sqrt{2mE}/\hbar$

$\psi_{II} = C e^{-\kappa x} + D e^{\kappa x}$ | $\kappa = \sqrt{2m(V_0 - E)}/\hbar$

$\psi_{III} = E e^{ikx}$

probability of reflection $= |B/A|^2$

(2) a) yes one can use particle in the box eigenfunction as a bases set for this problem.

b) Particle in the box eigenfunctions are not really suitable as a bases set for the harmonic oscillator problem.

However, if we choose our box large enough, we could obtain accurate results for the first few harmonic oscillator levels.

(3)

a) $[a^+, b] = 0$

b) $E_{00} = \frac{1}{2}\hbar\omega_1 + \frac{1}{2}\hbar\omega_2$

c) $q_1^2 q_2 = \frac{1}{2\omega_1}\frac{1}{\sqrt{2\omega_2}}(a+a^+)^2(b+b^+)$

where $\hbar$ and m have been set $= 1$

(4) $\sin(a\phi)$ is not an eigenfunction of L_z

It is an eigenfunction of the energy

$$-\frac{\hbar^2}{2\mu r^2}\frac{d^2}{d\phi^2}\sin(a\phi) = \frac{\hbar^2 a^2}{2\mu r^2}\sin(a\phi)$$

$$\hat{L}_z \phi - \phi \hat{L}_z = \hbar/i$$

(5) Amanda specified $\psi = 0.84\psi_1 + 0.55\psi_2$

Probability of observing E_1 is $(0.84)^2 = 0.70$

or 70%

What is the average of x?

$$\langle \psi | x | \psi \rangle = \langle c_1\psi_1 + c_2\psi_2 | x | c_1\psi_1 + c_2\psi_2 \rangle$$

$$= 2c_1c_2 \langle \psi_1 | x | \psi_2 \rangle$$

ψ_1 = the 0^{th} level, ψ_2 = the first excited state

so $\langle \psi_1 | x | \psi_2 \rangle = \langle 0 | x | 1 \rangle = \sqrt{\frac{\hbar}{2m\omega}}$

$$\langle \psi | x | \psi \rangle = 2(0.84)(0.55)\sqrt{\frac{\hbar}{2m\omega}}$$